Phase II monitoring of auto-correlated linear profiles using multivariate linear mixed model

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Abstract

In the last few decades, profile monitoring in univariate and multivariate environment has drawn a considerable attention in the area of statistical process control. In multivariate profile monitoring, it is required to relate more than one respons

e variable to one or more explanatory variables. In this paper, the multivariate multiple linear profile monitoring problem is addressed under the assumption of existing autocorrelation among observations. Multivariate linear mixed model (MLMM) is proposed to account for the autocorrelation between profiles. Then two control charts in addition to a combined method are applied to monitor the profiles in phase II. Finally, the performance of the presented method is assessed in terms of average run length (ARL). The simulation results demonstrate that the proposed control charts have appropriate performance in signaling out-of-control conditions.

Keywords: Average run length (ARL), multivariate exponential weighted moving average covariance (MEWMC) chart, multivariate linear mixed model (MLMM), within profile correlation, multivariate multiple linear regression profiles, phase II

1. Introduction

Profile monitoring is very useful when quality of products or processes can be characterized by a functional relationship between a dependent variable and one or more explanatory variables. Profiles can be classified generally as linear, polynomial, nonlinear, or waveform families. In univariate applications, one response variable is modeled as a function of one or more explanatory variables. However, in multivariate applications, one can model a vector of response variables in terms of one or more explanatory variables. Noorossana et al. (2010.a) proposed the use of three control charts for phase II monitoring of multivariate simple linear profiles. In another study by Noorossana et al. (2010.b), the performance of four methods were investigated for monitoring functional relation between six explanatory variables and six responses in a calibration application in phase I. Eyvazian et al. (2011) considered four statistical control charts to analyze issues related to monitoring multivariate multiple linear regression by extending the model proposed by Noorossana et al. (2010.b). In addition, they proposed a change point method based on the likelihood ratio approach to determine the location of shifts. Zou et al. (2012) developed a lasso-based methodology for monitoring general multivariate linear profiles. Their proposed control chart is capable of determining the shift direction based on the observed profile data. Ayoubi et al. (2014) utilized maximum likelihood estimation (MLE) method to identify the time of a monotonic change in the mean of response variables of multivariate linear profiles in Phase II. Amiri et al. (2014) proposed a method for diagnosing outlying profiles and out-of-control parameters in multivariate multiple linear regression profile structure in Phase II. Zhang et al. (2015) motivated by a real-data application in semiconductor industries, developed a Phase I modelling and monitoring framework based on the regression-adjustment technique and functional principal component analysis (FPCA) for multivariate profile data. Ayoubi et al. (2016) applied MLE method to estimate change point without any assumptions about the change type in multivariate multiple linear profiles in Phase II. Kazemzadeh et al. (2015) used MLE method to estimate step and linear drift changes in the regression parameters of multivariate linear profiles in Phase II. Paynabar et al. (2016) applied a change point approach to monitor and diagnose multichannel profiles in Phase I. Ghashghaei and Amiri (2017a) developed two control charts for simultaneous monitoring of mean vector and covariance matrix in multivariate multiple linear regression profiles in Phase II. Ghashghaei and Amiri (2017b) proposed four joint control schemes for simultaneous monitoring of mean vector and covariance matrix in multivariate multiple linear regression profiles in Phase II. Ghashghaie et al. (2018) extended EWMA-SC and GWMA-SC control charts to a multivariate case to monitor multivariate multiple linear regression profiles in Phase II.

Most of the studies in the profile monitoring assume there is no correlation structure among observation within profiles. This assumption is, however, unrealistic since some data, which may be collected over time or space, exhibit serial or spatial correlation particularly when the observations are gathered in short time intervals or close spatial distances. Within profile correlation (WPC), which violates the independence assumption of the traditional control charts, may result in increasing type I error rates. Some studies suggest the utilization of model-based approaches to deal with autocorrelation in profiles. Noorossana et al. (2008) modified three different existing methods in the literature to eliminate the effect of autocorrelation in simple linear regression profile in phase II. Soleimani et al. (2009) considered a simple linear profile in phase II and assumed there is a first order autoregressive model among observations of a profile. They have also used a remedial measure for transforming Y-values in order to cope with the autocorrelation effect. Also Soleimani et al. (2013a) investigated monitoring of multivariate simple linear profiles in phase II. They applied a corrective measure based on the transformation method to handle autoregressive moving average (ARMA) correlation structure within profiles. Koosha and Amiri (2013) proposed two remedies approach to account for the autocorrelation within logistic profiles in phase I. Keramatpour et al. (2014) used a remedial measure to eliminate the effect of autocorrelation in phase II monitoring of first-order autoregressive (AR (1)) polynomial profiles. Soleimani and Hadizadeh (2014) applied a remedial measure based on a transformation method to remove the generalized autoregressive conditional heteroscedasticity (GARCH) structure within multivariate profiles in phase II. Cheng and Yang (2016) proposed approaches to monitor the profile of the linear regression model with ARMA errors both in Phase I control scheme and Phase II monitoring application. Maleki et al (2017) studied Phase I monitoring and change point estimation of auto-correlated Poisson profiles where the response values within each profile are correlated. Hadizadeh and Soleimani (2017) considered GARCH (1,1) model within the simple linear profile in phase II. They utilized two estimation methods to extract the GARCH effect. Maleki et al. (2017) introduced a Markov model in phase II monitoring of binary profiles in which the response values within each profile are auto-correlated. They used a metaheuristic algorithm to estimate model parameters. Taghipour et al. (2018) applied a transformation method to remove the autocorrelation effect on phase I monitoring of multivariate profiles which follow the ARMA (1,1) model.

