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RESEARCH PAPER

A Game-Theoretic Approach to Greening, Pricing, and Advertising Policies in a Green Supply Chain

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ABSTRACT
In this paper, greening, pricing, and advertising policies in a supply chain will be examined with government intervention. The supply chain has two members. First, a manufacturer seeking to determine the wholesale price and the greening level; second, a retailer that has to determine the advertising cost and the retail price. The government is trying to encourage the manufacturer to green the production using subsidies. By using the game theory, at first, the demand function and the profit functions of both members are introduced; then, in a dynamic game, their Stackelberg equilibrium is calculated. Sensitivity and parameter analysis is made for more illustration of the problem. This study found the supply chain profit function behavior, and results showed that if the sensitivity of demand-price was less than a specific value, the manufacturer would not participate in greening policies

KEYWORDS: *sustainable supply chain, greening policies, advertising, pricing, Stackelberg, game theory.*

1. Introduction

Climate change has become a public concern in the 21st century due to the pollution caused by human beings. Therefore, environmental care, which is called "Greening", is also considered by the manufacturers. In addition, when social aspects come in, we face three sides of a triangle, called sustainability. Traditional supply chain management can be used to address only economic issues; however, with sustainable supply chain management, environmental, social, and economic dimensions are considered in an integrated manner. This study focuses on the environmental and economic dimensions. Due to increased public awareness, green products have taken a big part in customer attention and have an effect on the demand for products. Greening in companies can be of two kinds. One of them is at the operations of producing a product, and the other one is the design of the plant, procedures, and transportation of the product [1].

In addition, advertising the product or service is an important marketing strategy, which can increase the demand and profit of the supply chain. Some components of the supply chain such as manufacturers or retailers may take the advertising decisions.

In such cases, with several decision-makers, we face multi-factor decision-making where each factor's decision affects others' decisions. One way to examine such issues is to use game theory. Game theory is the solution by which each factor can find the best decision in the presence of other factors' decision.

In this paper, a game-theoretic approach to greening policies, pricing, and advertising is presented in a two-stage supply chain including a manufacturer and a retailer. The research questions in this study include:

What is the greening level of the manufacturer at equilibrium?

What is the impact of demand parameters on the greening level?

What are the impacts of greening on the wholesale price, retail price, and chain profit?

Will the wholesale price or retail price lead to the maximum profit?

How much should the retailer spend on advertising?

In the following, Section two will review the literature on greening, advertising, and pricing. The third section deals with the definition of the problem. Formulation of the problem is presented in Section four. The fifth section contains the

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Stackelberg equilibrium. Section six deals with parameter analysis. Numerical results are presented in the seventh section, and Section eight comprises the conclusion.

2. Literature Review

In this section, the studies about greening, advertising, and pricing in different supply channels are reviewed using game theory.

Few articles on the game theory have addressed the greening issue so far. Sinayi & Rasti-Barzoki [2] considered a two-tier model that consists of a supply chain and government. In their research, consumer surplus was considered as an indicator of the social dimension of sustainability. They found that cooperation between the manufacturer and the retailer always leads to greener products.

Swami and Shah [1] considered a production company in their study that sold its products through a retailer to customers. The manufacturer and retailer are making efforts to green the production. They considered the manufacturer as the leader in a Stackelberg game and examined the impact of different parameters, such as the amount of greening efforts and greening costs, on pricing and decision making. They found that the ratio of the optimal greening efforts was equal to the ratio of their green sensitivity ratios and greening cost ratios. Further, the integrated channel makes higher profits and efforts compared to the case of the decentralized channel. Xu et al. [3] presented a game-theoretic model to study the relationship between companies and customers in the green supply chain of home appliances.

They proposed a Nash equilibrium with two strategies and took into account the government. Every company provides green and non-green products, and each client has two choices of acceptance or rejection. They put forward some countermeasures, which can be adopted by the government to improve the construction of the green supply chain of the home appliance industry. Li et al. [4] studied a dual-channel supply chain, each of which produces green products for its customers. They reviewed pricing and greening strategies in two centralized and decentralized modes in a Stackelberg game. They found that when the greening cost was greater than a threshold, the manufacturer did not open a direct channel. Moreover, they found that the retail price in the centralized green supply chain was higher than that in the decentralized supply chain. Sheu and Chen [5] examined the impact of government financial interventions on competition in a three-level green supply chain using a game-theoretic model. The results of their research showed that the government should ensure that profits from green products would not be negative through taxes and grants. Zhang et al. [6] examined cooperative and non-cooperative games in a two-level green supply chain with green and non-green products and complete information. The results of the game equilibrium show that, under different production conditions, the system performance in cooperative mode is higher than the other.

Another major decision in a supply chain is pricing, which usually involves decisions about the wholesale price of the central manufacturer and the retailer's retail price. Although pricing and advertising are the factors that have a significant impact on market demand and, consequently, the profit of each member of the supply chain. There are still a small number of analytical models that make decisions about these variables simultaneously. Wang [7] studied the welfare implications of resale price maintenance (RPM) in a model, where a manufacturer sells its products to the customer through competitive retailers. These retailers are heterogeneous in marginal distribution costs and geographic locations. He found that, with RPM, the manufacturer made more profit. Karray and Zaccour [8] considered price competition in their model and used coop advertising as an effective competitive strategy for the manufacturer. This manufacturer operates in the presence of retailers selling their goods with different brands, although national advertising decisions are not yet taken into account. Yue et al. [9] considered the price elasticity of the customer demand function, while the manufacturer, regardless of retailer, directly determined the consumer price according to the retailer offer price.

