An Approach to Optimizing the Water Resources Management Problem in a Fuzzy Environment

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KEYWORDS
Water resources management; Fuzzy numbers; Membership function; Fuzzy arithmetic; Fuzzy constraints; Two stage; Policy analysis; Uncertainty.

ABSTRACT
Fully fuzzy linear programming is applied to water resources management due to its close connection with human life, which is considered to be of great importance. This paper investigates the decision-making concerning water resources management under uncertainty based on two-stage stochastic fuzzy linear programming. A solution method for solving the problem with fuzziness in relations is suggested to prove its applicability. The purpose of the method is to generate a set of solutions for water resources planning that helps the decision-maker make a tradeoff between economic efficiency and risk violation of the constraints. Finally, a numerical example is given and is approached by the proposed method.

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1. Introduction
Water Resources Management is an integrating concept for a number of water sub-sectors such as hydropower, water supply and sanitation, irrigation and drainage, and environment (Gasinov and Yenilmez[7]). The water resources management includes (Jairaj and Vedula[10]):
- The quantitative and qualitative exploration of water resources;
- Water requiring inventory records;
- Measurement and matching of the water resources and water needs (demands) in a special system;
- Decision support depending on the results.

Up to now, fuzzy set theory has been applied to broad fields. Fuzzy set theory introduced by Zadeh [28] creates a model that is set up using approximately known data. Fuzzy numerical data can be represented by means of fuzzy subsets of the real line, known as fuzzy numbers. For the fuzzy set theory development, we may refer to the papers of Kaufmann [12] and Dubois and Prade [3]. They extended the application of algebraic operations of real numbers to fuzzy numbers by using a fuzzy principle. Fuzzy linear constraints with fuzzy numbers were studied by Dubois and Prade [3].

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within inexact two-stage stochastic programming. A two-stage optimization framework for planning reservoir operations was proposed by Wang and Adams [23], where hydrologic uncertainty and seasonal reservoir inflows have been modeled as in a periodic Markov process. Through a two-stage dynamic programming approach, a long-term hydrothermal scheduling of multi-reservoir systems was examined by Ferrero et al. [5]. Bellman and Zadeh [1] introduced the concept of a maximizing decision-making problem. Zhao et al. [29] introduced a complete solution set for fuzzy linear programming problems using linear and nonlinear membership functions. For water resources management, Huang and Loucks [9] proposed inexact two-stage stochastic programming. An interactive fuzzy resolution method for solving linear programming problems with fuzzy parameters was proposed by Jimenez et al. [11]. For developing water resources management, Huang and Loucks [9] proposed inexact two-stage stochastic programming. An interactive fuzzy resolution method for solving linear programming problems with fuzzy parameters was proposed by Jimenez et al. [11]. For developing water resources management, Huang and Loucks [9] proposed inexact two-stage stochastic programming. An interactive fuzzy resolution method for solving linear programming problems with fuzzy parameters was proposed by Jimenez et al. [11].


2. Preliminaries

Some basic concepts and results related to fuzzy numbers as well as some of their arithmetic operations, triangular fuzzy numbers, and some of algebraic operations are recalled in this section (Kaufmann and Gupta [13], Sakawa [19] and Zimmermann[30]).

Definition 1.
A fuzzy number \( \tilde{p} \) is a mapping:
\[
\mu_{\tilde{p}} : R \rightarrow [0, 1],
\]
with the following properties:

(i) \( \mu_{\tilde{p}}(x) \) is an upper semi-continuous membership function;

(ii) \( \tilde{p} \) is a convex set, i.e.,
\[
\mu_{\tilde{p}}(\lambda x + (1-\lambda) y) \geq \min\{\mu_{\tilde{p}}(x), \mu_{\tilde{p}}(y)\}
\]
for all \( x, y \in R, 0 \leq \lambda \leq 1; \)

(iii) \( \tilde{p} \) is normal, i.e., \( \exists x_0 \in R \) for which \( \mu_{\tilde{p}}(x) = 1; \)

Suppose that \( (\tilde{p}) = \{x : \mu_{\tilde{p}}(x) > 0\} \) is the support of a fuzzy set \( \tilde{p} \).

