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Study of Multi-Objective Nonlinear Programming in Optimization of The Rough Interval Constraints

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KEYWORDS

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ABSTRACT

This paper studies a multi-objective nonlinear programming problem with rough intervals in the constraints. The problem was investigated by taking maximum value range and minimum value range inequalities as constraints conditions; hence, it was converted into two classical multi-objective nonlinear programming problems, called lower and upper approximation problems. All of the lower and upper approximation problems were solved by using the weighting method, where an optimal rough interval solution was obtained. The stability set of the first kind corresponding to the optimal rough interval solution was determined. Finally, a numerical example was given for the sake of illustration.

1. Introduction

Multi-objective analysis assumes that objectives are generally in conflict. Therefore, unless a well-defined utility function exists, there is no a single optimal solution, but rather a set of nondominated or non-inferior solutions from which a best compromise solution must be detected [2].

Osman et al. [10] proposed a method for solving the problem of identifying the best compromise solution to multi-objective programming. Osman and El-Banna [9] suggested an algorithm for obtaining the subset of the parametric space with the same corresponding $\alpha -$pareto optimal solution. Sakawa and Yano [19] introduced the concept of $\alpha -$Pareto optimality of the fuzzy parametric program. Rommelfanger et al. [18] solved the multi-objective linear optimization problem using the interactive method, where the coefficients of the objectives and/ or the constraints are known exactly, yet imprecisely. Khalifa [5] proposed an interactive approach to solve a multi- objective nonlinear programming problem with fuzzy parameters in the objective functions. Sakawa [20] developed interactive methods for solving multi-objective optimization problems. Tabucanon[22] treating multi-criteria decision-making. Niakan et al. [8] optimized the location of hubs under uncertainty through a proposed multi-objective mixed integer model. Sultan et al. [21] suggested an approach based on the iterative goal programming method introduced by Dauer and Krueger [3] to solve a bi-level linear programming problem whose objective functions have different fuzzy goals.

Pawlak [15] proposed the rough set theory, the purpose of which is to maximize or minimize an objective function over a certain set of feasible solutions. However, in many practical situations, the decision-makers (DM) are not qualified

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enough to specify the objective and/or feasible set precisely, yet can perform the task in a rough sense (Osman et al. [11- 12]. Pawlak and Slowinski [16] developed the rough set approach to the multi-attribute objective decision-making problem. Based on the rough set, Youness [24] classified the feasible set into the mathematical programming and named it rough programming. Xu and Yao [23] discussed a class of random rough linear MOP problems. Zaher et al. [25] introduced three types of multi-criteria decision-making methods based on the rough intervals concept.

Hamzehee et al. [4] studied the linear programming problem involving rough interval in the coefficients. Osman et al. [13] introduced a duality of multi-objective convex programming problems involving rough parameters. Khalifa [6] studied fractional programming problem with inexact rough intervals. Atteya [1] characterized and solved the multi-objective programming problems with some imprecision in their formulation. Osman et al. [14] applied fuzzy goal programming for solving fully rough multi-level multi-objective linear programming.

The rest of the paper is organized as follows: Section 2 presents some preliminaries required in the paper. Section 3 investigates the problem formulation and solution concepts. Section 4 deals with the stability set of the first kind corresponding to the obtained rough solution. Section 5 introduces a numerical example for the sake of illustration. Finally, some concluding remarks are reported in Section 6.

2. Preliminaries

In this section, the definition of rough interval and some of their arithmetic operations needed in the problem are reviewed (Lu et al. [7]).

Definition1. Let x denote a compact set of real numbers. A rough interval x^R is defined as follows:

$$x^R = [x^{(UAI)} : x^{(LAI)}] \quad (1)$$

where $x^{(UAI)}$ and $x^{(LAI)}$ are upper and lower approximation intervals of x^R , respectively.

Let $RI(\Re) = \{[a^{(UAI)} : a^{(LAI)}] : a^{(LAI)} \subseteq a^{(UAI)}, a^{(LAI)} \subseteq \Re = (-\infty,\infty), a^{(UAI)} \subseteq \Re\}$ be the set of all rough intervals on $\Re$.

Definition2. For rough intervals x^R and y^R, when $x^R \geq 0$ and $y^R \geq 0$, we have:

$$x^R (+) y^R = [x^{(UAI)} + y^{(UAI)} : x^{(LAI)} + y^{(LAI)}], \quad (2)$$

$$x^R (-) y^R = [x^{(UAI)} - y^{(LAI)} : x^{(LAI)} - y^{(UAI)}], \quad (3)$$

$$x^R (\times) y^R = [x^{(UAI)} \times y^{(UAI)} : x^{(LAI)} \times y^{(LAI)}] \quad , \quad (4)$$

$$x^R (/) y^R = [x^{(UAI)} / y^{(UAI)} : x^{(LAI)} / y^{(LAI)}]. \quad (5)$$

where $* \in \{+,-,\times,/\}$ is the binary operation on rough intervals.