Their model did not consider wholesale and retail prices as a decision variable for the supply chain members. Szmerekovsky and Zhang [10] in their research showed that if demand decreased exponentially with retail price and increased with national and local advertising in a multiplicative way, the optimal solution for the manufacturer was to refuse to pay a refund for the local advertising. Several articles such as Xie and Neyret [11], Xie and Wei [12], and Yan [13] assumed that demand decreased linearly with retail price, and found that the manufacturer usually benefited from providing a percentage refund for local advertising costs.

SeyedEsfahani et al. [14] introduced a demand function with new parameters, which can cover both the convex and concave demand curves. Their research was developed by Aust and Buscher [15]. They developed the issue by relaxing the constraints on the profit margin of the manufacturer and the retailer. One of the prevailing assumptions in these studies is that the demand function is considered as separable and multiplicative by advertising and pricing. In addition, in all of these studies, static and definitive models have been studied in which players' decisions are assumed to be for a period only.

There are, of course, probabilistic and dynamic research studies that consider the issue of advertising. For example, He et al. [16] suggested a Stackelberg differential game and provided advertising for the manufacturer and retailer with a feedback format. Many types of research that consider linear and nonlinear demand functions and suggest non-fixed price elasticity; however, studies with exponential demand function provide fixed price elasticity.

Jørgensen and Zaccour [17] and Aust and Buscher [18] in their recent research conducted a comprehensive review of coop advertising in terms of game types, variables, and objective functions.

Jafari et al. [19] studied pricing and ordering decisions in a dual-channel supply chain with a monopolistic manufacturer and duopolistic retailers. They assumed that the market was controlled by the manufacturer and established different game-theoretic models including Bertrand, Collusion, and Stackelberg.

Their work results showed that various games did not have any effects on the manufacturer's responses; however, the retail prices given by the Collusion game were higher than those by the other games. Saghaeeian, A. & Ramezanian [20] developed a bi-level programming model for the Stackelberg game. They considered a two-stage supply chain that consisted of manufacturers, distribution centers, and retailers. In their study, there was a competition among products of the two-stage supply chain.

On the other hand, in a supply chain, the cost of advertising is one of the basic decisions. In the literature, many research studies have investigated advertising and its various forms of development. Wang et al. [21] contributed to the issue of cooperative advertising with one manufacturer and two retailers. They examined four structures, including two Stackelberg games and two Nash games and showed how cooperative advertising policy could undergo changes in different competitive conditions. Karray and Amin [22] examined the impact of cooperative advertising on a channel with retailers competition. They considered both pricing and advertising variables and two games in their work. The first game has a condition that the manufacturer and retailers do not use coop advertising, and the second game involves using coop-advertising.

Contrary to the results provided for one-manufacturer and one-retailer channels, they found that coop-advertising might not be profitable for the retailers or for the channel. He et al. [23] studied a manufacturer who sold products through a retailer. In this study, the retailer attempted to compete with other retailers by undertaking promotional expenditures such as advertising. The manufacturer also participated in retailer advertising with a participation rate. To calculate the participation rate, a Stackelberg game between the manufacturer and the retailer and a Nash game between the retailer and other rival retailers are considered.

They showed that a retailer could be supported by its manufacturer only when a subsidy threshold was crossed. The manufacturer also offered more support when the corresponding retailer competed with one other retailer. Esmaeili et al. [24] studied advertising and pricing in a three-level supply chain with one supplier, one manufacturer, and one retailer. They employed three approaches including the Nash, Stackelberg, and Cooperative game, and concluded that cooperative game had the highest advertising expenditure. Esmaeili et al. [25], in another study, considered advertising, pricing, and service decisions.

Their supply chain had a manufacturer and one retailer. They used the Nash, Stackelberg, and Cooperative game, and found that the cooperative game yielded the highest profits. Ezimadu [26] examined the cooperative advertising in a supply chain involving a manufacturer, a distributor, and a retailer with the Stackelberg game.

Tab. **1** shows the category of reviewed articles in terms of game types, players, competition, and various objective functions. The closest paper to this study is [27] which studied pricing and advertising in a two-tier supply chain. They found that if the price elasticity was less than a certain amount, the manufacturer should not participate in local advertising.

The contribution of this paper is to develop their work by:

- Considering the greening level for manufacturer production besides pricing and advertising decisions, simultaneously;

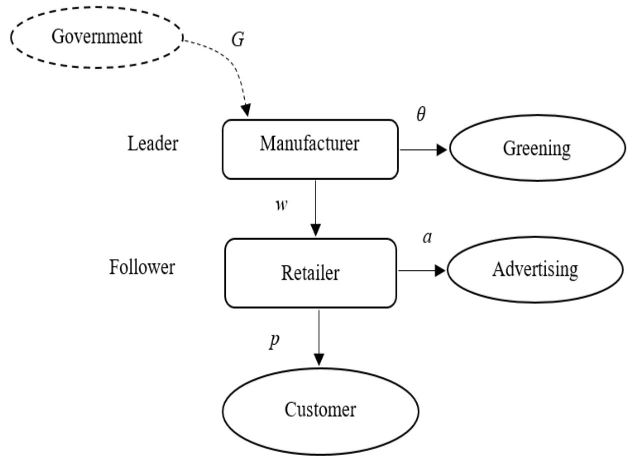
- Considering the subsidy parameter of the government and its effect on greening and advertising by developing a game-theoretic model.

