A Mathematical Model for Integrating Cell Formation Problem with Machine Layout

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KEYWORDS	ABSTRACT
Cell formation problem, Machine layout, Cell utilization, Mathematical model.	*This paper deals with the cellular manufacturing system (CMS) that is based on group technology concepts. CMS is defined as identifying the similar parts that are processed on the same machines and then grouping them as a cell. The most proposed models for solving CMS are focused on cell formation problem while machine layout is considered in few papers. This paper addresses a mathematical model for the joint problem of the cell formation problem and the machine layout. The objective is to minimize the total cost of inter-cell and intra-cell (forward and backward) movements and the investment cost of machines. This model has also considered the minimum utilization level of each cell to achieve the higher performance of cell utilization. Two examples from the literature are solved by the LINGO Software to validate and verify the proposed model.*

1. Introduction

Cellular manufacturing (CM) is an application of the group technology (GT) philosophy to designing manufacturing systems. The main idea of GT is to improve productivity of manufacturing system by grouping parts and products with similar characteristics into families and forming production cells with a group of dissimilar machines and processes. Comprehensive summaries and taxonomies of studies devoted to part-machine grouping problems were presented in [1], [2], [3] and [4]. One of the first problems encountered in implementing CM is cell formation problem (CFP). In the last three decades of research on CFP, researchers have mainly used zero - one machine component incidence matrix as the input data for the problem. Many approaches that have been applied to the CFP include genetic algorithms [5] and [6], tabu search [7] and [8], neural network [9], mathematical programming [10] and [11] and simulated annealing [12] and [13]. Despite a large number of published papers on CFP, very few authors have considered operation sequence in calculating inter-cell material movement and intra-cell material movement. CFP methods, without using operation sequence data, may calculate inter-cell movement based on the number of cells that a part will visit in the manufacturing process. However, the number of cells visited by the part can be less than the actual number of inter-cell movements since the part may travel back and forth between cells. Such movements may not be accurately reflected without properly using operation sequence data. Different data structures provide different sets of information and enable the cell designers to make appropriate use of them while solving the CFP. A zero one incidence matrix offers advantages of computational simplicity for solving the CFP. However, it is not possible to address issues pertaining to machine utilization, inter-cell workload and layout of machines within each identified cell. On the other hand, using additional data pertaining to setup time, process time and production volumes considering

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machine capacity enable cell designers to address these issues using a much more complex solution methodology. Use of sequence data for CFP provides additional information to the cell designer. Sequence data identifies the order in which jobs are processed in a manufacturing system. Therefore, this information could be used not only for identifying part families and machine groups but also to arrive at the layout of machines within each cell based on dominant flow patterns within each cell. Despite this simple truth, traditionally, the cell design problem and the layout problem are treated in a discontinuous fashion. Thus, only a few studies have attempted to resolve these decisions concurrently [14], [15] and [16]. Besides using sequence data for CFP, it is important to address layout of machines as the required information for identification of manufacturing cells.

In the next section we motivate our research by a review of the literature pertaining to the use of sequence data for CFP and methodologies.

2. Literature Review

Vakharia and Wemmerlov [17] presented a heuristic approach for the machine cell design, where machines within each cell are arranged along a linear flow line. Irani et al. [18] used maximal spanning arborescence as a graphic structure to integrate the machine grouping, the intra-cell layout, and the inter-cell layout. Sequential, two-phase mathematical programming models were proposed to decompose the joint problem. Liao [19] proposed a sequential three-stage procedure, to determine the best part routing, machine cells, and inter-cell layout for a line-type cellular manufacturing system. Approaches that are more complicated have also been developed to address the joint problem. Arvindh and Irani [20] developed an iterative approach to design a cellular manufacturing system where one iterative loop deals with machine and part grouping, and another iterative loop varies the number of cells to find the best design. Akturk and Balkose [21] described a multi-criterion clustering approach that considers manufacturing attributes, operational sequences, and within cell layout. Nair Jayakrishnan and Narendran [22] addressed the task of identifying machine-cells and component families on the basis of production-sequence data. They defined a new similarity coefficient that captures the ordinal character of the data matrix and introduced a quantitative criterion, with a weighing factor for assessing the quality of the solution based on a non-hierarchical clustering algorithm. Suresh et al. [23] utilized fuzzy neural network approach to cell formation using sequence data. Lee and Chiang [24] considered the joint clustering-layout problem where machine cells are to be located along the bi-directional linear flow layout. They seek to minimize the actual inter-cell flow cost, instead of the typical measure that minimizes the number of inter-cell movements. A three-phase approach, using the cut tree network