Since $x^{(UAI)}, x^{(LAI)}, y^{(UAI)}$, and $y^{(LAI)}$ are conventional intervals, Equations (2)-(5) are transferred into the following functions in the case of : $x^{(UAI)} = [x^{-(UAI)}, x^{+(UAI)}],\ y^{(UAI)} = [y^{-(UAI)}, y^{+(UAI)}]$;

Equations (2)- (5) can be rewritten as follows: $x^{(LAI)} = [x^{-(LAI)}, x^{-(LAI)}], y^{(LAI)} = [y^{-(LAI)}, y^{+(LAI)}]$,

$$x^R (+) y^R = [[x^{-(UAI)} + y^{-(UAI)}, x^{+(UAI)} + y^{+(UAI)}] : [x^{+(LAI)} + y^{+(LAI)}, x^{-(LAI)} + y^{-(LAI)}]] \quad (6)$$

$$x^R (-) y^R = [[x^{-(UAI)} - y^{+(UAI)}, x^{+(UAI)} - y^{-(UAI)}] : [x^{-(LAI)} - y^{+(LAI)}, x^{+(LAI)} - y^{-(LAI)}]] \quad (7)$$

$$x^R (\times) y^R = [[x^{-(UAI)} \times y^{-(UAI)}, x^{+(UAI)} \times y^{+(UAI)}] : [x^{-(LAI)} \times y^{-(LAI)}, x^{+(LAI)} \times y^{+(LAI)}]] \quad (8)$$

$$x^R (\div) y^R = [[x^{-(UAI)} \div y^{+(UAI)}, x^{+(UAI)} \div y^{-(UAI)}] : [x^{-(LAI)} \div y^{+(LAI)}, x^{+(LAI)} \div y^{-(LAI)}]] \quad (9)$$

Definition3. A function $f:\Re^n \to RI(\Re)$ is said to be a rough interval function (because $f(x)$ is a rough interval in $\Re$). Similarly, we denote the rough interval function f with the following $f(x)=\left[f^{(UAI)}(x):f^{(LAI)}(x)\right]$, where for every $x\in\Re^n$, $f^{(UAI)}, f^{(LAI)}$ are real upper and lower approximation intervals and $x^{(LAI)}\subseteq x^{(UAI)}$.

Definition4. To interpret the meaning of optimizing the rough interval, the partial order relation is introduced as follows:

Let $x^R=[[x^{+(UAI)},x^{-(UAI)}]:[x^{+(LAI)},x^{-(LAI)}]]$, and $y^R=[[y^{+(UAI)},y^{-(UAI)}]:[y^{+(LAI)},y^{-(LAI)}]]$

be two rough intervals; then, we say that:

$x^R(\le)y^R$ if and only if $x^{+(UAI)}\le y^{+(UAI)}$ and $x^{-(UAI)}\le y^{-(UAI)}$ (10)

$x^R(<)y^R$ if and only if $x^R(\le)y^R$ and $x^R\ne y^R$. (11)

3. Problem Formulation and Solution Concepts

Let us consider the following multi-objective nonlinear programming problem with rough intervals in the constraints

$$(P_{b^R})\qquad \min F(x) \tag{12}$$

subject to

$$x\in X(b^R)=\left\{x\in\Re^n : M(x)\le b^R, b^R\in[b^{(UAI)}:b^{(LAI)}]\right\}, \tag{13}$$

where $F:\Re^n\to\Re^m$, $M:\Re^n\to\Re^r$ are convex functions on $\Re^n$, and $F=(f_1,f_2,...,f_m)^T$, $M=(g_1,g_2,...,g_r)^T$, and $b^R=(b_1^R,b_2^R,...,b_r^R)$ represent vectors of rough intervals in the constraints.

Definition5. (Rough efficient solution). x^* is said to be a rough efficient solution of the (P_{b^R}) problem if $f_i(x^*)\le f_i(x)$ with $f_i(x^*)\le f_i(x)$ for at least one $i=1,2,...,m.$

According to the operations of rough interval (10)-(11), each inequality in (13) can be transformed into 2^{r+1} inequalities such as :

$$M(x)\le b_j^{(UAI)}, \text{ and } M(x)\le b_j^{(IAI)}, j=1,2,...,r \tag{14}$$

Let D_j stand for a set of solutions to j inequality, $D^{(UAI)}=\bigcup_{j=1}^{2r+1} D_j$, and $D^{(UAI)}=\bigcap_{j=1}^{2r+1} D_j$.