Tab. 1. Categorization of game-theoretic articles

Article	Year	Game types: Stackelberg	Game types: Nash	Game types: Bargain hunting	Players: Manufacturer	Players: Retailer	Players: Customer	Players: Government	Competition: Manufacturer	Competition: Retailer	Objective function: Greening	Objective function: Pricing	Objective function: Advertising
H. Wang [7]	2005			●	●	●						●	
J. Yue et al. [9]	2006	●			●	●						●	●
S. Karray and G. Zaccour [8]	2006	●			●	●			●			●	●
J. G. Szmerekovsky and J. Zhang [10]	2009	●			●	●						●	●
X. He et al. [16]	2009	●			●	●						●	●
J. Xie and A. Neyret [11]	2009	●	●		●	●						●	●
J. Xie and J. C. Wei [12]	2009	●			●	●						●	●
R. Yan [13]	2010	●			●	●						●	●
A. Xu et al. [3]	2011		●		●		●	●			●		
S.-D. Wang et al. [21]	2011	●	●		●	●				●			●
M. M. SeyedEsfahani et al. [14]	2011	●	●		●	●						●	●
J.-B. Sheu and Y. J. Chen et al. [5]	2012		●		●	●		●			●	●	
X. He et al. [23]	2012	●	●		●	●				●		●	●
G. Aust et al. [15]	2012	●	●		●	●						●	●
S. Swami and J. Shah [1]	2013	●			●	●					●	●	
C.-T. Zhang et al. [6]	2014	●			●	●		●			●	●	
S. Karray and S. H. Amin [22]	2015	●			●	●				●		●	●
L. Zhao et al. [27]	2016	●			●	●						●	●
Esmaeili et al. [24]	2016	●	●		●	●						●	●
B. Li et al. [4]	2016	●			●	●					●	●	
H. Jafari et al. [19]	2016	●			●	●						●	
Saghaeeian, A., & Ramezanian [20]	2017	●			●	●				●		●	
Esmaeili et al. [25]	2017	●	●		●	●						●	●
Sinayi & Rasti-Barzoki [2]	2018	●			●	●		●			●	●	
Ezimadu [26]	2019	●			●	●							●
This study		●			●	●					●	●	●

3. Problem Definition

Here, a manufacturer in a two-stage green supply chain is considered that delivers products only through the medium of a retailer to consumers. Due to the importance of environmental issues and the public and government attention to this issue, green production positively affects customer demand. Manufacturers and retailers are in a dynamic game in which the manufacturer acts as the leader for the retailer. These two chain members seek to determine the green policies, selling prices, and cost of advertising to maximize their profits. Meanwhile, the government has set a subsidy for the manufacturer to encourage him to green production. Fig. 1 shows the structure of the problem.



Continue of Tab. 2. Categorization of the game-theoretic articles

Fig. 1. Structure of the problem

The demand function is considered cumulative in relation to advertising (like [27]) and greening level (like [28]). All information about the parameters and behavior of the demand function is known (Game with complete information).

3-1. Notations

The notations used in this article are:

Indexes

m Manufacturer index

r Retailer index

Parameters

A Market potential

b Sensitivity of demand-price

c Production cost

k_r Effectiveness of retailer advertising expenditures

α Expansion effectiveness coefficient of the greening

λ Investment parameter for greening

G Manufacturer subsidy by Government

Decision variables

w Wholesale price

p Retail price

a Retailer advertising cost

θ Level of greening

Demand and profit functions

D Demand function

π Profit function

3-2. Assumptions

The assumptions that are considered in this article are:

All parameters, variables, and functions are positive (Similar to [27]).

The shortage is not allowed (Similar to [29]).

The manufacturer is the leader and the retailer is a follower (Similar to [1, 27]).

The level of greening is related to production operations and product structure (Similar to [28]).

Greening does not affect the cost of production (Similar to [1, 28]).

4. Problem Formulation

According to the definition of the problem and the assumptions presented before, the demand function is considered as follows:

$$D(p,a,\theta) = A - bp + \alpha\theta + k_r\sqrt{a} \tag{1}$$

and manufacturer and retailer profit functions are as follows:

$$\pi_m(w,\theta) = (w - c + G)(A - bp + \alpha\theta + k_r\sqrt{a}) - \lambda\theta^2 \tag{2}$$

$$\pi_r(p,a) = (p - w)(A - bp + \alpha\theta + k_r\sqrt{a}) - a \tag{3}$$

where $p>w>c$.

5. Stackelberg equilibrium

This study faces a game with two players whose decisions are sequential. Thus, the Stackelberg equilibrium is used. This section presents the Stackelberg equilibrium with respect to the manufacturer and the retailer. In order to obtain the equilibrium, the manufacturer is considered as the leader and the retailer as its follower, and it is solved by backward induction.