model, is developed to solve this joint problem. Chiang and Lee [25] developed a genetic-based algorithm with the optimal partition approach for the cell formation in bi-directional linear flow layout, where the objective is to minimize the actual inter-cell flow cost, instead of the typical measure that optimizes the number of inter-cell movements. Boulif and Atif [26] addressed a branch-and-bound- enhanced genetic algorithm for cell formation problem using a graph partitioning formulation of this problem. They considered some of the natural data inputs and constraints encountered in real life production systems, such as operation sequence, maximum number of cells, maximum cell size, and machine cohabitation and non-cohabitation. Wu et al. [15] used a hierarchical genetic algorithm to form manufacturing cells and determine the group layout of a CMS simultaneously. They have also developed a new group mutation operator to increase the mutation probability. Mahdavi et al. [16] developed a heuristic algorithm based on flow matrix for cell formation and layout design. The objective was to make use of the valuable information about the flow patterns of various jobs in a manufacturing system and obtain relevant performance measures for the cell design and layout problem. In this paper, the cell formation and layout design are considered simultaneously. We propose a new mathematical model that utilizes the sequence data as input to the problem and identifies machine cells and the layout of machines within each cell. The objective is to minimize the total costs of inter-cell and intra-cell (forward and backward) movements and the investment cost of machines in CM using sequence data. Due to different minimum utilization level for each cell, the proposed model presents different scenarios of part-machine grouping. The approach is illustrated by examples that are solved by the LINGO Software and computational results are reported and analyzed.

3. Problem Formulation

We consider several factors such as routing (sequence), machine capacity, demand, and layout type in the problem formulation. Routing is often presented in a machine/part/sequence matrix, where the component, a_{isj} of the matrix indicates the operational sequence (operation s) of part type i to be processed by machine type j. Since the machine layout type (e.g., single row, U shape, multiple rows, or other configurations) has significant impact on the part transfer cost, it needs to be considered in the CFP. Figure 1 shows the routing of part types based on sequence data and the types of material handling cost considering distance between machines. The location shows the place and layout of machines in cells. For example, the location of machines in line layout in cell 1 is:
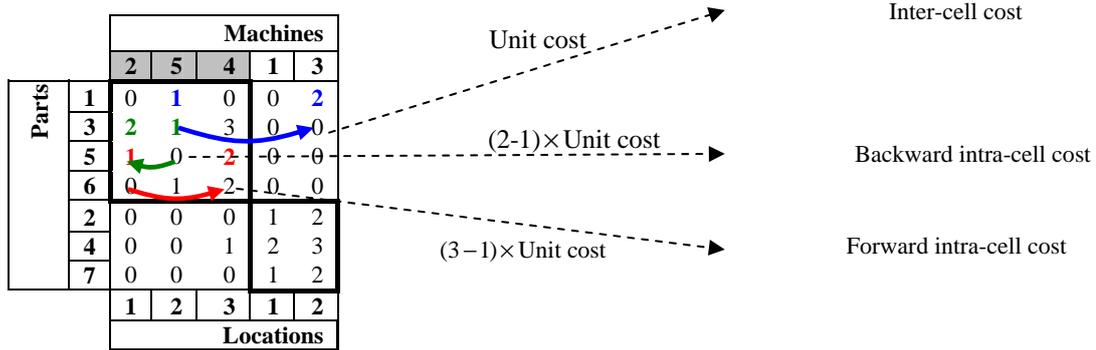
Machine 2 → Machine 5 → Machine 4

Fig. 1. An integrated CMS

In this section, we formulate a mathematical model based on sequence data in CMS. The proposed model deals with the minimization of the integrated inter-cell and intra-cell (forward and backward) movements cost and the investment cost of machines.

3.1. Assumptions

The problem is considered under the following assumptions.

1- The number of cells is known.

2- The upper bound and lower bound of the cells size are known.

