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A Three-Stage Model for Location Problem Under Fuzzy Environments

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KEYWORDS

Fuzzy environment;
Non-linear mathematical programming;
Multiple objectives.

ABSTRACT

This study contains a three-level mathematical model with a number of suppliers in fixed locations, candidate distribution centers, and affected areas at certain points. A mixed integer nonlinear mathematical programming model is presented here for the open location problem, while a split delivery of demand is considered. In this study, a fuzzy environment is taken into account for our presented model to be in an uncertain environment. The objective is considered for cost minimization, minimization of the maximum travel time of vehicles, and minimization of demands. Finally, to this end, this study uses a fuzzy Multiple Optimal Linear Programming. To make the proposed model and solution approach applicable, numerical examples are provided.

1. Introduction

Unexpected events and natural disasters (floods, earthquakes, hurricanes, etc.) and their consequences require current societies to plan for assistance in such crisis. This plan is filled with challenges such as damage to infrastructure, trans- portation, limited time and resources, difficulties in coordination between different factors, and so on. Therefore, compared to conventional logistics, providing assistance in crisis and emergency logistics is complicated and challenging [24]. In the incidence of natural disasters at the time of critical condition, demand for logistic goods and services increases, and quick distribution of essential facilities can be effective in minimizing the damage and fatal accidents. Therefore, the affected areas shall be supported by various emergency items such as tents, water, etc., which are needed quickly in crisis. Emergency aid processes include the transfer of the needed goods from different suppliers (Red Crescent, airports, local suppliers, etc.) by local distribution centers to the damaged areas. Therefore, one of the important logistic strategies to improve performance and reduce latency is the location and establishing of distribution centers near the affected areas. If distribution centers are situated in appropriate locations from the network, which could cover demand in these conditions appropriately, it would be very important in the successful rescue operation. In all the cases listed, poor selection of suitable locations will increase the probability of capital loss and ultimately will lead to many human losses. Of other logistic activities that are

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time, and customer satisfaction, and they solved it by fuzzy multi-objective programming. In addition, Sheu (2007) investigated a combined fuzzy clustering approach to optimize multi-objective dynamic programming. The weighting method for converting the distribution to one objective is applied to achieve cost minimization and maximization of demand coverage rate.

Ozdamar and Yi (2007) provided an integrated location-distribution model to coordinate logistical operation and unloading in disaster conditions. The purpose of the model was to maximize service levels through immediate access to the affected areas and location of temporary emergency units in appropriate points. The sub-problem of location included the facilitation of limited medical resources and access to balance in the rate of service, among medical centers. Medical staff can move between distribution centers; however, the total number of these people remain fixed over time.

The coverage radius for relief items in locating the humanitarian relief facilities is done in a study conducted by Black and Beamon (2008). One of the main features of the provided model is to consider and apply the budget constraints before and after the disaster. In addition, upper and lower limits were considered for the time of response to the demand by any supply center, and it is suggested that the relief time cannot exceed this limit. Maximizing the total demand covered by constructed distribution centers is the only objective function of the model.

Yi and Kumar (2007) provided ant colony optimization algorithm to solve logistic problems in disaster relief activities in the responding phase. In this study, sending goods to distribution centers deployed in the affected areas and transferring of victims to relief centers have been considered simultaneously. The objective function is to minimize the weighted sum of unmet demand for total goods. In their model, vehicle routes are determined; however, locating distribution centers is not considered. Vitoriano et al. (2011) provided a multi-criterion optimization model based on cost, time, and priority for the distribution of humanitarian relief. This model helps select vehicles and design the routes; however, locating distribution centers is not considered. Lin et al. (2011) provided a multi-period, multi-product and multi-vehicle logistic model for logistical planning of major commodities with priority in disaster response phase. The model has two objectives: the first objective minimizes the unmet demand, and the second one minimizes travel time. Berkoune et al. (2012) presented a mathematical model for planning transportation of goods in the response phase where he tried to minimize the travel time of vehicles carrying goods. Eshghi and Najafi (2013) proposed a multi-objective, multi-product, multi-period, and randomized model to achieve logistical management of relief items and injured people.

Disaster network explained in their research included affected centers, hospitals, and transfer centers of relief items. Objectives of the model include minimization of the total number of non-serviced people, the total number of unmet demands, and the total number of transportation vehicles required. Their investigation took into consideration uncertainty in the sent items, the number of affected persons, and the capacity of suppliers and hospitals. For this purpose, a robust approach was developed in the model to face uncertainty, and a solution was proposed based on hierarchical objective functions.