5-1. The retailer

In this Stackelberg game, the problem of the retailer is to maximize its profit as follows:

$$Max\ \pi_r(p,a) = (p - w)(A - bp + \alpha\theta + k_r\sqrt{a}) - a \tag{4}$$

5-1-1. Proposition 1. retail price and advertising cost

The retailer's decision variables are:

$$p^{St} = \frac{-2A - 2bw + k_r^2 w - 2\alpha\theta}{-4b + k_r^2} \tag{5}$$

$$a^{St} = \frac{k_r^2(A - bw + \alpha\theta)^2}{(-4b + k_r^2)^2} \tag{6}$$

Equations (5) and (6) show the retail price and retailer advertising expenditure in equilibrium, respectively, and are used to maximize the retailer profit.

5-2. The manufacturer

After calculating the retailer variables, now, the manufacturer function is updated by substituting Equations (5) and (6) into Equation (2):

$$Max\ \pi_m(w,\theta) = (-c + G + w)\left(\frac{(2b - k_r^2)(A - bw + \alpha\theta)}{4b - k_r^2} + k_r^2\sqrt{\frac{k_r^2(A - bw + \alpha\theta)^2}{(-4b + k_r^2)^2}}\right) - \theta^2\lambda \tag{7}$$

5-2-1. Proposition 2. wholesale price and greening level

The manufacturer decision variables are used in Equations (8) and (9) as follows:

$$w^{St} = \frac{b(-c + G)\alpha^2 + (A + b(c - G))(4b - k_r^2)\lambda}{b(-\alpha^2 + 8b\lambda - 2k_r^2\lambda)} \tag{8}$$

$$\theta^{St} = \frac{(A + b(-c + G))\alpha}{-\alpha^2 + 8b\lambda - 2k_r^2\lambda} \tag{9}$$

Equations (8) and (9) show the greening level and wholesale price of the manufacturer, respectively, in equilibrium and to maximize the manufacturer profit.

5-3. Feasibility conditions for wholesale price

The wholesale price is positive under the mentioned conditions in Relations (28) and (30).

5-4. Feasibility conditions for the greening level

The greening level must be positive; hence, we have:

$$b < \frac{A}{c-G},\ G < c \tag{10}$$

On the other hand, considering conditions from Relation (28) under which the manufacturer function is concave, the optimum value of θ with respect to b has a vertical asymptote. To calculate this vertical asymptote, the denominator of θ is considered to be equal to zero ($-\alpha^2 + 8b\lambda - 2k_r^2\lambda = 0$). Hence, we can derive that $b = \frac{\alpha^2 + 2k_r^2\lambda}{8\lambda}$. Therefore, θ can be positive if:

$$b > \frac{\alpha^2+2k_r^2\lambda}{8\lambda} \quad (11)$$

5-5. Feasibility conditions for retail price

The retail price is positive under the mentioned conditions in Relations (28) and (30).

6. Parameter Analysis

This section concentrates on the effects of demand parameters at the greening level.

6-1. Greening level range

Rang of the greening level is related to the wholesale price and it must be in such a way that the profits become positive.

6-1-1. Proposition 3

The greening level is in this range:

$$\frac{b\,w-A}{\alpha} < \theta < \frac{b\,w}{\alpha} \quad (i)$$

6-2. Effect of demand parameters on greening level

6-2-1. Proposition 4

$$\frac{\partial\theta}{\partial b} < 0 \quad (ii)$$

$$\frac{\partial\theta}{\partial A} > 0 \quad (iii)$$

$$\frac{\partial\theta}{\partial k_r} > 0 \quad (iv)$$

$$\frac{\partial\theta}{\partial G} > 0 \quad (v)$$

Greening level has an inverse relationship with "sensitivity of demand-price" (ii) and has a direct relationship with "market potential" (iii), "effectiveness of retailer advertising expenditures" (iv), and " Manufacturer subsidy by Government " (v).

6-3. Effect of demand parameters on the wholesale price

6-3-1. Proposition 5

$$\frac{\partial w}{\partial b} < 0 \quad (vi)$$

$$\frac{\partial w}{\partial A} > 0 \quad (vii)$$

$$\frac{\partial w}{\partial k_r} > 0 \quad (viii)$$

$$\frac{\partial w}{\partial G} > 0 \text{ if } \frac{\alpha^2}{8b-2k_r^2} < \lambda < \frac{2\alpha^2}{8b-2k_r^2} \quad (ix)$$

$$\frac{\partial w}{\partial G} < 0 \text{ if } \frac{2\alpha^2}{8b-2k_r^2} < \lambda$$

6-4. Analysis

Part (i) means that greening level increases with wholesale price. This part shows the interval of θ and can help the manufacturer set the greening level. θ and *w* decrease with *b* (ii),(vi) and increase with *A* (iii),(vii). Thus, if demand-price elasticity is high, the manufacturer cannot increase the greening level and its because of the drop in wholesale price. On the other hand, when the market potential is high, the market has a larger demand and needs a higher level of θ. Therefore, it is easier for the manufacturer to increase the greening level. Moreover, the manufacturer can set a higher wholesale price. Greening level and wholesale price increase with the effectiveness of advertising expenditure (iv),(viii). Because of the greater demand caused by advertising, the need for greening and wholesale price increase. Part (v) means that the greening level increases with government subsidy, which is the government purpose. However, the relation between the wholesale price and government subsidy depends on investment parameter for greening (λ). As (ix) shows, if the cost factor of advertising is larger than $\frac{2\alpha^2}{8b-2k_r^2}$, wholesale price decreases with *G*. Otherwise, the manufacturer increases its price even by government subsidy.