3- Consecutive operations of each part type are performed on different machines in a given sequence. Moreover, sequence of operations is important in the calculation of intercellular and intracellular material handling cost since it gives a more accurate count of the number of times that a part either has to move between cells or between machines(forward and backward movements) within the same cell.

4- The processing times for all operations of part types on different machine types are known and deterministic.

5- Parts are moved between and within cells. Inter-cell movement is incurred whenever consecutive operations of the same part type are carried out in different cells. For instance, assume that the operation s of part type i is processed on machine type j in cell k. If the next operation, $s + 1$, of part type i is processed on any machine but in another cell, then there is an inter-cell movement. The intra-cell movement is incurred whenever consecutive operations of the same part type are processed in the same cell. For instance, say that the operation s of part type i is processed on machine type j in cell k. If the next operation, $s + 1$, of part type i is processed on any machine but within the same cell, then there is an intra-cell movement. To the best of our knowledge, all studies considering this movement supposed that intra-cell movement occurs between two different machine types. But, in reality, it can occur between same machine types on different locations in one cell. We have considered this concept in our model. Moreover, in the manufacturing systems, the backward movement incurs more expenses, so its cost

is assumed greater than forward movement cost in the proposed model.

6- We assume the type of layout is linear and all machine types should be assigned to locations which have same dimensions. Hence, the distance between two machines assigned to two different locations is calculated by subtracting location numbers of those machines from each other. For instance, in Figure 2, cell k has 7 locations with linear layout type. This Figure shows the effect of operation sequence and intra-cell layout on forward and backward movements within a cell. Operations 1 and 2 of part type 1 must be processed on machine type 3 in location 1, hence there is no intra-cell movement. But operation 3 is processed with machine type 3 in location 5. Because of operations 2 and 3 of part type 1 are processed by two machines of type 3 located in different locations 1 and 5, then a forward intra-cell movement occurs. The movement distance of these consecutive operations is equal to the distance between locations 1 and 5. Operations 4 and 5 of part type 2 are processed on machine type 5 in location 7 and machine type 4 in location 2, respectively. Then a backward intra-cell movement happens.

	Cell k						
Location	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Machine	3	4	1	2	3	1	5
Par type 1	1,2			4		3	
Part type 2		1,5	2			3	4

Fig. 2. Operation sequence and intra-cell layout in cell k

7- The value of cell utilization is set by the designer considering his experiences. This setting is based on a trade off among all cells to obtain the best configuration of machines in cells. Meanwhile in cellular manufacturing systems, cells are formed in different sizes, therefore investigating cell utilization in all cells is important due to similarity coefficient. So, decision maker set minimum utilization for each cell to satisfy the formed cells in the cellular manufacturing system.

8- The demand for each part type is given.

9- The capacity of each machine type is known.

10- There are several machines of each type with identical duplicates to satisfy capacity requirements and reduce/eliminate inter-cell movement. The number of duplicates of each machine type is constant over the planning horizon.

11- The investment cost of each machine type is known.

3.2. Indexing Sets

i index for part type ($i =1, 2,…, P$)

j, j' index for machine type ($j, j' =1,2...,M$)

k index for cells ($k =1, 2,…, C$)

s index for operations ($s =1, 2,…,OP_i$)

l, l' index for location of machine type (l , $l' =1, 2,…, L_k$).

3.3. Parameters

$\gamma^{\text{inter-cell}}$: material handling cost between cells.

$\gamma_f^{\text{intra-cell}}$: forward material handling cost within cells.

$\gamma_b^{\text{intra-cell}}$: backward material handling cost within cells.

Min_ut$_k$: minimum utilization of cell k.

L_k: lower bound of the number of machine type in cell k.

U_k: upper bound of the number of machine type in cell k.

N_j : number of machines of type j available for allotment to cells**.**

t_{isj}: processing time of sth operation of part type i with machine type j.

D_i: demand quantity of part type i.

T_j: the capacity of machine type j.

C_j : investment cost of machine type j.

$d_{ll'k}$: Distance between location l and l' in cell k.

r_{ij}: 1 if part type i is to be processed on machine type j; 0 otherwise.

a_{isj}: 1 if operation s of part type i is to be processed on machine type j; 0 otherwise.