Let \( F_{n}(R) \) denote the set of all compact fuzzy numbers on \( R \), that is, for any \( g \in F(R), g \) satisfies the following:

1. \( \exists x \in R : g(x) = 1; \)
2. For any \( 0 < \alpha \leq 1, g_{\alpha} = [g_{\alpha}^{L}, g_{\alpha}^{U}] \) is a close interval number on \( R \).

Definition 2.
The \( \alpha \) – level set of \( \tilde{p} \in F(R), 0 \leq \alpha \leq 1 \) is denoted by \( (\tilde{p})_{\alpha} \) and is defined as follows:
\[
(\tilde{p})_{\alpha} = \left\{ x \in R : \mu_{\tilde{p}}(x) \geq \alpha, 0 < \alpha \leq 1 \right\}
\]

\[
\text{closure}(\text{support}(\tilde{p})), \quad \alpha = 0
\]
Definition 3.
A fuzzy number $\widetilde{A}$ on $R$ is called triangular fuzzy number, if there exist real $a, b, c$, and $b,c \geq 0$ such that:

$$\widetilde{A}(x) = \begin{cases} \frac{x - b}{b}, & b - a \leq x \leq a, \\ \frac{-x + a + c}{c}, & a \leq x \leq a + c, \\ 0, & \text{elsewhere} \end{cases}$$

Let the triangular fuzzy number denoted by $\widetilde{A} = (a, b, c)$ and $F(R)$ be the set of all $L - R$ fuzzy numbers on $R$.

Definition 4.
$\widetilde{A} = (a, b, c)$ is called non-negative triangular fuzzy number if $a \geq 0$.

Definition 5.
Let $\widetilde{A} = (a, b, c) \geq 0$, $\widetilde{B} = (d, e, f) \geq 0$, and $x \in R$; the formulas for the addition, subtraction, scalar multiplication, and multiplication can be defined as follows:

1. Addition:
$$\widetilde{A} \oplus \widetilde{B} = (a, b, c) \oplus (d, e, f) = (a + d, b + e, c + f).$$

2. Subtraction:
$$\widetilde{A}(-)\widetilde{B} = (a, b, c) \ominus (d, e, f) = (a - d, b - e, c - f).$$

3. Multiplication:
$$\widetilde{A} \otimes \widetilde{B} = \begin{cases} (a, b, c, f), & a \geq 0 \\ (a f, b e, c d), & a < 0, c \geq 0 \\ (a f, b e, c d), & c < 0 \end{cases}$$

4. Scalar multiplication:
$$x \widetilde{A} = \begin{cases} (x a, x b, x c), & x \geq 0, \\ (x c, x b, x a), & x < 0. \end{cases}$$

Remark 1. Let $\widetilde{0} = (0, 0, 0)$ represent a zero triangular fuzzy number.

Remark 2. $\widetilde{A} \geq \widetilde{0}$ if and only if $a \geq 0, a - b \geq 0, a + c \geq 0$.

3. Water Resources Management Problem
In this section, some of the notations needed in the problem formulation are introduced.

3-1. Notations
The following notations are needed in the formulation

- $f$: A benefit of system ($$/\text{m}^3$$);
- $B_j$: Net benefit to user $j$ per $\text{m}^3$ of water allocated ($$/\text{m}^3$$);
- $T_j$: Allocation target for water that is promised to the user $j$ ($\text{m}^3$);
- $S_{jQ}$: Shortage of water to user $j$ when the seasonal flow is $Q$ ($\text{m}^3$);
- $T_{j\text{max}}$: Maximum allowable allocation amount for user $j$ ($\text{m}^3$);
- $m$: Total number of water users;
- $i$: Water user, $i = 1, 2, 3$, where $i = 1$ for municipality, $i = 2$ for the industrial user, and $i = 3$ for the agricultural sector.