7. Numerical Results

In this section, the variables and the manufacturer and retailer profit functions in equilibrium are discussed numerically. At first, a numerical example is provided and, then, the sensitivity analyses are discussed. All calculations and plots have been done by Wolfram Mathematica software version 10.4 by a computer equipped with corei5 CPU 2.3 GHz and 8 GB RAM.

7-1. Example

Now, a numerical example is provided to clarify the subject.

Tab shows the value of the parameters.

Tab. 2. Parameter values

Parameter	Value
A	100
b	4
c	10
k_r	3
α	2
λ	3
G	8

By solving the problem, we can obtain the decision variable values as in
Tab. .

Tab. 3. Optimum decision variables

Decision variable	Value
w	14.71
p	29.24
θ	4.84
a	474.78
π_m	668.21
π_r	369.27

As shown, in equilibrium, retail price is higher than the wholesale price due to retailer profit margin and advertising cost, which is the retailer's responsibility. When the retailer costs are a=474.78 on advertising, the greening level of the supply chain is θ=4.84, and both supply chain members are profitable.

7-2. Sensitivity analysis

In this section, sensitivity analyses of decision variables are presented.

7-2-1. Greening level and profit functions

Fig. **2** shows the manufacturer and retailer profit function graph in terms of advertising expenditure and greening level. The sum of these profits is also shown, which indicates the chain profit ($\pi_m + \pi_r$).

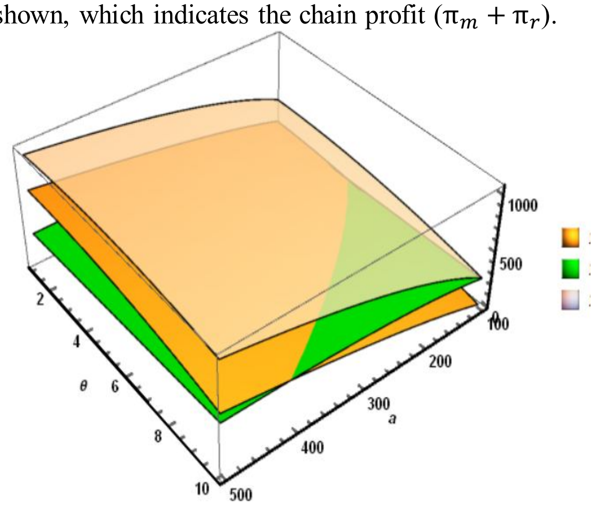


Fig. 2. Manufacturer and retailer profit functions in terms of *a* and θ

According to Fig. **2**, the manufacturer profit increases with respect to advertising expenditure and decreases with respect to the greening level. Retailer profit is almost monotonic with respect to advertising expenditure; however, it increases with respect to the greening level.

- Because of the greater importance of supply chain profit than manufacturer or retailer profit, the manufacturer must increase the greening level and retailer must increase advertising to maximize supply chain profit.

7-2-2. Greening level and demand price sensitivity

Fig. **3** illustrates the level of greening with respect to demand-price sensitivity.

- It is implied that in a market with higher sensitivity of demand-price, the implementation of greening is more difficult for the manufacturer.

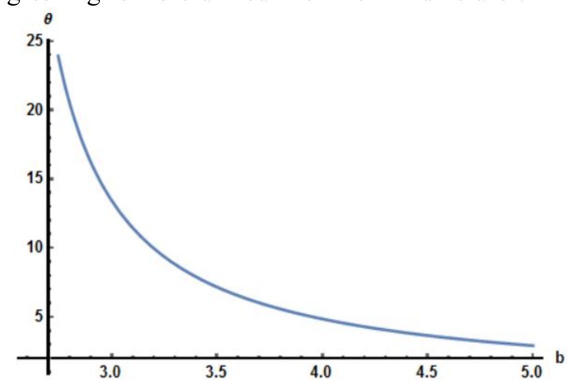


Fig. 3. Greening level with respect to demand sensitivity of price changes

7-2-3. Greening level and pricing

Fig. **4** shows the relationship between the greening level and wholesale and retail price. Both prices increase with θ. If the manufacturer raises the greening level, the retail price increases with a larger slope and, following that, the purchase value of the product decreases; therefore, it can be stated that

- retail price is more important than wholesale price when we need to increase the level of greening.

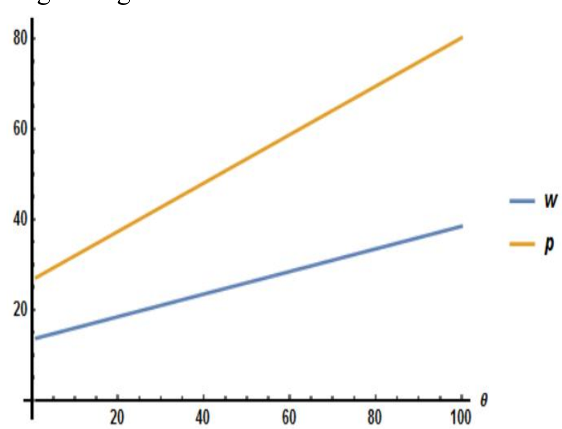


Fig. 4. Impact of θ on wholesale and retail price

7-2-4. Greening level and demand function

The demand function has a decreasing trend with respect to *p*; however, it has an increasing trend with respect to θ (Fig. **5**). In other words, customers have a positive feeling about green products.