3.4. Decision Variables

$$X_{isk} = \begin{cases} 1 & \text{if } s^{th} \text{ operation of part } i \text{ is assigned to cell } k, \\ 0 & \text{otherwise.} \end{cases}$$

$$Y_{jkl} = \begin{cases} 1 & \text{if machine type } j \text{ is assigned to cell } k \text{ in location } l, \\ 0 & \text{otherwise.} \end{cases}$$

$$Z_{ik} = \begin{cases} 1 & \text{if part type } i \text{ is assigned to cell } k, \\ 0 & \text{otherwise.} \end{cases}$$

3.5. Mathematical Model
3.5.1 Objective Function

We propose the objective function as follows:

$Min\ Z =$

$$\gamma^{\text{inter-cell}} \times \left[\sum_{i=1}^{P} (OP_i - 1) - \left(\sum_{i=1}^{P} \sum_{k=1}^{C} \sum_{s=1}^{OP_i-1} \sum_{j=1}^{M} \sum_{j'=1}^{M} \sum_{l=1}^{L_k} \sum_{l'=1}^{L_k} X_{isk} X_{i,s+1,k} Y_{jkl} Y_{j'kl'} \right) \right]$$

$$+ \gamma_f^{\text{intra-cell}} \left(\sum_{i=1}^{P} \sum_{k=1}^{C} \sum_{s=1}^{OP_i-1} \sum_{j=1}^{M} \sum_{j'=1}^{M} \sum_{l=1}^{L_k} \sum_{l'=l+1}^{L_k} d_{ll'k} X_{isk} X_{i,s+1,k} Y_{jkl} Y_{j'kl'} \right) +$$

$$\gamma_b^{\text{intra-cell}} \left(\sum_{i=1}^{P} \sum_{k=1}^{C} \sum_{s=1}^{OP_i-1} \sum_{j=1}^{M} \sum_{j'=1}^{M} \sum_{l'=1}^{L_k} \sum_{l=l'+1}^{L_k} d_{ll'k} X_{isk} X_{i,s+1,k} Y_{jkl} Y_{j'kl'} \right) + \sum_{k=1}^{C} \sum_{j=1}^{M} \sum_{l=1}^{L_k} C_j Y_{jkl}$$

3.5.2. Constraints

$$\sum_{j=1}^{M} \sum_{l=1}^{L_k} Y_{jkl} \geq L_k \qquad\qquad \forall k \qquad (1)$$

$$\sum_{j=1}^{M} \sum_{l=1}^{L_k} Y_{jkl} \leq U_k \qquad\qquad \forall k \qquad (2)$$

$$\sum_{k=1}^{C} \sum_{l=1}^{L_k} Y_{jkl} \leq N_j \qquad\qquad \forall j \qquad (3)$$

$$\sum_{j=1}^{M} Y_{jkl} \leq 1 \qquad\qquad \forall k,l \qquad (4)$$

$$\sum_{k=1}^{C} Z_{ik} = 1 \qquad\qquad \forall i \qquad (5)$$

$$\sum_{j=1}^{M} \sum_{k=1}^{C} \sum_{l=1}^{L_k} X_{isk} Y_{jkl} a_{isj} = 1 \qquad\qquad \forall i,s \qquad (6)$$

$$\sum_{i=1}^{p} \sum_{s=1}^{OP_i} Y_{jkl} X_{isk} t_{isj} D_i \leq T_j \qquad\qquad \forall j,k,l \qquad (7)$$

$$\sum_{i=1}^{P} \sum_{j=1}^{M} \sum_{l=1}^{L_k} Z_{ik} Y_{jkl} r_{ij} \geq \min\_\text{ut}_k \sum_{i=1}^{P} \sum_{j=1}^{M} \sum_{l=1}^{L_k} Z_{ik} Y_{jkl} \qquad (8)$$
$$\forall k$$

$$X_{isk}, Z_{ik}, Y_{jkl} \in \{0,1\} \qquad\qquad \forall i,j,l,s,k \qquad (9)$$

The objective function considers minimizing the total cost of inter-cell and intra-cell (forward and backward) movements and investment cost of machines. The first term computes the total cost of inter-cell movements. This cost is incurred when consecutive operations of the same part type are carried out in different cells. OP_i is the number of operations of part type i and OP_i-1 indicates the total number of movements of part type i.