Some authors, by contrast, presented the methods which are not based on corrective measures or elimination autocorrelation effects. Instead, they assigned some structure to variance-covariance matrix of residuals to consider correlation within profiles. Jensen et al. (2008) presented the use of linear mixed models (LMM) to monitor the linear profiles in order to account for any correlation structure within profiles with focus in phase I. Jensen and Birch (2009) proposed the use of nonlinear mixed models to monitor nonlinear auto-correlated profiles in phase I. Qiu et al.

(2010) described within-profile spatially correlation by a nonparametric mixed-effects model. In fact they focused on phase II profiles and considered possible step shifts in the fixed-effects term. Amiri et al. (2010) used linear mixed model method for auto-correlated polynomial profiles in phase I for an automotive industry case. Narvand et al. (2013) utilized three control charts to monitor fixed effect term of the linear mixed models in simple linear auto-correlated profiles on phase II. Soleimani et al. (2013b) extended Jensen et al. (2008)'s work to phase II of simple linear profile monitoring. Abdel-Salam et al. (2013) applied a semiparametric procedure that combines parametric and non-parametric profiles in phase I and account for autocorrelation within profiles. Zhang et al. (2014) proposed Gaussian process (GP) to model the correlation within simple linear profiles in phase II. Li et al. (2018) developed multivariate Gaussian process (MGP) to model multivariate profiles in the presence of correlation. They used a pairwise estimation strategy to estimate multivariate profiles parameters in phase I.

In this paper, the problem of the existing autocorrelation within multivariate multiple linear profiles (MMLP) in phase II is investigated. For the sake of considering autocorrelation within profiles, a multivariate linear mixed model (MLMM) is proposed. Two control charts are developed for monitoring the changes in the mean vector and covariance matrix as well as a combined chart for simultaneous monitoring of both. The performance of the presented method is investigated through numerical simulation in terms of ARL under step shifts.

The remainder of the paper is organized as follows. Our proposed modelling and assumptions is described in Section 2. In Section 3, monitoring schemes are explained in detail. Section 4 is assigned to evaluation of methods using a numerical example. Our concluding remarks is given in section 5.

2. Multivariate linear mixed model

The multivariate multiple linear mixed model for the k^{th} auto-correlated profile is defined as

$$\mathbf{Y}_k = \mathbf{X}_k\mathbf{B} + \mathbf{Z}_k\mathbf{b}_k + \mathbf{E}_k \quad k = 1,2,...,m, \tag{1}$$

where $\mathbf{Y}_k$ is a $n_k \times q$ matrix, each row of $\mathbf{Y}_k$ (y_i, i=1,...,n) is a q dimensional vector. $\mathbf{X}_k$ and $\mathbf{Z}_k$ are $n_k \times p$ and $n_k \times q$ known matrices related to covariate of fixed and random effects, respectively. $\mathbf{B}$ is a $(p+1)\times q$ matrix indicating the regression coefficient of fixed effects, $\mathbf{b}_k$ is the coefficient matrix of random effects of size $r \times q$ and $\mathbf{E}_k$ is the random error matrix in which it is assumed that each row, say ε_i, has a q-variate normal distribution with mean vector zero and variance-covariance matrix $\mathbf{\Sigma}_\varepsilon$. Jensen et al. (2008) mentioned that if the errors are correlated, covariance matrix between errors is often assumed to be a simple form, such as compound symmetry (CS) or autoregressive (AR), in order to reduce the number of covariance parameters that need to be estimated. An AR(p) indicates an autoregressive model of order p. The autocorrelation among observations t and t' of profile k^{th} is defined by $R_k = [\gamma_{|t-t'|}(\varphi)]$ for t,t′=1,...,n, where $\gamma_{|t-t'|} = \varphi_1\gamma_{|t-t'|-1} + \varphi_1\gamma_{|t-t'|-2} + ... + \varphi_1\gamma_{|t-t'|-p}$, γ_0=1 and φ_p is autocorrelation coefficient. For more details see Schabenberger and Pierce (2001). In this paper, the number of observations are taken to be the same for all profiles, i.e. $n_k = n$, for all $k = 1,2,...,m$. Moreover for each row of matrix $\mathbf{b}_k$ denoted by $b_j(j=1,...,r)$, $b_j \sim N_q(0,\boldsymbol{\phi})$, where $\boldsymbol{\phi}$ similar to $\mathbf{\Sigma}_\varepsilon$ is a $q \times q$ positive definite covariance matrix.

To handle the multivariate model, it is more convenient to consider matrices as column vectors. In the vector form, it is assumed that the matrix rows are concatenated in a single column vector. For instance $n \times q$ matrix $\mathbf{Y}$ can be written as a $(nq) \times 1$ column vector $\mathbf{Y}_v$ and consequently the l^{th} element of the column vector $\mathbf{Y}_v$ ($\boldsymbol{Y}_v^l$) corresponds to $[Y_{ij}]$ for $l = (i-1) \times q + j$, in which $1 \le i \le n,\ 1 \le j \le n$ and $1 \le l \le nd$.