- In all conditions with retail prices, increasing the green level will raise demand.

As is observed, with an increase in *b*, the demand level decreases and, according to the increasing retail price, the demand reduction rate increases.

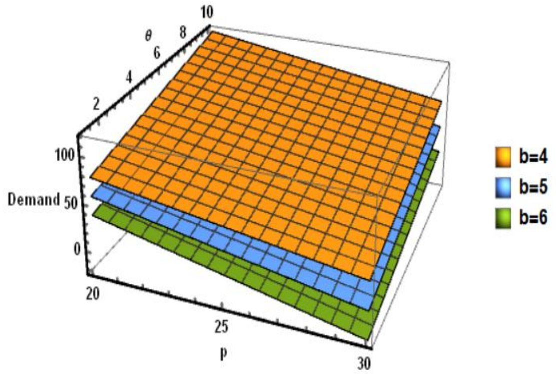


Fig. 5. Demand function with respect to *p* and θ

7-2-5. Greening cost

Fig. **6** shows the manufacturer profit regarding the level of greening at three levels of λ.

- There is a limitation on the investment parameter of greening and, after that, the manufacturer profit and greening level decrease, because of the expenditure of greening is imposed on the manufacturer. In this case, λ=3 makes the maximum profit with a higher greening level.

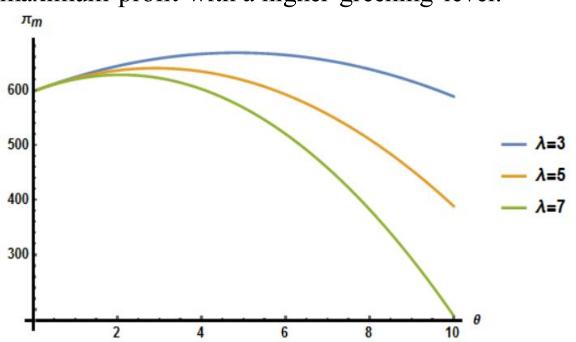


Fig. 6. Manufacturer profit with respect to θ and λ

7-2-6. Manufacturer greening implementation

Now, it is considered here that the manufacturer receives government subsidies, but does not participate in greening policies. Fig. **7** shows the behavior of the manufacturer profit function with respect to different values of α and λ. As can be seen, manufacturer profit in the "not implementing" mode in all conditions is less than that in the "implementing" mode. This improvement is also evident in retailer profit (Fig. **8**).

- Manufacturer contributing to greening policies with government subsidies is a win-win game.

This is good and viable for the supply chain profit and the environment.

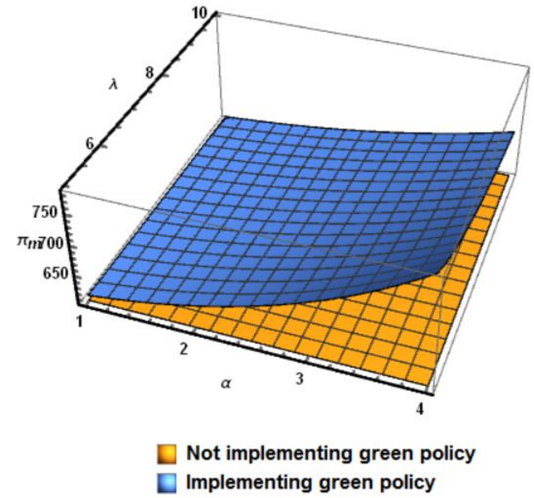


Fig. 7. Implementing and not implementing green policies compared for manufacturer's profit regarding α and λ

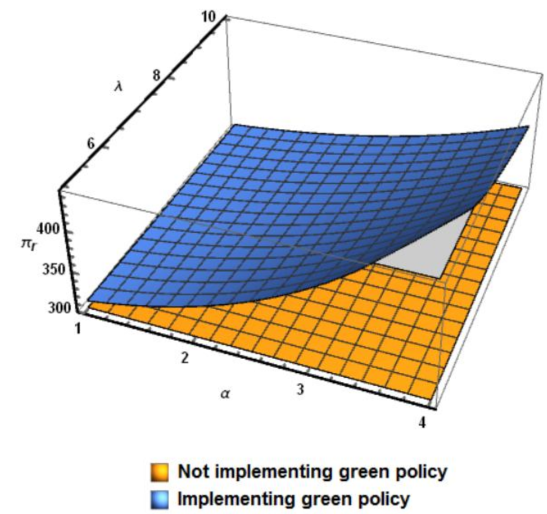


Fig. 8. Implementing and not implementing green policies compared for retailer profit regarding α and λ

7-3. Managerial insights

This section attempts to propose some useful management tips due to sensitivity analyses.

By increasing the advertising expenditure, the level of greening can be increased, while the overall supply chain cost is kept almost stable. Thus, it is recommended that the retailer raise the expenditure on advertising and the manufacturer involve the retailer in the chain profit with a participation rate.

If the sensitivity of demand price is less than $\frac{\alpha^2+2k_r^2\lambda}{8\lambda}$, the manufacturer should not consider greening in its production plan.