Bozorgi-Amiri et al. (2013) developed a multi-objective robust stochastic programming model for relief logistic in the conditions of uncertainty. In this research, not only demand was considered, but also supply and purchase and transportation costs were considered as uncertain parameters. Their model includes two stages. The first stage is concerned with determining the distribution center locations and required an inventory of any relief items under storage, and the second stage is concerned with determining the level of goods transferred from relief distribution centers to affected areas. Their model is based on the assumption that disaster information does not depend on time and routing the vehicles.

Wang et al. (2014) provided a multi-objective model for open locating-routing problem for distribution after the earthquake. The considered disaster network in their study included distribution centers and affected areas. In the presented model, emergency repair of roads and damaged communication channels were not considered. They used a non-dominated sorting genetic algorithm (NSGA-II) to solve the model. Zhan et al. (2014) provided the vehicle allocation problem in relief logistic to ensure efficiency and equity in the decision-making process about issues such as vehicle routing and allocation of relief. The considered network is a two-echelon distribution centers to critical areas is designed for rapid distribution of the emergency aid. Split delivery of demand required as demand in the critical area is larger than the capacity of the vehicle, and each critical area can be served more than once and by different vehicles. Heterogeneous vehicles are considered at different speeds and capacities. It should be noted that any vehicle is allowed to transport multiple types of assistance to each allocation, and various types of aid are allowed at the same time in one vehicle load. In addition, after the completion of operation when the vehicles serve the last node of the route, they do not need to return to their origin. Therefore, the route for the vehicles is considered open. Intended objectives in the problem include the minimization of the total cost including fixed cost of creating distribution centers, travel cost of the vehicle, and costs of goods transported from suppliers to distribution centers. The second objective is the minimization of maximum travel time on the route (maximum travel time means the latest completion time of the service among all critical areas), and the third objective is to minimize unmet demand.

Tab. 1. Sets and indices

H	Set of suppliers 1,…,h
N	Set of disaster areas 1, . . . , n
M	Set of candidate DCs n+ 1,…,n+ m V Set
of node 1,…,n+ m	
K	Set of vehicles 1,…,k
L	Set of relief 1,….,l
E	Set of available traffic links (i,j), i,j v, i = j i, j
	Indices of nodes i,j v
l	Indices of relief
k	Indices of vehicles

Tab. 2. Parameters

$\tilde{f}_i$	Fixed cost of establishing DC i, $\forall i \in M$
e_{ij}	Distance of link (i, j), $\forall (i, j) \in E$
S_{hil}	Transportation cost per unit of relief l from supplier h to distribution i $\tilde{D}_{il}$
	Quantity of relief l demanded by disaster area i
sv_l	Unit volume of relief l, $\forall l \in L$
O_{hl}	Amount of relief l available in supplier h
$\tilde{Q}_{il}$	Maximum capacity of the distribution center i from relief l $\tilde{c}_k$
	Transportation cost per kilometer of vehicle k
v_k	Normal speed of terrestrial vehicle k
CA_k	Loading capacity of terrestrial vehicle k

Tab. 3. Decision variables

y_i	1, if candidate DC i is opened, 0, else, $\forall i \in M$
x_{ijk}	1, if i precedes j in route of vehicle k, 0, else
R_{ijk}	1, if i is on route of vehicle k, 0, else
P_{ik}	1, if the last demand point serviced by vehicle k is node i $\in N$; 0,else W_{hil}
	Quantity of relief l transported from supplier h to distribution center i dev_{il}
	Amount of unsatisfied demand relief type l at node i at the end of the
operation	
q_{jlk}	Quantity of relief l distributed by k to demand point i

3-1. Mathematical model

$$z \quad Min \sum_{i \quad M} \tilde{f}_i y_i \sum_{k \quad K} \sum_{(i,j) \quad E} \tilde{c}_k d_{ij} x_{ijk} \sum_{h \quad H} \sum_{i \quad M} \sum_{l \quad L} \tilde{S}_{hil} Whil \tag{1}$$

$$z_2 \quad \min\max \sum_{(i,j) \quad E} \frac{e_{ij} x_{ijk}}{v_k}, k \quad K \tag{2}$$