Therefore, we can write the model in equation (1) in a vector form as

$$vec(\mathbf{Y}_k) = (\mathbf{X}_k \otimes \mathbf{I}_q) vec(\mathbf{B}) + (\mathbf{Z}_k \otimes \mathbf{I}_q) vec(\mathbf{b}_k) + vec(\mathbf{E}_k) \quad \text{k=1,2,...,m}, \tag{2}$$

where vec is vectorization operator, the symbol $\otimes$ stands for the kronecker product and $\mathbf{I}_q$ is identity matrix of size q. In equation (2) $vec(\mathbf{b}_k) \sim N_{rq}(0, \boldsymbol{\Psi})$ and $\boldsymbol{\Psi}$ is a $rq \times rq$ block diagonal covariance matrix equals to $\mathbf{I}_r \otimes \boldsymbol{\phi}$. Moreover $vec(\mathbf{E}_k)$ is independent of $vec(\mathbf{b}_k)$ and has a nq-variate normal distribution with mean 0 and $nq \times nq$ covariance matrix $\mathbf{R} \otimes \boldsymbol{\Sigma}_\varepsilon$.

After converting matrices to the vector form, the distribution of the response vector $\boldsymbol{Y}_k$ is given by

$$\boldsymbol{Y}_k \sim N((\mathbf{X}_k \otimes \mathbf{I}_q) vec(\mathbf{B}_k), \mathbf{V}_k), \tag{3}$$

where $\mathbf{V}_k$ or briefly $\mathbf{V}$ is variance-covariance matrix of response vector $\boldsymbol{Y}$ and is defined as

$$\mathbf{V} = (\mathbf{Z}_k \otimes \mathbf{I}_q) \boldsymbol{\Psi} (\mathbf{Z}_k \otimes \mathbf{I}_q)^T + \mathbf{R}_k \otimes \boldsymbol{\Sigma}_\varepsilon . \tag{4}$$

When the model parameters are known, the estimates of the fixed effect coefficient using maximum likelihood estimation (MLE) method ($\hat{\mathbf{B}}$) is given by

$$vec(\hat{\mathbf{B}}_k) = [(\mathbf{X}_k \otimes \mathbf{I}_q)^T \mathbf{V}_0^{-1} (\mathbf{X}_k \otimes I_q)]^{-1} (\mathbf{X}_k \otimes \mathbf{I}_q)^T \mathbf{V}_0^{-1} vec(\mathbf{Y}_k) \tag{5}$$

where V_0 and Ψ_0 are known parameters.

3. Proposed control charts for phase II monitoring scheme

In this section, two multivariate control charts will be proposed to monitor the mean and the covariance matrix in phase II. It is assumed that the IC profile model parameters are known or have been accurately estimated from Phase I analysis.

3-1. MEWMA method

When process is in control, $vec(\hat{B})$ has a $(p+1)q$-variate normal distribution with mean β and covariance matrix $\Sigma_{vec(\hat{B})}$ given by (see appendix A)

$$\beta = E(vec(\hat{\mathbf{B}}_k)) = (B_{01}, B_{11}, ..., B_{p1}, B_{02}, B_{12}, ..., B_{p2}, ..., B_{0q}, B_{1q}, ..., B_{pq})^T \tag{6}$$

and

$$\boldsymbol{\Sigma}_{vec(\hat{B}_k)} = ((\mathbf{I}_q \otimes \mathbf{X})^T \mathbf{V}_0^{-1} (\mathbf{I}_q \otimes \mathbf{X}))^{-1}, \tag{7}$$

respectively. Lowry et al. (1992) presented a multivariate exponentially weighted moving average (MEWMA) control chart to monitor the regression parameters. In this paper, their control chart is utilized in order to monitor the fixed effects.

The control chart statistic for k^{th} profile is a T^2 given by

$$\mathrm{T}^2_{Z_k} = \boldsymbol{Z}_k^T \boldsymbol{\Sigma}_Z^{-1} \boldsymbol{Z}_k \tag{8}$$

where

$$\boldsymbol{Z}_k = \lambda(vec(\hat{\mathbf{B}}_k) - \beta) + (1-\lambda)\boldsymbol{Z}_{k-1} \tag{9}$$

is a multivariate normal random vector with covariance matrix

$$\mathbf{\Sigma}_z = \frac{\lambda}{2-\lambda}\mathbf{\Sigma}_{vec(\hat{\mathbf{B}}_k)} \,. \tag{10}$$

The parameter λ is the smoothing parameter satisfying $0 < \lambda \leq 1$, and $\mathbf{Z}_0$ is a $((p+1)q)\times 1$ vector with zero entries. The upper control limit for the MEWMA chart is selected to achieve a specified in-control ARL (also denoted as ARL $_0$).

3-2. MEWMC method

Hawkins et al. (2008) proposed a multivariate exponentially weighted moving covariance (MEWMC) chart to detect both increases and decreases in marginal variability of a multivariate normal process. This property is crucial when some components exhibit variance increases while the others possess compensating decreases. To describe more, consider the out of control situation for covariance matrix in which increasing the largest eigenvalue is accompanied by decreasing the smallest eigenvalue with the same factor. Consequently, generalized covariance matrix remains unaltered which leads to undetectable changes. The proposed chart is based on the assumption of individual observations, however an extension to *n*-observation is presented here for our multivariate multiple profile monitoring scheme.