If the sensitivity of demand price is larger than $\frac{\alpha^2+2k_r^2\lambda}{8\lambda}$, the manufacturer must implement greening, because the manufacturer profit in green production is larger than that in non-greening with all changes in greening parameters.

If the investment parameter for greening is in range $\left[\frac{\alpha^2}{8b-2k_r^2}, \frac{2\alpha^2}{8b-2k_r^2}\right]$, increasing the government subsidy will increase the wholesale price, and this leads to demand reduction as well as customer satisfaction. Therefore, it is not recommended that the government raise the value of subsidies.

If the investment parameter for greening is larger than $\frac{2\alpha^2}{8b-2k_r^2}$, increasing the value of subsidy will decrease the wholesale price and lead to greater demand and greening level. Therefore, it is recommended that the government raise the value of subsidies.

Retail price is more important than the wholesale price to determine the greening level.

In a market with higher sensitivity of demand-price, the implementation of greening is more difficult for the manufacturer.

8. Conclusion

In this paper, greening policies, pricing, and advertising were addressed simultaneously in a two-stage supply chain with government intervention. This supply chain involves a manufacturer and a retailer, where the retailer makes decisions based on the manufacturer's decisions. The government is trying to encourage the manufacturer to green its products by subsidy. Because of several factors in such a study and factor interactions, the game theory was used to determine the value of decision variables in equilibrium. At first, the demand and profit function of each factor was introduced; then, in a Stackelberg game, the equilibrium values of the decision variables were obtained. The illustration of the problem was obtained by an example, and several sensitivity analyses were carried out on the decision variables. It was found that the manufacturer would not perform the greening unless the sensitivity of demand-price was greater than the threshold. Moreover, increasing the greening level and advertising together maximized the supply chain profit.

For future research, it is suggested that the role of government be considered as an agent. The sensitivity of demand-price is considered as an exponential as well and compared with this paper. In addition, retailer competitions can be considered in additional works.

Appendix

The proofs of the propositions are given in this section.

Proof of proposition 1

The retailer's problem is a function of p and a, which must be solved simultaneously. The first and second derivatives of p are:

$$\partial \pi_r / \partial p = A + k_r\sqrt{a} - 2bp + bw + \alpha\theta \tag{12}$$

$$\partial^2 \pi_r / \partial p^2 = -2b \tag{13}$$

These derivatives for a are as follows:

$$\partial \pi_r / \partial a = -1 + \frac{k_r(p-w)}{2\sqrt{a}} \tag{14}$$

$$\partial^2 \pi_r / \partial a^2 = -\frac{k_r(p-w)}{4a^{3/2}} \tag{15}$$

By computing the second-order partials, the Hessian matrix of the retailer function can be formed:

$$\mathrm{H}(p,a) = \begin{bmatrix} -2b & \frac{k_r}{2\sqrt{a}} \\ \frac{k_r}{2\sqrt{a}} & -\frac{(p-w)k_r}{4a^{3/2}} \end{bmatrix} \tag{16}$$

By checking the principal minors, we have:

$$D_1(p,a) = |-2b| = -2b < 0 \tag{17}$$

$$D_2(p,a) = \begin{vmatrix} -2b & \frac{k_r}{2\sqrt{a}} \\ \frac{k_r}{2\sqrt{a}} & -\frac{(p-w)k_r}{4a^{3/2}} \end{vmatrix} = -\frac{k_r^2}{4a} + \frac{bk_rp}{2a^{3/2}} - \frac{bk_rw}{2a^{3/2}} \tag{18}$$

$\mathrm{H}(p,a)$ must be a negative definite matrix for the retailer function to be concave. $D_1(p,a)$ is negative; therefore, $D_2(p,a)$ must be positive and we have:

$$a < \frac{4b^2p^2 - 8b^2pw + 4b^2w^2}{k_r^2} \tag{19}$$

Now, through the calculation of p and a, as in Equations (5) and (6), simultaneous solving of $\partial\pi_r/\partial p = 0$ and $\partial\pi_r/\partial a = 0$ can be achieved.

Proof of proposition 2

The first and second derivatives of π_m with respect to w and θ are:

$$\partial\pi_m/\partial w = (-c+G+w)\left(\frac{(2b-k_r^2)(A-bw+\alpha\theta)}{4b-k_r^2} + k_r^2\sqrt{\frac{k_r^2(A-bw+\alpha\theta)^2}{(-4b+k_r^2)^2}}\right) - \theta^2\lambda \tag{20}$$

$$\partial^2\pi_m/\partial w^2 = -\frac{2b\left(Ak_r^3 + 8b^2\sqrt{\frac{k_r^2(A-bw+\alpha\theta)^2}{(-4b+k_r^2)^2}} - bk_r^2\left(k_rw + 6\sqrt{\frac{k_r^2(A-bw+\alpha\theta)^2}{(-4b+k_r^2)^2}}\right) + k_r^3\left(\alpha\theta + k_r\sqrt{\frac{k_r^2(A-bw+\alpha\theta)^2}{(-4b+k_r^2)^2}}\right)\right)}{(-4b+k_r^2)^2\sqrt{\frac{k_r^2(A-bw+\alpha\theta)^2}{(-4b+k_r^2)^2}}} \tag{21}$$