The typical two-stage stochastic programming for the water resources management problem, introduced by Huang and Loucks [9], Wang and Huang [25], is considered.

$$\max f = \sum_{j=1}^{n} B_j T_j - E \left[ \sum_{j=1}^{n} C_j S_{jQ} \right]$$ (1)

subject to
$$\sum_{j=1}^{n} (T_j - S_{jQ})(1 + \delta) \leq Q,$$ (2)

(Water availability constraints)
$$S_{jQ} \leq T_j \leq T_{j\text{max}}; \forall j,$$ (3)

(Water-allocation target constraints)
$$S_{jQ} \geq 0; \forall j$$ (4)
An Approach to Optimizing the Water Resources Management Problem in a Fuzzy Environment

3-2. Fuzzy model for water resources management problem

Consider the fuzzy model for problems (5)- (8) as follows:

\[
\text{max } f(S_y, B, \tilde{C}, \tilde{T}) = \sum_{j=1}^{n} \tilde{B}_j \otimes \tilde{T}_j - \sum_{i=1}^{m} \sum_{j=1}^{n} p_i (\tilde{C}_j \otimes S_y)
\]

Subject to

\[
S_y \in \tilde{X} = \left\{ S_y \left| \left( \sum_{j=1}^{n} (\tilde{T}_j - S_y) (1 + \tilde{\delta}) \right) \leq \tilde{q}_j ; \forall j \right. \right\}
\]

where \( \tilde{B}_j, \tilde{C}_j, \tilde{\delta}, \tilde{q}_j, \tilde{T}_j \) and \( \tilde{T}_j \) are triangular fuzzy numbers.

Definition 6.

(Optimal fuzzy solution). \( S_y^{*} \) that satisfies the conditions in (9) is called a fuzzy optimization solution.

By using the representation of the fuzzy number as mentioned before, Problem (9) becomes

\[
\text{max } f(S_y, B, \tilde{C}, \tilde{T}) = \sum_{j=1}^{n} (c_j, b_j, t_j) \otimes (u_j, v_j, w_j) - \sum_{i=1}^{m} \sum_{j=1}^{n} p_i ((d_j, e_j, g_j) \otimes S_y)
\]

Subject to

\[
\sum_{j=1}^{n} ((u_j, v_j, w_j) - S_y) (1 + \delta_1, \delta_2, \delta_3) \leq (q_1, q_2, q_3) \forall j
\]

\[
S_y \in \tilde{X} = \left\{ S_y \left| \left( \sum_{j=1}^{n} (u_j, v_j, w_j) \right) \leq \left( T_{j1}, T_{j2}, T_{j3} \right) \right. \right\} \forall j
\]

Based on the arithmetic operations of fuzzy numbers, Problem (10) can be rewritten as follows:

\[
\text{max } f(S_y, B, \tilde{C}, \tilde{T}) = \sum_{j=1}^{n} (c_j, b_j, t_j) \otimes (u_j, v_j, w_j) - \sum_{i=1}^{m} \sum_{j=1}^{n} p_i ((d_j, e_j, g_j) \otimes S_y)
\]

Subject to
In problem (11), \( \tilde{B}_j \), \( \tilde{C}_j \), and \( \tilde{T}_j \) are the fuzzy variables of \( R \) characteristics based on membership functions \( \mu_{\tilde{B}_j} \), \( \mu_{\tilde{C}_j} \), and \( \mu_{\tilde{T}_j} \), respectively.

**Definition 7.**

On the account of the extension principle,
\[
\mu\left(f(S^*, \tilde{B}, \tilde{C}, \tilde{T}) \right) \leq f(S^*, \tilde{B}, \tilde{C}, \tilde{T}) = \sup \min \left( \mu_{\tilde{B}}(B), \mu_{\tilde{C}}(C), \mu_{\tilde{T}}(T) \right),
\]

4. **Characterization of \( \alpha \) – Fuzzy Optimal Solution Problem (11)**

For deducing the \( \alpha \) – fuzzy optimal solution for problem (11), let us consider the following \( \alpha \) -parametric problem:

**Model 1:**
\[
\max f(S_{ij}, B, C, T) = \sum_{j=1}^{n} B_j T_j - \sum_{i=1}^{n} \sum_{j=1}^{n} p_i (C_j S_{ij})
\]
Subject to
\[
S_{ij} \in X', B_j \in \left( \tilde{B}_j \right)_\alpha, C_j \in \left( \tilde{C}_j \right)_\alpha, T_j \in \left( \tilde{T}_j \right)_\alpha
\]
where \( \left( \tilde{B}_j \right)_\alpha \), \( \left( \tilde{C}_j \right)_\alpha \), and \( \left( \tilde{T}_j \right)_\alpha \) denote the \( \alpha \) – cut sets of \( \tilde{B}_j \), \( \tilde{C}_j \), and \( \tilde{T}_j \), respectively. Based on the convexity of the problem, \( \mu_{\tilde{B}_j}(B_j) \), \( \mu_{\tilde{C}_j}(C_j) \), and \( \mu_{\tilde{T}_j}(T_j) \) are real intervals that are denoted by \( \left[ \tilde{B}_j(\alpha) \right] , \left[ \tilde{C}_j(\alpha) \right] , \left[ \tilde{T}_j(\alpha) \right] \) respectively. Let \( \Omega_\alpha \), \( \Delta_\alpha \), and \( \Psi_\alpha \) be the set of \( 1 \times n \) matrices \( B = (B_j) \), \( C = (C_j) \), and \( T = (T_j) \) with \( B_j \in \left[ \tilde{B}_j(\alpha) \right] , \left[ \tilde{C}_j(\alpha) \right] , \left[ \tilde{T}_j(\alpha) \right] \), \( C_j \in \left[ \tilde{B}_j(\alpha) \right] , \left[ \tilde{T}_j(\alpha) \right] \), and \( T_j \in \left[ \tilde{T}_j(\alpha) \right] \). It is obvious that problem (14) may be rewritten as follows:

**Model 2.**
\[
\max f(S_{ij}, B, C, T) = \sum_{j=1}^{n} B_j T_j - \sum_{i=1}^{n} \sum_{j=1}^{n} p_i (C_j S_{ij})
\]
Subject to
\[
S_{ij} \in X', B_j \in \Omega_\alpha, C_j \in \Delta_\alpha, T_j \in \Psi_\alpha.
\]

**Definition 8.** \( S^*_{ij} \in X' \) is called an \( \alpha \) –
\[ f(S^*_y, B, C, T) \geq f(S_y, B, C, T); \forall S_y \in X', B_j \in \Omega_a, C_j \in \Delta_a, T_j \in \Psi_a \]

**Theorem 1.** \( S^*_y \in \tilde{M} \) is an \( \alpha \)-fuzzy optimal solution to problem (9) if and only if \( S^*_y \in X' \) is an \( \alpha \)-parametric optimal solution to problem Model 2.

**Proof. Necessity:** Suppose that \((X', Y') \in \tilde{M}\) is the \( \alpha \) - fuzzy optimal solution for problem (2), and \((X', Y') \in M\) is not an \( \alpha \)-parametric optimal solution for problem Model 2. Then, there are \( S^*_y \in \tilde{X} \) and \( g \in \Omega_a, l \in \Delta_a, h \in \Psi_a \) such that
\[ f(S^*_y, g,l,h) \geq f(S_y^*, g,l,h). \]
Since \( g \in \Omega_a, l \in \Delta_a, h \in \Psi_a \), we have
\[ \mu(f(S^*_y, g,l,h)) \geq f(S_y^*, g,l,h) \geq \alpha. \]
This contradicts the \( \alpha \)-fuzzy optimal solution of \( S^*_y \in \tilde{X} \) for problem (9).