Definition6. Suppose that $g_j(x)\le[b_j^{(UAI)}:b_j^{(LAI)}], j=1,2,...,r$. Then, inequality $g_j(x)\le b_j^R, j=1,2,...,r$ is called the characteristic formula of $g_j(x)\le[b_j^{(UAI)}:b_j^{(LAI)}], j=1,2,...,r$. $b_j^R\in[b_j^{(UAI)}:b_j^{(LAI)}]$.

Dfinition7. For each constraint inequality $g_j(x)\le[b_j^{(UAI)}:b_j^{(LAI)}], j=1,2,...,r$, if there exists one characteristic formula such that its set of solution is the same as $D^{(UAI)}$ or $D^{(LAI)}$, then we call this characteristic formula as the maximum value range inequality or minimum value range inequality, respectively.

Theorem1. Suppose that $g_j(x)\le[b_j^{(UAI)}:b_j^{(LAI)}], j=1,2,...,r$. Then, $g_j(x)\le b_j^{(UAI)}, j=1,2,...,r$, and $g_j(x)\le b_j^{(LAI)}, j=1,2,...,r$ are maximum value range inequality and

By taking the maximum and minimum values range inequalities as constrained conditions in response to objective function $F(x)$, problem (P_{b^R}) can be reduced into the following two classical multi-objective linear programming (MOLP) problems as follows:

$$(P_{b^{(UAI)}})\qquad \min F(x) \tag{15}$$

subject to

$$x\in X^{(UAI)}=\left\{x\in\Re^n : g_j(x)\le b_j^{(UAI)}, j=1,2,...,r\right\}, \text{and} \tag{16}$$

$$(P_{b^{(UAI)}})\qquad \min F(x) \tag{17}$$

subject to

$$x\in X^{(LAI)}=\left\{x\in\Re^n : g_j(x)\le b_j^{(LAI)}, j=1,2,...,r\right\}, \tag{18}$$

Definition8. A point $x \in \Re^n$ is said to be surely a feasible solution to Problems (17)- (18) if it satisfies Constraints (18).

Definition9. A point $x \in \Re^n$ is said to be a possible feasible solution to Problems (15)-(16) if it satisfies Constraints (16).

Problems $(P_{b^{(UAI)}})$ and $(P_{b^{(UAI)}})$ can be resolved by using the weighting problems as:

$$(P_{b^{(UAI)}})_w \quad \min\left\{\sum_{i=1}^{m} w_i f_i(x):\ x \in X^{(UAI)}\right\},$$

where $w \geq 0, w \neq 0$, and (19)

$$(P_{b^{(UAI)}})_w \quad \min\left\{\sum_{i=1}^{m} w_i f_i(x):\ x \in X^{(LAI)}\right\}$$

,where $w \geq 0, w \neq 0$ (20)

In this paper, assume that problems $(P_{b^{(UAI)}})_w$, and $(P_{b^{(UAI)}})_w$ are stable (Rockafellar [17]).

We see that $(\hat{x},\hat{b})$ is a surely Pareto optimal solution to Problems (17)- (18) and a possible Pareto optimal solution to Problems (15)- (16) if there exists $\hat{w} \geq 0$ such that $(\hat{x},\hat{b})$ is the unique surely and possible optimal solutions to Problems (18) and (16), respectively.

Suppose that the optimal solutions corresponding to (19) and (20) are:

$x_1^{'}, x_2^{'},..., x_n^{'};\ F_1^{'}$, $\quad x_1^{''}, x_2^{''},..., x_n^{''};\ F_2^{''}$.

Then, the optimal solution to rough interval multi-objective nonlinear programming Problem (12)- (13) is as follows:

$$\text{Min}\, Z = \left[F_1^{'} : F_2^{''}\right]$$

$$\begin{bmatrix} x_1 \\ x_2 \\ . \\ . \\ . \\ x_n \end{bmatrix} = \begin{bmatrix} \left[x_1^{'} : x_1^{''}\right] \\ \left[x_2^{'} : x_2^{''}\right] \\ . \\ . \\ . \\ \left[x_n^{'} : x_n^{''}\right] \end{bmatrix} \tag{21}$$

This optimal solution is a rough interval solution and contains more information for the DMs.