$$\partial\pi_m/\partial\theta = (-c+G+w)\left(\alpha + \frac{2b\alpha}{-4b+k_r^2} + \frac{k_r^3\alpha(A-bw+\alpha\theta)}{(-4b+k_r^2)^2\sqrt{\frac{k_r^2(A-bw+\alpha\theta)^2}{(-4b+k_r^2)^2}}}\right) - 2\theta\lambda \tag{22}$$

$$\partial^2\pi_m/\partial\theta^2 = -2\lambda \tag{23}$$

We can form the Hessian matrix:

$$\mathrm{H}(w,\theta) = \begin{bmatrix}\Delta_1 & \Delta_2\\ \Delta_2 & -2\lambda\end{bmatrix} \tag{24}$$

where Δ_1, Δ_2 are:

$$\Delta_1 = \partial^2\pi_m/\partial w^2 \text{ from Equation (21)} \tag{25}$$

$$\Delta_2 = \frac{\alpha\left(Ak_r^3 + 8b^2\sqrt{\frac{k_r^2(A-bw+\alpha\theta)^2}{(-4b+k_r^2)^2}} - bk_r^2\left(k_rw + 6\sqrt{\frac{k_r^2(A-bw+\alpha\theta)^2}{(-4b+k_r^2)^2}}\right) + k_r^3\left(\alpha\theta + k_r\sqrt{\frac{k_r^2(A-bw+\alpha\theta)^2}{(-4b+k_r^2)^2}}\right)\right)}{(-4b+k_r^2)^2\sqrt{\frac{k_r^2(A-bw+\alpha\theta)^2}{(-4b+k_r^2)^2}}} \tag{26}$$

Principal minors are:

$$D_1(w,\theta) = |\Delta_1| = \Delta_1 \tag{27}$$

$D_1(w,\theta)$ is negative when:

$$b > \frac{k_r^2}{4}, w < \frac{A+\alpha\theta}{b} \tag{28}$$

Considering Relation (28), the second principal minor is:

$$D_2(w,\theta) = \begin{vmatrix}\Delta_1 & \Delta_2\\ \Delta_2 & -2\lambda\end{vmatrix} = \frac{4b^2(-\alpha^2 + 8b\lambda - 2k_r^2\lambda)}{(-4b+k_r^2)^2} \tag{29}$$

$D_2(w,\theta)$ is positive when:

$$\frac{w}{\theta} > \frac{\alpha}{b}, \lambda > \frac{\alpha^2}{8b - 2k_r^2} \tag{30}$$

With the conditions developed, the manufacturer function is concave, and Equations (8) and (9) maximize it.

Proof of proposition 3

It can be derived from Proposition 4 and Relations (28) and (30).

Proof of proposition 4

(ii) $b < \frac{A}{c-G} \xrightarrow{b>\frac{k_r^2}{4}} A > \frac{k_r^2}{4}(c-G) \rightarrow \frac{\partial\theta}{\partial b} = -\frac{8\left(A+b(-c+G)\right)\alpha\lambda}{\left(-\alpha^2+8b\lambda-2k_r^2\lambda\right)^2} + \frac{(-c+G)\alpha}{-\alpha^2+8b\lambda-2k_r^2\lambda} < 0,$

(iii) $b > \frac{\alpha^2+2k_r^2\lambda}{8\lambda} \rightarrow \frac{\partial\theta}{\partial A} = \frac{\alpha}{-\alpha^2+8b\lambda-2k_r^2\lambda} > 0,$

(iv) $A > b(c-G) \rightarrow \frac{\partial\theta}{\partial k_r} = \frac{4(A+b(-c+G))k_r\alpha\lambda}{(-\alpha^2+8b\lambda-2k_r^2\lambda)^2} > 0,$

(v) It can be concluded from Relations (28) and (30) that
$\frac{\partial\theta}{\partial G} = \frac{b\alpha}{-\alpha^2+8b\lambda-2k_r^2\lambda} > 0.$

Proof of Proposition 5

It can be concluded from Relations (28) and (30) that:

(vi) $\frac{\partial w}{\partial b} = \frac{\lambda((4b^2(c-G)-Ak_r^2)\alpha^2-2A(-4b+k_r^2)^2\lambda)}{b^2(\alpha^2+2(-4b+k_r^2)\lambda)^2} < 0,$

(vii) $\frac{\partial w}{\partial A} = \frac{(4b-k_r^2)\lambda}{b(-\alpha^2+8b\lambda-2k_r^2\lambda)} > 0,$

(viii) $\frac{\partial w}{\partial k_r} = \frac{2(A+b(-c+G))k_r\alpha^2\lambda}{b(\alpha^2+2(-4b+k_r^2)\lambda)^2} > 0.$

(ix) Considering:

$$\frac{\alpha^2}{8b-2k_r^2} < \lambda < \frac{2\alpha^2}{8b-2k_r^2} \xrightarrow{b>\frac{k_r^2}{4}} \frac{\partial w}{\partial G} = \frac{1}{2}\left(-1-\frac{\alpha^2}{\alpha^2-8b\lambda+2k_r^2\lambda}\right) > 0,$$

if,

$$\lambda > \frac{2\alpha^2}{8b-2k_r^2} \xrightarrow{b>\frac{k_r^2}{4}} \frac{\partial w}{\partial G} < 0.$$

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