The proposed MEWMC chart is based on the multi-standardized data whose in-control distribution, when process is in control is $N_{nq}(0,\mathbf{I}_{nq})$. For this purpose, we should firstly find a matrix ***A*** with the property $\mathbf{A}\mathbf{V}_0\mathbf{A}^T = \mathbf{I}_{nq}$ and then transform the process readings as $vec(\mathbf{U}_k) = \mathbf{A}(vec(\mathbf{Y}_k) - vec(\boldsymbol{\mu}_0))$ where $vec(\boldsymbol{\mu}_0)$ is in control values of $vec(\mathbf{XB})$. Matrix **A** can be defined as inverse Cholesky root matrix of $\mathbf{V}_0$. Unlike most of the other control charts which are only based on either the trace or the determinant, the MEWMC statistic is defined such that it has the advantages of both operators as follows.

$$C_k = tr(\mathbf{S}_k) - \log|\mathbf{S}_k| - q \,, \tag{11}$$

where

$$\mathbf{S}_k = \lambda vec(\mathbf{U}_k)vec(\mathbf{U}_k^T) + (1-\lambda)\mathbf{S}_{k-1} \tag{12}$$

and $vec(\mathbf{U}_k)$ is a $nq \times 1$ random vector in the k^{th} profile, $\mathbf{S}_0 = \mathbf{I}_{nq}$ and λ is smoothing parameter satisfying $0 < \lambda \le 1$.

A signal is given at profile k if $\mathbf{C}_k$ is greater than a pre-specified control limit that is set off to achieve a specific ARL_0 value.

Hawkins et al. (2008) also suggest to employ the MEWMA alongside the MEWMC to monitor the mean vector, the covariance matrix, or both. The combined chart, namely MEWMAC involves maintaining a MEWMA and a MEWMC and signalling when either of these charts crosses its control limit.

4. Simulation study

In order to evaluate performance of the proposed control charts, a multivariate multiple numerical example utilized by Eyvazian et.al. (2011) is considered here where the within correlation structure among observations is added to the example. The resulted model contains the random effects as follows.

$$Y_1 = 3 + 2X_1 + X_2 + Z_1 b_{11} + Z_2 b_{12} + \varepsilon_1,\ Y_1 = 2 + X_1 + X_2 + Z_1 b_{21} + Z_2 b_{22} + \varepsilon_2 \tag{1}$$

A sample of four observations for y_1 and y_2 are generated using the pairs of (2,1), (4,2), (6,3) and (8,2) as values for independent variables (x_1, x_2). Moreover, it is assumed Z is contained within X with columns $\mathbf{Z} = \begin{bmatrix} 2 & 1 \\ 4 & 2 \\ 6 & 3 \\ 8 & 2 \end{bmatrix}$. It is also assumed $vec(\mathbf{b}) \sim N_{2\times 2}(0, \boldsymbol{\phi} \otimes \mathbf{I}_2)$ with

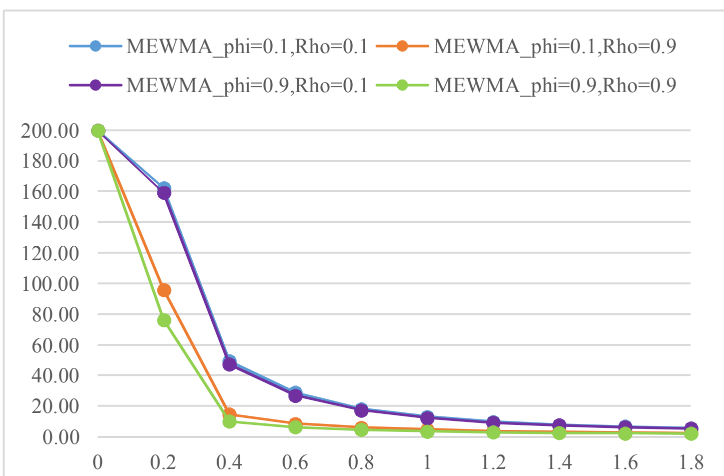
$\boldsymbol{\phi} = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.1 \end{pmatrix}$ and vector $(\varepsilon_1, \varepsilon_2)$ has bivariate normal distribution with mean zero and covariance matrix $\boldsymbol{\Sigma}_\varepsilon = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_2\sigma_1 & \sigma_2^2 \end{pmatrix}$ where $\sigma_1 = \sigma_2 = 1$. To evaluate the effect of different correlation values on response variables, two ρ values 0.1 and 0.9 are taken into account for weak and strong correlation, respectively. Also an AR(1) structure is deployed to regard correlation among observations in each profile. To investigate the autocorrelation effects on our simulation, two different values for the autocorrelation coefficient φ_1 is regarded, $\varphi_1 = 0.1$ for weak and $\varphi_1 = 0.9$ for strong correlations, respectively.

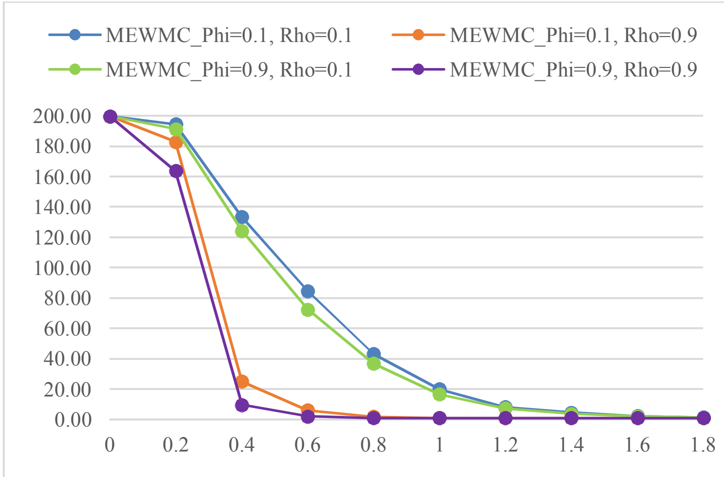
The value of smoothing constant in MEWMA chart, namely λ is set to 0.2. In this paper, all control chart schemes are designed to have an in-control ARL of approximately 200 and each ARL value is estimated using 5000 replications. The upper control limits (UCL) of MEWMA, MEWMC are 17.5 and 9.28, respectively.