**Sufficiency:** \( S^*_y \in X' \) is the \( \alpha \)-parametric optimal solution for problem Model 2, and \( S^*_y \in \tilde{X} \) is not an \( \alpha \)-fuzzy optimal solution for problem (9). Then, there are \( S^*_y \in \tilde{X} \) such that:
\[ \mu(f(S^*_y, \bar{C}, \bar{D})) \geq f(S^*_y, \bar{C}, \bar{D}) \geq \alpha, \text{ i.e.,} \]
\[ \sup_{B', C, T} \min_{\mu_{\tilde{B}}(B), \mu_{\tilde{C}}(C), \mu_{\tilde{T}}(T)} f(S^*_y, B', C, T) \geq f(S^*_y, B, C, T) \geq \alpha. \quad (14) \]

For the supremum to exist, there are \( \phi \in B', \psi \in C', \) and \( \varphi \in T' \) with \( \min(\mu_{\tilde{B}}(\phi), \mu_{\tilde{C}}(\psi), \mu_{\tilde{T}}(\varphi)) < \alpha \), then
\[ \sup_{B', C, T} \min_{\mu_{\tilde{B}}(B), \mu_{\tilde{C}}(C), \mu_{\tilde{T}}(T)} \mu_{\tilde{B}}(\phi) < \alpha. \]
This contradicts (14). Then, there are \( \phi \in B' \), \( \psi \in C' \), and \( \varphi \in T' \) that satisfy
\[ \min(\mu_{\tilde{B}}(\phi), \mu_{\tilde{C}}(\psi), \mu_{\tilde{T}}(\varphi)) \geq \alpha, \text{ i.e.,} \]
\[ \phi \in \Omega_a, \psi \in \Delta_a \text{ and } \varphi \notin \Psi_a \quad (15) \]

Based on (14) and (16), the contradiction of the optimality of \( S^*_y \in M \) for problem Model 2 is clear. Corresponding to Model 2, the following two-level mathematical programming models are structured as follows:

**Model 1-2-1.**
\[ \max f^+(S_y, B, C, T) = \sum_{j=1}^{n} (B_j T_j) - \sum_{i=1}^{m} \sum_{j=1}^{n} p_i \left( C_j \right) S_y \]

Subject to
\[ \begin{align*}
\sum_{j=1}^{n} (T_j) - S_y (1 + \delta_1) & \leq q_1; \forall j \\
\sum_{j=1}^{n} (T_j) - S_y (1 + \delta_1 - \delta_2) & \leq (q_1 - q_2); \forall j \\
\sum_{j=1}^{n} (T_j) - S_y (1 + \delta_1 + \delta_3) & \leq (q_1 + q_3); \forall j \\
S_y & \leq \max \left\{ T_j \right\}; \forall j \\
S_y & \leq \min \left\{ T_j \right\}; \forall j \\
S_y & \leq \max \left\{ T_j \right\}; \forall j \\
B_j & \in \Omega_a, C_j \in \Delta_a, T_j \in \Psi_a; \\
S_y & \geq 0; \forall j
\end{align*} \]

**Model 1-2-2.**
\[ \max f^-(S_y, B, C, T) = \sum_{j=1}^{n} (B_j T_j) - \sum_{i=1}^{m} \sum_{j=1}^{n} p_i \left( C_j \right) S_y \]

Subject to
\[ \begin{align*}
\sum_{j=1}^{n} (T_j) - S_y (1 + \delta_1) & \leq q_1; \forall j \\
\sum_{j=1}^{n} (T_j) - S_y (1 + \delta_1 - \delta_2) & \leq (q_1 - q_2); \forall j \\
\sum_{j=1}^{n} (T_j) - S_y (1 + \delta_1 + \delta_3) & \leq (q_1 + q_3); \forall j \\
S_y & \leq \max \left\{ T_j \right\}; \forall j \\
S_y & \leq \min \left\{ T_j \right\}; \forall j \\
S_y & \leq \max \left\{ T_j \right\}; \forall j \\
B_j & \in \Omega_a, C_j \in \Delta_a, T_j \in \Psi_a; \\
S_y & \geq 0; \forall j
\end{align*} \]
Step 6: Use any software to obtain the solutions of Model 2.1 and Mode 2.2, hence the optimal fuzzy solution of the problem (9).