4. Stability Set of The First Kind

4-1. Stability set of the first kind for problem $(P_{b^{(UAI)}})_w$

Definition10. (The solvability set). 1. The solvability set of Problems (15)-(16) denoted by V_1 and defined by $V_1 = \{b^{(UAI)} \in R^r : (x, b^{(UAI)})$ is a surely Pareto optimal solution to Problems (15)- (16)$\}$.

2. The solvability set of problem (19) is denoted by V_2 and is defined by $V_2 = \{(w, b^{(UAI)}) \in R^{m+r} :\ (x^*, b^{*(UAI)})$ which is the surely optimal solution to Problem (19)$\}$.

Definition11. 1. Suppose that $b^{(UAI)} \in V_1$ with a corresponding surely Pareto optimal solution $\hat{x}$ and rough parameter $b^{(UAI)}$ of Problems (15)-(16). Then, the stability set of the first kind corresponding to $(\hat{x}, \hat{b}^{(UAI)})$ which is denoted by $S_1(\hat{x}, \hat{b}^{(UAI)})$ and is defined as follows:

$S^1(\hat{x}, \hat{b}^{(UAI)}) = \{(w, b^{(UAI)}) \in V_1 :\ (\hat{x}, \hat{b}^{(UAI)})$ is a surely Pareto optimal solution to Problems (15)-(16)$\}$.

2. Suppose that $(w, b^{(UAI)}) \in V_2$ with a corresponding surely optimal solution $(\hat{x}, \hat{b}^{(UAI)})$ to Problem (15). Then, the stability set of the first kind for Problem (19) denoted by $S_I^1(\hat{x}, \hat{b}^{(UAI)})$ and defined as

$S_I^1(\hat{x}, \hat{b}^{(UAI)}) = \{(w, b^{(UAI)}) \in V_1 :\ (\hat{x}, \hat{b}^{(UAI)})$ is the surely optimal solution to Problem(19)$\}$. (22)

4-2. Determination of the stability set of the first kind for the problem $(P_{b^{(UAI)}})_w$

Let a certain $w \in R^m$ with a corresponding $(x^*, b^{*(UAI)})$ as a surely optimal solution to the problem such that the Kuhn-Tucker conditions of Problem (19) take the following form:

$$\sum_{i=1}^{m} w_i \frac{\partial (f_i(\hat{x}))}{\partial x_\delta} + \sum_{j=1}^{r} u_j \frac{\partial h_j(\hat{x}, \hat{b}_j^{(UAI)})}{\partial x_\delta} = 0, \delta = 1, 2, \ldots, m, \tag{23}$$

$$\sum_{j=1}^{r} u_j \frac{\partial h_j(\hat{x}, \hat{b}_j^{(UAI)})}{\partial b_\beta} = 0, \beta = 1, 2, \ldots, r, \tag{24}$$

$$\sum_{i=1}^{m} w_i = 1, \tag{25}$$

$$u_j(h_j(\hat{x},\hat{b}_j^{(UAI)})=0, j \in I, h_j(\hat{x},\hat{b}_j^{(UAI)})<0, j \notin I, \quad (26)$$

$$u_j \geq 0, j \in I, u_j = 0, j \notin I. \quad (27)$$

The stability set of the first kind $S_1(\hat{x},\hat{b}_j^{(UAI)})$ can be determined according to the value of $u_j\,(j=1,2,...,r)$ which solves (23)- (24), and (23) and determine which is positive or zero.

Let $u_j = 0, j \in I = \{1,2,...,r\}$, and $u_j > 0, j \notin I$ solves (23)- (25), and (27). Also, let

$$U = \{I : u_j = 0, j \in I, u_j > 0, j \in I, solves\ (23)-(25), and\ (27)\} \quad (28)$$

Hence,

$$S_1(x^*, b^{*(UAI)}) = \bigcup_{I \in U} S_I^1(x^*, b^{*(UAI)}) \quad (29)$$

Similarly, the stability set of the second kind can be determined as follows:

$$S_2(x^*, b^{*(UAI)}) = \bigcup_{I \in U} S_I^2(x^*, b^{*(LAI)}) \quad (30)$$

where

$$S_I^2(x^*, b^{*(LAI)}) = \{ (w, b^{(LAI)}) \in V_2 : (\hat{x}, \hat{b}^{*(LAI)}) \text{ is the possible optimal solution to Problem (20)} \quad (31)$$

Thus, the stability set of the first kind of Problems (12)-(13) corresponding to the rough optimal solution (x^R, b^R) is

$$S(x^R, b^R) = \left(\bigcup_{\in UI} S_I^1(x^*, b^{*(UAI)})\right) \cap \left(\bigcup_{\in UI} S_I^2(x^*, b^{*(LAI)})\right). \quad (32)$$

5. Numerical Example

Consider the following problem

$\min\ (x_1^2 + x_2^2,\ x_1^2 + 2x_2)$

Subject to

$x_1 + x_2 \leq b^R,$

$x_1 \geq 0,\ x_2 \geq 0.$

with $b^R = [b^{(UAI)} : b^{(LAI)}],\ b^{(LAI)} \subseteq b^{UAI)}$, and $b^R = [[1,5]:[3,4]]$.