Table reports the simulated out-of-control ARL values when β_{01} shifts to $\beta_{01} + \lambda\sigma_1$ for different between-responses and within-profile correlations. This table reveals that under constant correlation coefficient between responses, the control charts performance gets better when the autocorrelation coefficient increases. Moreover, all control charts have a better performance under the conditions where the autocorrelation is strong compared to the case when autocorrelation is weak. In addition, when correlation among responses is weak, the MEWMA performs better under small shifts ($\lambda < 1.4$), however for strong correlation level ($\rho = 0.9$) the results are different. In this condition, the MEWMC is superior to the MEWMA for $\lambda > 0.8$ when WPC coefficient is small and for $\lambda > 0.4$ when WPC coefficient is large. Moreover, as we expected the combined method reveals that the OOC is faster than the other two methods.

Table1. The simulated out-of-control ARL values under the shifts from β_{01} *to* $\beta_{01}+\lambda\sigma_1$

Control Chart	parameters value	λ 0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
MEWMA		162.39	89.94	49.55	29.02	18.33	13.31	10.03	8.00	6.82	5.76
MEWMC	$\rho=0.1$ $\varphi=0.1$	194.66	178.48	133.54	84.85	43.19	19.97	8.36	4.61	2.41	1.51
MEWMAC		88.23	60.44	38.52	22.34	13.88	8.60	5.42	3.31	2.19	1.45
MEWMA		96.11	32.73	14.78	8.81	6.28	4.92	4.03	3.46	3.04	2.72
MEWMC	$\rho=0.9$ $\varphi=0.1$	182.99	93.48	25.10	6.09	2.04	1.14	1.01	1.00	1.00	1.00
MEWMAC		65.76	24.50	10.38	4.22	1.83	1.12	1.01	1.00	1.00	1.00
MEWMA		159.48	86.22	47.32	27.22	17.63	12.51	9.42	7.65	6.47	5.52
MEWMC	$\rho=0.1$ $\varphi=0.9$	191.49	169.52	124.56	72.64	36.92	16.67	7.45	3.74	2.14	1.40
MEWMAC		91.36	58.65	36.41	21.22	12.52	7.92	4.74	3.02	1.90	1.37
MEWMA		76.44	21.52	10.17	6.46	4.77	3.81	3.20	2.77	2.46	2.23
MEWMC	$\rho=0.9$ $\varphi=0.9$	163.99	54.97	9.65	2.19	1.09	1.00	1.00	1.00	1.00	1.00
MEWMAC		53.26	16.20	5.21	2.00	1.11	1.00	1.00	1.00	1.00	1.00