6. Numerical Example
Consider the problem introduced by Wang and Huang [25] with triangular fuzzy numbers as: Table 1. Economic data (S/ m³) and seasonal flows (in 10⁶ m³) at different probability levels

<table>
<thead>
<tr>
<th>Activity</th>
<th>Probability</th>
<th>Seasonal flow(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>User</td>
<td>0.3</td>
<td>(2,3,4)</td>
</tr>
<tr>
<td>Municipal(i=1)</td>
<td>0.5</td>
<td>(7,9,13)</td>
</tr>
<tr>
<td>Industrial(i=2)</td>
<td>0.5</td>
<td>(14,16,20)</td>
</tr>
<tr>
<td>Agricultural(i=3)</td>
<td>0.2</td>
<td>(0.15,0.20,0.40)</td>
</tr>
</tbody>
</table>

Subject to

\[
\begin{align*}
\sum_{j=1}^{n} \left( (T_{ij})_{\alpha} - S_{ij} \right)(1 + \delta_{i} - \delta_{2}) & \leq q_{i}^{*}; \forall
\\
\sum_{j=1}^{n} \left( (T_{ij})_{\alpha} - S_{ij} \right)(1 + \delta_{i} + \delta_{3}) & \leq (q_{i}^{*} + q_{ij}^{*}) \forall
\\
S_{ij} & \in X_{2} = \left\{ S_{ij} \in \mathbb{R}^{+} : S_{ij} \leq (T_{ij})_{\alpha} \leq (T_{ij})_{\alpha}^{\max}; \forall j \right\}
\\
B_{j} & \in \Delta_{a} = \Delta_{a}^{C_{j}} \cup \Delta_{c}, T_{j} \in \Psi_{a},
\\
S_{ij} & \geq 0; \forall i, j
\end{align*}
\]

5. Solution Method
In this section, a solution procedure for solving the problem (9) is introduced as in the following steps:

Step 1: Formulate the problem (9).

Step 2: Transform the problem (9) into the problem (10) and the corresponding problem (11).

Step 3: Ask the decision-maker to specify \(\alpha(0 < \alpha < 1)\).

Step 4: Convert the problem (11) into Model 2.1.

Step 5: Use the arithmetic operations of fuzzy numbers to obtain the following two auxiliary models: Model 2.1, and Model 2.2.

Model 2.1.

\[
\begin{aligned}
\max f^{+}(S, B, C, T) &= 644.25 - \left( 70.5S_{11} + 17.25S_{12} + 14.25S_{13} + 117.5S_{21} + 28.75S_{22} + 23.75S_{23} + 475S_{31} + 11.55S_{32} + 9.5S_{33} \right)
\end{aligned}
\]
An Approach to Optimizing the Water Resources Management Problem in a Fuzzy Environment

Tab. 2. The solution of Model 1.1

<table>
<thead>
<tr>
<th>Optimal policy</th>
<th>Optimum value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{13} = S_{12} = S_{21} = S_{23} = S_{31} = S_3$</td>
<td>$f^+ = 181.224$</td>
</tr>
<tr>
<td>$S_{13} = 8.0526$</td>
<td></td>
</tr>
<tr>
<td>$S_{22} = 9.1053$</td>
<td></td>
</tr>
<tr>
<td>$S_{33} = 9.1053$</td>
<td></td>
</tr>
</tbody>
</table>

Tab. 4. The fuzzy solution of the problem

<table>
<thead>
<tr>
<th>Optimal policy</th>
<th>Optimum value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{11} = S_{12} = S_{21} = S_{23} = S_{31} = S_3$</td>
<td>$f = (0, 0, 308.2354)$</td>
</tr>
<tr>
<td>$S_{13} = 8.0526$</td>
<td></td>
</tr>
<tr>
<td>$S_{22} = 9.1053$</td>
<td></td>
</tr>
<tr>
<td>$S_{33} = 9.1053$</td>
<td></td>
</tr>
</tbody>
</table>

7. Concluding Remarks

In this paper, the water resources management problem was studied under fuzzy environment. Two auxiliary models were obtained from the proposed approach. Each model was solved using Lingo package computer. The advantage of the approach was significant for its use in interactive methods for making any comment by related managers and achieving the solutions logically. Finally, fully fuzzy linear programming for water resources management is recommended while considering minimal $S_j$ by which the water-allocation target, $T_j$, is not met when the seasonal flow with the probability of $p_i$ is $q_i$.

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