By using the weighting method, the problem becomes

$\min\left(w_1(x_1^2 + x_2^2) + w_2(x_1^2 + 2x)\right)$

Subject to

$x_1 + x_2 \leq b^R,$

$x_1 \geq 0,\ x_2 \geq 0.$

Insert $w_1 = 0.56$, $w_2 = 0.44$. Then, the problem becomes

$(P_{b^{(UAI)}})_w\ \min\ (x_1^2 + 0.56x_2^2 + 0.88x_2)$

Subject to

$x_1 + x_2 \leq b^R,$

$x_1 \geq 0,\ x_2 \geq 0,$

$b^{(UAI)} \subseteq [1,5].$

And

$(P_{b^{(UAI)}})_w\ \min\ (x_1^2 + 0.56x_2^2 + 0.88x_2)$

Subject to

$x_1 + x_2 \leq b^R,$

$x_1 \geq 0,\ x_2 \geq 0,$

$b^{(LAI)} \subseteq [3,4].$

The solution of $(P_{b^{(UAI)}})_w$ is

$(x^*, b^{*(UAI)}) = (2.0769, 2.9231, 5),\ f_1(x^*, b^{*(UAI)}) = 12.8580,\ f_2(x^*, b^{*(UAI)}) = 10.1597.$ In addition, the solution of $(P_{b^{(UAI)}})_w$ is

$(x^*, b^{*(LAI)}) = (1.359, 1.6410, 3),\ f_1(x^*, b^{*(LAI)}) = 4.5398,\ f_2(x^*, b^{*(LAI)}) = 5.1289.$

The solution of the problem is

$\begin{bmatrix} x_1^R \\ x_2^R \end{bmatrix} = \begin{bmatrix} [x_1^{'} : x_1^{''}] \\ [x_2^{'} : x_2^{''}] \end{bmatrix} = \begin{bmatrix} [1.359 : 2.0769] \\ [1.6410 : 2.9231] \end{bmatrix}$ such that the rough optimum value is

$\min Z = [F_1^{'} : F_2^{''}] = [[4.5398, 12.8580] : [5.1289, 10.1597]]$ The stability set of the first kind $S(2.0769, 2.9231, 5)$ corresponding to $(x^*, b^{*(UAI)})$ is

$$S_1(\hat{x}, \hat{b}_j^{(UAI)}) = \left\{ \begin{array}{l} (w, b^R) \in \Re^6 : \ 0.32 b^{+(UAI)} + 0.68 b^{+(LAI)} \geq 0.32 b^{-(UAI)} + 0.68 b^{-(LAI)} \geq 2.4 \\ , w_1 \leq 0.9, \ w_2 = 0.4 \end{array} \right\}$$

The stability set of the first kind corresponding to $(x^*, b^{*(LAI)})$ is

$$S_2(\hat{x}, \hat{b}_j^{(UAI)}) = \left\{ \begin{array}{l} (w, b^R) \in \Re^6 : \ 0.55 b^{+(UAI)} + 0.45 b^{+(LAI)} \geq 0.55 b^{-(UAI)} + 0.45 b^{-(LAI)} \geq 1.3 \\ , w_1 = 1, \ w_2 = 0 \end{array} \right\}$$

Hence,

$$S(x^R, b^R) = \left\{ \begin{array}{l} (w, b^R) \in \Re^6 : \ 0.32 b^{+(UAI)} + 0.45 b^{+(LAI)} \geq 0.32 b^{-(UAI)} + 0.45 b^{-(LAI)} \geq 1.3 \\ , w_1 = 1, \ w_2 = 0 \end{array} \right\}$$

6. Conclusions

In this paper, a multi-objective nonlinear programming (R-MONLP) problem with rough intervals in the constraints was considered as an extension of the flexibility of the standard MONLP problem. The advantage was that multi-objective problem with rough intervals allowed the DM to deal with the situation realistically. Furthermore, under roughness, the MONLP problem was solved easily, and a set of solutions was obtained rather than a single solution.

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