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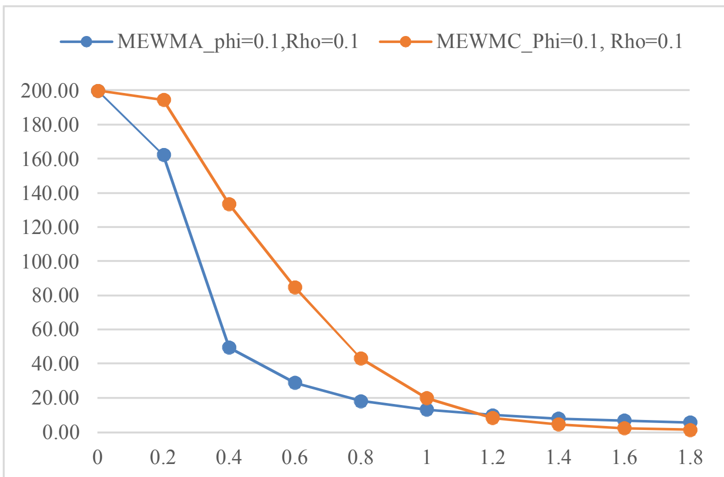
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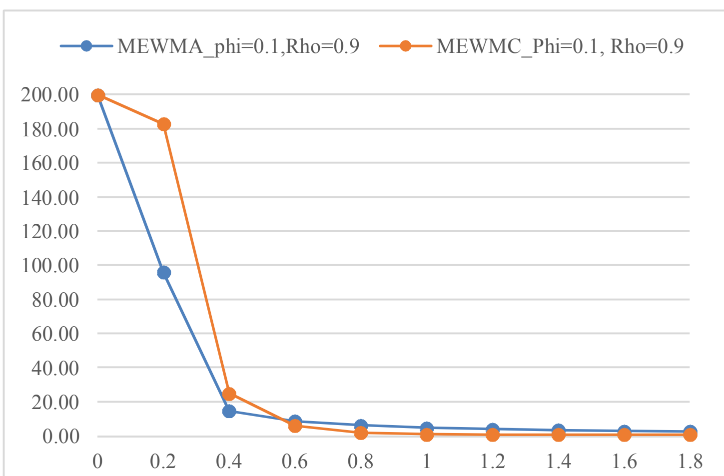
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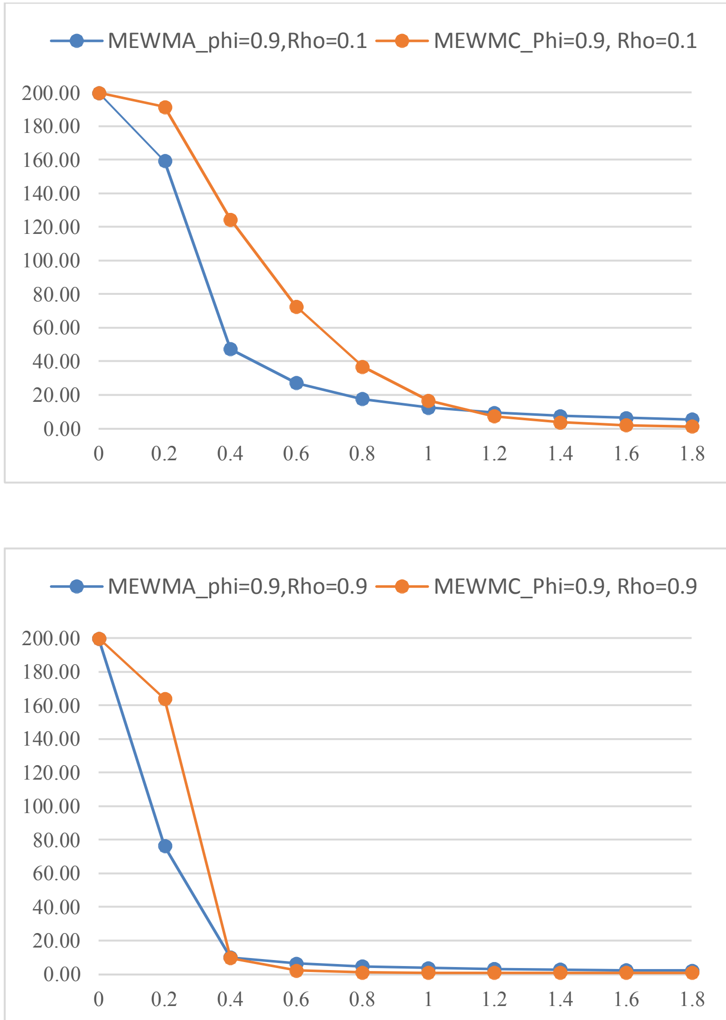

Figure 1. ARL comparisons under shifts in intercept

Figure 1 summarizes the comparison outcomes between out-of-control ARL values under different correlation levels when intercept shifts.

Table 2 denotes the simulated out of control ARL values for the shifts in the slope of the first profile in σ_1 units. The control charts superiority under such shifts are relatively similar to the ones in Table 1, however the control charts have a better performance for the slope shifts than the intercept. For the sake of argument, we avoid inserting the remaining figures corresponding to Tables 2-6. However, they are available upon request.

Table 2. The simulated out-of-control ARL values under the shifts from β_{11} *to* $\beta_{11}+\lambda\sigma_1$

Control Chart	parameters value	λ 0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
MEWMA	$\rho=0.1$	42.64	11.22	5.93	4.08	3.15	2.62	2.25	2.05	1.93	1.80
MEWMC	$\varphi=0.1$	118.50	11.65	1.72	1.01	1.00	1.00	1.00	1.00	1.00	1.00
MEWMAC		32.02	6.89	1.52	1.01	1.00	1.00	1.00	1.00	1.00	1.00
MEWMA	$\rho=0.9$	40.55	10.87	5.82	3.99	3.09	2.56	2.21	2.02	1.90	1.77
MEWMC	$\varphi=0.1$	119.82	11.39	1.59	1.00	1.00	1.00	1.00	1.00	1.00	1.00
MEWMAC		32.07	6.79	1.46	1.01	1.00	1.00	1.00	1.00	1.00	1.00
MEWMA	$\rho=0.1$	38.81	10.43	5.73	3.92	3.04	2.52	2.19	2.01	1.90	1.73
MEWMC	$\varphi=0.9$	113.41	10.63	1.45	1.01	1.00	1.00	1.00	1.00	1.00	1.00
MEWMAC		31.19	6.10	1.45	1.00	1.00	1.00	1.00	1.00	1.00	1.00
MEWMA	$\rho=0.9$	38.56	10.60	5.60	3.88	3.01	2.49	2.18	2.00	1.89	1.72
MEWMC	$\varphi=0.9$	113.11	10.18	1.40	1.00	1.00	1.00	1.00	1.00	1.00	1.00
MEWMAC		29.97	5.87	1.45	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table3. The simulated out-of-control ARL values under the shifts from σ_1 *to* $\gamma\sigma_1$