Therefore, the term $\sum_i (OP_i - 1)$ shows the total number of movements in the cellular manufacturing system. Moreover, the term $\sum_i \sum_k \sum_s \sum_j \sum_{j'} \sum_l \sum_{l'} X_{isk} X_{i,s+1,k} Y_{jkl} Y_{j'kl'}$ computes the total number of intra-cell movements in the manufacturing system. So, the first term calculates the total number of intercellular cost, i.e., the total number of inter-cell movements is equal to the total number of movements minus the total number of intra-cell movements. The second term of the objective function represents the forward intra-cell material handling cost, where the forward movement distance between machines j and j' assigned to locations l and l' in cell k is designated by $d_{ll'k}$. This cost is sustained when consecutive operations of the same part types are processed in the same cell but on two machines of different types or even same type in forward layout, for example machines j and j' assigned to locations l and l' in forward mood process the operations s and $s+1$ of part i. The third term of objective function represents the backward intra-cell material handling cost, where the backward movement distance between machines j and j' assigned to locations l and l' is given by $d_{ll'k}$.

This cost is sustained when consecutive operations of the same part types are processed in the same cell but on two machines of different types or even same type in backward layout, for example machines j and j' assigned to locations u and u' in backward mood process the operations s and $s+1$ of part type i. The fourth term represents the cost of all machines assigned to cells.

Inequalities (1) and (2) ensure the lower and upper bound considerations for the number of machines to be allocated to each cell. Inequality (3) ensures that the number of machines available for a given type is not bypassed. Inequality (4) ensures that each machine can be allocated to only one location of each cell at most. Equation (5) guarantees that each part must be assigned to only one cell. Constraint (6) guarantees that each operation will be assigned to a cell which contains the required machine type. Inequality (7) ensures that capacity limitation of each machine is satisfied.

Constraint (8) specifies minimum utilization of cells to achieve a feasible and better arrangement of machines and operations of parts. Relation (9) specifies that the decision variables are binary.

3.6. Linearization of the Proposed Model

Obviously, the objective function and constraints (6) - (8) are nonlinear. However, these terms can be linearized without much difficulty as they are products of binary variables. We need to introduce auxiliary variables to replace these nonlinear terms with additional constraints.

The required new variables can be defined by the following equations

$$O_{isklj} = X_{isk} Y_{jkl}$$

$$B_{i,s,k,l,l',j,j'} = O_{isklj} X_{i,s+1,k} Y_{j'kl'}$$

$$V_{iklj} = Z_{ik} Y_{jkl}$$

By considering the above equation, following constraints should be added to the mathematical model:

$$O_{isklj} - X_{isk} - Y_{jkl} + 1.5 \geq 0 \qquad \forall i,s,k,l,j \qquad (10)$$

$$1.5 O_{isklj} - X_{isk} - Y_{jkl} \leq 0 \qquad \forall i,s,k,l,j \qquad (11)$$

$$B_{i,s,k,l,l',j,j'} - O_{isklj} - X_{i,s+1,k} - Y_{j'kl'} + 2.5 \geq 0$$

$$\forall i,k,l,l',j,j', s = 1,..,OP-1 \qquad (12)$$

$$2.5 B_{i,s,k,l,l',j,j'} - O_{isklj} - X_{i,s+1,k} - Y_{j'kl'} \leq 0$$

$$\forall i,k,l,l',j,j', s = 1,..,OP-1 \qquad (13)$$

$$V_{iklj} - Z_{ik} - Y_{jkl} + 1.5 \geq 0 \qquad \forall i,k,l,j \qquad (14)$$

$$1.5 V_{iklj} - Z_{ik} - Y_{jkl} \leq 0 \qquad \forall i,k,l,j \qquad (15)$$

$$V_{iklj}, O_{isklj}, B_{i,s,k,l,l',j,j'} \in \{0,1\} \qquad \forall i,s,k,l,l',j,j' \qquad (16)$$

Based on defining the new binary variables, the linear mathematical model is as follows:

Min Z =

$$\gamma^{inter-cell} \times \left[\sum_{i=1}^{P} (OP_i - 1) - \left(\sum_{i=1}^{P} \sum_{k=1}^{C} \sum_{s=1}^{OP_i-1} \sum_{j=1}^{M} \sum_{j'=1}^{M} \sum_{l=1}^{L_k} \sum_{l'=1}^{L_k} B_{i,s,k,l,l',j,j'} \right) \right] +$$

$$\gamma_f^{intra-cell} \left(\sum_{i=1}^{P} \sum_{k=1}^{C} \sum_{s=1}^{OP_i-1} \sum_{j=1}^{M} \sum_{j'=1}^{M} \sum_{l=1}^{L_k} \sum_{l'=l+1}^{L_k} d_{ll'k} B_{i,s,k,l,l',j,j'} \right) +$$

$$\gamma_b^{intra-cell} \left(\sum_{i=1}^{P} \sum_{k=1}^{C} \sum_{s=1}^{OP_i-1} \sum_{j=1}^{M} \sum_{j'=1}^{M} \sum_{l'=1}^{L_k} \sum_{l=l'+1}^{L_k} d_{ll'k} B_{i,s,k,l,l',j,j'} \right) + \sum_{k=1}^{C} \sum_{j=1}^{M} \sum_{l=1}^{L_k} C_j Y_{jkl}$$

So that, constraints (1)-(5), (9)-(16) and the new version of constraints (6) - (8) are:

$$\sum_{j=1}^{M}\sum_{k=1}^{C}\sum_{l=1}^{L_k} O_{isklj} a_{isj} = 1 \qquad \forall i,s \qquad (17)$$

$$\sum_{i=1}^{p}\sum_{s=1}^{OP_i} O_{isklj} t_{isj} D_i \leq T_j \qquad \forall j,k,l \qquad (18)$$

$$\sum_{i=1}^{P}\sum_{j=1}^{M}\sum_{l=1}^{L_k} V_{iklj} r_{ij} \geq \min\_\mathrm{ut}_k \sum_{i=1}^{P}\sum_{j=1}^{M}\sum_{l=1}^{L_k} V_{iklj} \qquad \forall k \qquad (19)$$

Now, the objective function becomes a 0-1 integer linear programming model. All constraints in the model are also linear. The number of variables and number of constraints in the linearized model are presented in Tables 1 and 2, respectively, based on the variable indices.

Tab. 1. Number of variables in the linearized model

Variable name	Variable count
X_{isk}	$P \times OP \times C$
Y_{jkl}	$M \times C \times L$
Z_{ik}	$P \times C$
V_{iklj}	$P \times C \times L \times M$
O_{isklj}	$P \times OP \times C \times L \times M$
$B_{i,s,k,l,l',j,j'}$	$P \times (OP-1) \times C \times L^2 \times M^2$

Tab. 2. Number of constraints in the linearized model

Equation number	Constraint count	Equation number	Constraint count
(1)	C	(12)	$P \times (OP-1) \times C \times L^2 \times M^2$
(2)	C	(13)	$P \times (OP-1) \times C \times L^2 \times M^2$
(3)	M	(14)	$P \times C \times L \times M$
(4)	$C \times L$	(15)	$P \times C \times L \times M$
(5)	P	(16)	$P \times C \times L \times M [1 + OP + L \times M \times (OP-1)]$
(9)	$P \times OP \times C + P \times M + M \times C \times L$	(17)	$P \times OP$
(10)	$P \times OP \times C \times L \times M$	(18)	$C \times L \times M$
(11)	$P \times OP \times C \times L \times M$	(19)	C

4. Numerical Illustration

To verify the behavior of the proposed model, two numerical examples are presented to illustrate applicability of the proposed model when various values of cell utilization level are defined by the decision maker. These examples are solved by a branch and-bound (B&B) method with the LINGO 8.0 Software.

Example 1.

Table 3 shows the sequence data pertaining to the problem consisting of 5 machines and 7 parts in which each part type is assumed to have two or three operations that must be processed respectively as numbered in the order with the processing time shown

in the parentheses. For instance, the first operation of part 1 should be processed on machine 4 with processing time 0.51 hours. In Table 3, the last three columns include the machine information (i.e., number of each machine type available, capacity and investment cost for their single copy). The last row of this table presents the demand of each part. Table 4 shows the input parameters for solving the above problem with two different cell utilization levels. Also, the distance between the locations of a cell is shown in Table 5. run without the fourth term of objective function (without cost of machines). Tables 7 and 8 show the solution of the model with the cost of machine for the second and the third run.