Control Chart	parameters value	γ 1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8	3
MEWMA	$\rho=0.1$	88.91	45.74	26.72	17.57	12.73	9.66	7.95	6.48	5.55	4.88
MEWMC	$\varphi=0.1$	31.88	4.47	1.59	1.09	1.01	1.00	1.00	1.00	1.00	1.00
MEWMAC		25.40	4.39	1.65	1.10	1.01	1.00	1.00	1.00	1.00	1.00
MEWMA	$\rho=0.9$	73.38	28.08	15.26	9.54	6.63	5.12	4.05	3.45	2.96	2.58
MEWMC	$\varphi=0.1$	8.23	1.24	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00
MEWMAC		7.40	1.20	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
MEWMA	$\rho=0.1$	104.02	54.51	33.91	22.36	16.39	12.32	9.87	8.33	6.94	6.03
MEWMC	$\varphi=0.9$	40.73	6.47	1.97	1.23	1.03	1.00	1.00	1.00	1.00	1.00
MEWMAC		30.35	6.71	2.07	1.22	1.03	1.01	1.00	1.00	1.00	1.00
MEWMA	$\rho=0.9$	82.06	34.21	18.26	11.62	8.29	6.33	5.21	4.23	3.65	3.24
MEWMC	$\varphi=0.9$	9.80	1.42	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00
MEWMAC		8.66	1.43	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 3 shows how the ARL values change when the standard deviation (SD) in the first profile shifts to $\sigma_1+\lambda\sigma_1$. The MEWMC indicates not only significant changes in the ARL values but also has an appropriate performance under SD shifts. As the first row of each correlation set shows, shifts in the SD can create alarms in the MEWMA too, however the MEWMC indicates the OOC condition significantly faster than the MEWMA. In contrast to the previous tables, all control charts perform better under weak within-profile autocorrelation and strong between-response correlation. Moreover, the performance of the control charts gets worse with increasing φ from 0.1 to 0.9.

Table 4, Table 5, and Table 6 report the shifts of the SD in the first profile alongside the shift in the intercept of the first profile, the slope of the first profile, and the SD of the second profile, respectively. It is apparent from Table that both the MEWMA and the MEWMC charts give out-of-control signal under simultaneous shifts. However, the MEWMC has superiority to the MEWMA in all correlation and shift levels. Although for $\rho = 0.1$ increasing the within-correlation coefficient leads to a worse performance in all charts, for $\rho = 0.9$ it results in a better performance. In Table 5 and Table 6, similar to the cases in which σ_1 shifts, the MEWMC is superior to the MEWMA. In both tables, stronger correlation within profiles while $\rho = 0.9$ worsens the performance of the charts.

Table 4. The simulated out-of-control ARL values under the simultaneous shifts from β_{01} to $\beta_{01} + \lambda\sigma_1$ and σ_1 to $\sigma_1 + \lambda\sigma_1$

Control	parameters	λ									
Chart	value	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
MEWMA	$\rho = 0.1$	75.77	31.65	17.08	11.34	8.38	6.56	5.30	4.56	3.94	3.48
MEWMC	$\varphi = 0.1$	30.86	4.24	1.45	1.08	1.01	1.00	1.00	1.00	1.00	1.00
MEWMAC		21.85	3.79	1.49	1.07	1.00	1.00	1.00	1.00	1.00	1.00
MEWMA	$\rho = 0.9$	46.34	15.73	8.44	5.84	4.35	3.49	2.95	2.60	2.28	2.03
MEWMC	$\varphi = 0.1$	7.19	1.24	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
MEWMAC		6.80	1.21	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
MEWMA	$\rho = 0.1$	84.64	34.69	18.89	12.75	9.46	7.42	6.15	5.25	4.55	4.09
MEWMC	$\varphi = 0.9$	37.66	5.57	1.65	1.10	1.01	1.00	1.00	1.00	1.00	1.00
MEWMAC		27.44	4.90	1.76	1.14	1.03	1.00	1.00	1.00	1.00	1.00
MEWMA	$\rho = 0.9$	43.27	13.98	7.98	5.54	4.30	3.58	3.02	2.64	2.43	2.25
MEWMC	$\varphi = 0.9$	8.81	1.21	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00
MEWMAC		7.44	1.26	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 5. The simulated out-of-control ARL values under the simultaneous shifts from β_{11} to $\beta_{11} + \lambda\sigma_1$ and σ_1 to $\sigma_1 + \lambda\sigma_1$

Control Chart	parameters value	λ 0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2
MEWMA	$\rho = 0.1$	30.37	9.15	5.06	3.56	2.78	2.32	2.03	1.82	1.63	1.45
MEWMC	$\varphi = 0.1$	23.10	2.21	1.04	1.00	1.00	1.00	1.00	1.00	1.00	1.00
MEWMAC		14.52	1.99	1.03	1.00	1.00	1.00	1.00	1.00	1.00	1.00
MEWMA	$\rho = 0.9$	27.55	8.20	4.59	3.22	2.50	2.08	1.79	1.59	1.41	1.27
MEWMC	$\varphi = 0.1$	7.10	1.08	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
MEWMAC		6.41	1.09	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
MEWMA	$\rho = 0.1$	30.67	9.11	5.02	3.56	2.78	2.32	2.05	1.83	1.63	1.45
MEWMC	$\varphi = 0.9$	29.24	2.35	1.03	1.00	1.00	1.00	1.00	1.00	1.00	1.00
MEWMAC		15.60	2.18	1.04	1.00	1.00	1.00	1.00	1.00	1.00	1.00
MEWMA	$\rho = 0.9$	28.54	8.33	4.67	3.27	2.59	2.14	1.85	1.64	1.47	1.32
MEWMC	$\varphi = 0.9$	8.43	1.13	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
MEWMAC		7.03	1.14	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 6. The simulated out-of-control ARL values under the simultaneous shifts from σ_1 to $\gamma\sigma_1$ and σ_2 to $\gamma\sigma_2$