Tab.3. 5 x 7 machines- part matrix in [27]

j \ i	1	2	3	4	5	6	7	N_j	C_j	T_j
1	0	1 (0.33)	0	2 (0.72)	0	0	1 (0,57)	2	600	200
2	0	0	2 (0.52)	0	1 (0.62)	0	0	2	900	350
3	2 (0.31)	2 (0.44)	0	3 (0.37)	0	0	2 (0.22)	2	750	200
4	1 (0.51)	0	1 (0.63)	0	0	1 (0.52)	0	2	700	200
5	0	0	3 (0.4)	1 (0.61)	2 (0.35)	2 (0.25)	0	2	600	350
D_i	80	110	140	95	120	80	135			

Tab. 4. Parameter setting for example 1

Parameter	Cell I	Cell II
L_k	2	2
U_k	4	4
Forward intra cell movement unit cost	3	
Backward intra cell movement unit cost	11	
Inter cell movement cost	35	
Min_Ut_k (first run) (without machine cost)	0.4	0.4
Min_Ut_k (second run) (with machine cost)	0.4	0.4
Min_Ut_k (third run) (with machine cost)	0.4	1

Tab. 5. The location distance in a cell.

		Location			
		1	2	3	4
	1	-	1	2	3
Location	2	1	-	1	2
	3	2	1	-	1
	4	3	2	1	-

Tab.7. The cell formation with *min_ut₁* = 0.4, *min_ut₂* = 0.4 (second run)

		Machine				
		4	2	5	1	3
	3	1	2	3		
P	5	0	1	2		
a	6	1	0	2		
r	1	1			0	2
t	2				1	2
	4			1	2	3
	7				1	2

Tab. 6. The cell formation of the first run without machine cost

		Machine							
		4	5	1	3	3	4	2	5
	1	1	0	0	2				
P	2	0	0	1	2				
a	4	0	1	2	3				
r	6	1	2	0	0				
t	7	0	0	1	2				
	3					0	1	2	3
	5					0	0	1	2

Tab. 8. The cell formation with *min_ut₁* = 0.4, *min_ut₂* = 1 (third run)

		Machine				
		4	2	5	1	3
	1	1	0	0		2
P	3	1	2	3		
a	5	0	1	2		
r	6	1	0	2		
t	2				1	2
	4			1	2	3
	7				1	2

Tab. 9. 7×14 machine- part matrix

i \\ j	1	2	3	4	5	6	7	D_i
1	0	3(.25)	0	0	0	2(.67)	1(.52)	200
2	2(.43)	0	0	0	1(.56)	0	0	130
3	0	0	0	1(.78)	0	0	0	90
4	0	2(.66)	0	0	0	0	1(.35)	85
5	1(.45)	3(.26)	0	0	0	0	2(.77)	110
6	0	3(.24)	0	0	0	2(.18)	1(.55)	125
7	0	1(.85)	0	2(.42)	0	0	0	145
8	1(.33)	0	0	0	2(.66)	3(.15)	0	110
9	1(.86)	0	0	2(.4)	0	0	3(.24)	95
10	0	0	1(.72)	0	0	0	2(.6)	60
11	0	0	2(.64)	1(.21)	0	0	0	80
12	0	0	2(.5)	1(.15)	0	0	0	190
13	0	0	1(.9)	0	0	0	0	65
14	1(.32)	0	0	0	2(.3)	0	0	115
N_j	2	2	2	2	2	2	2	
C_j	400	550	320	600	240	520	200	
T_i	250	300	250	360	300	340	400	

Example2.
In solving this example, we consider 7 different types of machines, 14 part types from the literature (Wu et al. 2007). The input data of this example are given in Tables 9 and 10.

The cells generated and the part assignment to various cells for minimum cell utilization (0.5, 0.5, and 0.5) is given in Table 11. For second run in example 2, the material handling cost between cells has been increased for the proposed model to 110 and the results have been shown in Table 12.

Tab. 10. Parameter setting for example 2.

Parameter	Cell I	Cell II	Cell III
L_k	2	2	2
U_k	4	4	4
Forward intra cell movement unit cost		7	
Backward intra cell movement unit cost		20	
Inter cell movement cost		70	
Min_Ut_k (first run)	0.5	0.5	0.5
Min_Ut_k (second run)	0.5	0.5	0.5

Tab. 11. The cell formation with $min\_ut_1 = 0.5$, $min\_ut_2 = 0.5$, $min\_ut_3 = 0.5$

		Part													
		1	4	5	6	7	10	3	11	12	13	2	8	9	14
M	7	1	1	2	1	0	2							3	
a	6	2	0	0	2	0	0						3		
c	2	3	2	3	3	1	0								
h	4					2		1	1	1	0			2	
i	3						1	0	2	2	1				
n	1			1								2	1	1	1
e	5											1	2	0	2

Tab. 12. The cell formation for second run with the increased inter cell movement cost

		Part												
		3	9	10	11	12	13	2	8	14	1	4	5	6	7
M	4	1	2	0	1	1	0								2
a	3	0	0	1	2	2	1								
c	7	0	3	2	0	0	0								
h	1		1					2	1	1			1		
i	5							1	2	2					
n	7										1	1	2	1	0
e	6								3		2	0	0	2	0
	2										3	2	3	3	1

5. Discussion

Table 13 includes obtained results from the Lingo solver in terms of the intra-cell forward and backward, inter-cell, machine investment cost and objective function value (OFV) for first and second runs in Example 1. The results show that if we exclude the machine investment cost in the objective function, then we face with more real cost in the system. This is due to using more machines for reducing inter-cell movement costs. Moreover, the number of voids has been increased, while the number of exceptional elements is zero. Therefore, with same lower bound of cell utilization, we achieved better results for the cell formation on total real cost and voids, when we consider the machine investment cost in the objective function. Total real cost includes material handling cost and machine investment cost, whenever the objective function is either with or without machine investment cost. In the first run the value of OFV is 33, but we added the machine investment costs resulting the total real cost of 33+5600=5633. In the second run the value of OFV equals to the total real cost of 3644.

The utilization concept is considered as number of non-zero components of each cell divided by whole components of that cell, and it is useful for decision maker.

By changing utilization of cell II from 0.4 to 1, the solution of Table 8 is formed, and new part family has been obtained. The results of two runs for Example 2 in Tables 12 and 13 illustrate the superiority of the proposed model, when the inter-cell material handling cost has been increased. Table 14 shows that the number of machines as a decision variable can be added to reduce the number of exceptional elements though the number of voids increases from 9 to 12.

Tab. 13. Computational results of 5×7 machine-part problem

	With machine cost	Without machine investment cost
Forward intra cell cost	24	33
Backward intra cell cost	0	0
Inter-cell cost	70	0
Machine investment cost	3550	5600
OFV	3644	33
Total real cost	3644	5633
Voids	3	12
Exceptional elements	2	0
Machine added	0	3

Tab.14. Computational results of 7×14 machine-part problem

	First run	Second run
Inter-cell cost	420	440
OFV	3754	3770
Voids	9	12
Exceptional elements	6	4
Machine added	0	1

6. Conclusions

In this paper, we proposed a new mathematical model which addresses the joint problems of the cell formation and machine layout in cellular manufacturing based on sequence data under cell utilization levels. Then, we used a transformation approach to convert the non-linear model to a linear programming. The previous methods to cellular manufacturing do not consider both the cell formation problem and layout design in a same linear mathematical model. We have considered inter-cell movement cost and forward and backward intra-cell movement cost parameters and also the minimum of the cell utilization, which is based on the designer view point. As it is known, in the manufacturing systems with linear layout of the machines, the parts backward movements incur more expenses than what we considered in the proposed model.

The advantages of this study with respect to the recent studies were as follows:

Consideration of machine layout in cellular manufacturing.

Calculation of forward and backward intra-cell material handling costs by considering the operation sequence and the distance between the locations assigned to machines.

Calculation of the cost of intra-cell material handling between same machine types in different locations accurately.

Two problems have been adopted from the literature and solved by the Lingo 8.0 at different cell utilization levels. We compared the results of cell formation between two cases, i.e., with and without machine investment cost as a term of objective function. Moreover, the role of machine replication in the system is shown in example 2. Application of meta-heuristics like tabu search, simulated annealing and genetic algorithm can be investigated in future researches to solve large-sized problems.

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