Control Chart	parameters value	γ 1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8	3
MEWMA	$\rho = 0.1$	53.12	22.96	12.87	8.42	6.28	4.93	4.00	3.34	2.92	2.58
MEWMC	$\varphi = 0.1$	12.47	1.79	1.06	1.00	1.00	1.00	1.00	1.00	1.00	1.00
MEWMAC		9.97	1.70	1.05	1.00	1.00	1.00	1.00	1.00	1.00	1.00
MEWMA	$\rho = 0.9$	58.73	26.20	14.61	9.79	7.18	5.47	4.49	3.79	3.23	2.86
MEWMC	$\varphi = 0.1$	15.17	1.90	1.07	1.00	1.00	1.00	1.00	1.00	1.00	1.00
MEWMAC		12.56	1.93	1.06	1.00	1.00	1.00	1.00	1.00	1.00	1.00
MEWMA	$\rho = 0.1$	65.42	29.31	16.72	11.09	8.02	6.18	5.03	4.22	3.52	3.06
MEWMC	$\varphi = 0.9$	15.75	2.38	1.10	1.00	1.00	1.00	1.00	1.00	1.00	1.00
MEWMAC		14.17	2.28	1.12	1.00	1.00	1.00	1.00	1.00	1.00	1.00
MEWMA	$\rho = 0.9$	68.45	30.98	17.72	11.89	8.57	6.72	5.30	4.52	3.84	3.40
MEWMC	$\varphi = 0.9$	18.11	2.28	1.13	1.01	1.00	1.00	1.00	1.00	1.00	1.00
MEWMAC		14.78	2.14	1.12	1.01	1.00	1.00	1.00	1.00	1.00	1.00

5. Conclusions

In this paper, we proposed a multivariate linear mixed model to deal with the correlation within the multivariate multiple profiles. In order to monitor the profiles in phase II, two multivariate control charts were introduced. The first one is the MEWMA chart which was employed to detect changes in the fixed effects and the second one is the MEWMC chart to monitor the covariance matrix. Furthermore, for simultaneous monitoring of the process mean and the covariance matrix, the MEWMA along with the MEWMC, were utilized. The performance of the presented control charts were evaluated through the ARL criterion under different correlation levels between responses and also among observations within each profile. The simulations revealed that both MEWMA and MEWMC perform better under presence of strong correlation between responses and strong autocorrelation within profiles when shifts occur in intercept and slope. However, when SD shifts, they have superior performance under strong correlation among responses and weak autocorrelation coefficient. On the other hand, when a shift occurs in the covariance matrix, the MEWMC signals faster than the MEWMA. The main reason for developing the combined method is to monitor process mean vector and covariance matrix, simultaneously. In our problem, since the MEWMA performs better than the MEWMC in detecting small shifts in intercept and slope, therefore it is concluded that when the MEWMAC is performed, the probability of coming OOC signal from the correct chart is more in case of small shifts than the large shifts. However, diagnosing the out-of-control parameters needs diagnostic methods which could be the topic of future studies.

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Appendix A

MLE estimator of the vector B equals to

$$vec(\hat{\mathbf{B}}_k) = [(\mathbf{X}_k \otimes \mathbf{I}_q)^T \mathbf{V}_0^{-1} (\mathbf{X}_k \otimes \mathbf{I}_q)]^{-1} (\mathbf{X}_k \otimes \mathbf{I}_q)^T \mathbf{V}_0^{-1} vec(\mathbf{Y}_k).$$

Let $[(\mathbf{X}_k \otimes \mathbf{I}_q)^T \mathbf{V}_0^{-1} (\mathbf{X}_k \otimes \mathbf{I}_q)] = \mathbf{W}$,

then

$$\hat{\mathbf{\Sigma}}_{vec(\hat{\mathbf{B}})} = E[vec(\hat{\mathbf{B}}_k).vec(\hat{\mathbf{B}}_k^T)] = E[\mathbf{W}^{-1} (\mathbf{X}_k \otimes \mathbf{I}_q)^T \mathbf{V}_0^{-1} vec(\mathbf{Y}_k).(\mathbf{W}^{-1} (\mathbf{X}_k \otimes \mathbf{I}_q)^T \mathbf{V}_0^{-1} vec(\mathbf{Y}_k))^T] =$$
$$\mathbf{W}^{-1} (\mathbf{X}_k \otimes \mathbf{I}_q)^T \mathbf{V}_0^{-1} E[vec(\mathbf{Y}).vec(\mathbf{Y}^T)] (\mathbf{W}^{-1} (\mathbf{X}_k \otimes \mathbf{I}_q)^T \mathbf{V}_0^{-1})^T =$$

$$\mathbf{W}^{-1} (\mathbf{X}_k \otimes \mathbf{I}_q)^T \mathbf{V}_0^{-1} \mathbf{V}_0 (\mathbf{W}^{-1} (\mathbf{X}_k \otimes \mathbf{I}_q)^T \mathbf{V}_0^{-1})^T = \mathbf{W}^{-1} \underbrace{(\mathbf{X}_k \otimes \mathbf{I}_q)^T \mathbf{V}_0^{-1} (\mathbf{X}_k \otimes \mathbf{I}_q)^T}_{\mathbf{W}} \mathbf{W}^{-1} =$$

$$\mathbf{W}^{-1} = [(\mathbf{X}_k \otimes \mathbf{I}_q)^T \mathbf{V}_0^{-1} (\mathbf{X}_k \otimes \mathbf{I}_q)]^{-1}.$$