$$p_{ik} \quad (0,1), \qquad i \quad N, k \quad K \qquad (31)$$

$$u_{ik} \quad (0,1), \qquad i \quad N, k \quad K \qquad (32)$$

Equation (1) is the first objective function that minimizes the distribution costs including the fixed costs of creating distribution centers, travel expenses of the vehicles, and the cost of transporting goods from suppliers to distribution centers.
Equation (2) as the second objective function minimizes the maximum travel time of the vehicles. The objective function (3) minimizes the total unmet demand.
Constraints (4) and (5) specify that only established distribution centers can obtain service. Constraint (6) ensures that every vehicle can travel through connection (i, j), if and only if node i is on the route of each vehicle. Constraint (7) specifies that the nodes at the end of the route of each vehicle must be serviced by the same vehicle. Equation (8) ensures that every vehicle must ultimately remain in a disaster area or distribution center. Constraint (9) shows that only one vehicle is selected for each route. Constraint (10) ensures that any vehicle serves once at most in any critical area. Constraint (11) ensures that any vehicle is sent from one distribution center at most. Constraint (12) ensures that the amount of aid transferred by any supplier of any goods to all distribution centers does not exceed the maximum amount. Constraints (13) and (14) are capacity constraints of distribution centers. Constraint (15) shows that the amount of relief distributed to each node does not exceed the amount demanded by that node. Constraint (16) ensures that there will be no shortage of goods. Constraint (17) ensures that the amount of all the relief distributed to disaster areas by a vehicle does not exceed their capacity. Constraint (18) ensures that every disaster area can be visited at least once. The assumption of split delivery in this constraint has been well illustrated. Constraints (19) - (24) are the limits of maintaining the flow, which also ensures the openness assumption of the routes (ensuring that each vehicle at any point is dispatched from that point, and the last node of the route does not return to the distribution center). Constraint (25) ensures that distribution centers are not related to each other. It means that goods are not exchanged between distribution centers. Constraint (26) is the constraint of elimination sub-tours. Constraints (27)-(32) correspond to non-negative integer values and numbers of zero, and one for decision variables. The model was presented given the certainty of parameters in Sections 3-5. In the real world, there is uncertainty in many of these parameters. To bring the model closer to real conditions in the future, the model has also been expanded in non-deterministic conditions. To develop the model, a robust optimization approach is used.

3-2. Uncertainty approach.

With respect to the above-mentioned consideration, a mixed integer programming model with fuzzy parameters is proposed. Next, the proposed model, by virtue of a new technique based on the possibilistic method [13, 21, 27], is converted to its commensurate deterministic version.
The commensurate adjuvant crisp model:
Suppose that $\tilde{c}$ is a triangular fuzzy number (TFN); Eq. (33) is the membership function of $\tilde{c}$:

$$_{c}(X) \quad \begin{cases} f_c(X) \quad \dfrac{x \quad c^P}{c^m \quad c^P} & if \quad c^p \quad x \quad c^m \\ 1 & if \quad x \quad c^m \\ g_c(X) \quad \dfrac{c^0 \quad c}{c^o \quad c^m} & if \quad c^m \quad x \quad c^p \\ 0 & if \quad c^p \quad x \quad or \quad x \quad c^p \end{cases} \qquad (33)$$

In the following FMP model, all parameters are defined as TFNs. The commensurate crisp-parametric problem can be written as follows (Jimenez et al., 2007). Thus, by considering these issues, the model will be presented as follows:

validity and usefulness of the model and the solution approach, several numerical tests are run, and the results are presented in this section. To this end, four different problems with different aspects were considered. The information related to the dimensions is shown in Table 1, and the information about the pa- rameters of the model is shown in Table 2. It should be noted that to generate the triangular fuzzy parameters according to Lai and Hwang (1992), three prominent points are obtained for each imprecise parameter. The most likely (C^m) value of each parameter is first provided randomly by utilizing the uniform distributions specified in Table 2. Thus, without loss of generality, two random numbers (r_1, r_2) are generated between 0.2 and 0.8 by applying uniform distribution. All problems considered in both deterministic and non-deterministic conditions were solved by software GAMS version 23.6 and Baron Solver.

Tab. 4. Dimensions of the Problem

Test problems	(h)	(M)	(N)	(K)	(L)
1	2	3	11	3	2
2	3	4	10	4	2
3	2	2	8	3	2
4	2	3	6	3	2

Tab. 5. Model Parameters

Parameters	Values
f_i	~uniform(10000,30000)
e_{ij}	~uniform(60,250)
O_{hl}	~uniform(14000,24000)
Q_{il}	~uniform(9000,12000)
D_{jl}	~uniform(800,2500)
V_k	~uniform(70,90)
CA_k	~uniform(23,39)
uv_l	~uniform(0.0123, 0.028)
c_k	uniform(3,5)
s_{hil}	uniform(8,10)

As is clear, to solve the problem in deterministic condition, distribution centers (1) and (2) are opened, to which the supplies of collected aid are sent, and vehicles tailored to the track status and demands of critical areas are assigned to distribution centers. As observed, the route is open for all vehicles, and they do not return to the distribution center. In addition, due to high-demand critical areas (7) and (8) that are larger than the remaining capacity of Vehicle 3 in the first stage of service, some of the remaining demand in Area 8 is met by Vehicle 1 in the next stage. Area 7 will meet again by vehicle 2. With careful consideration of non-deterministic condition, centers (1) and (3) have been opened for providing aid, and the critical area (3) due to high demand has been met in three stages by various vehicles.

The computational results are summarized in Tables 3 and 4 in both determin- istic and non-deterministic conditions based on three levels of (0.1, 0.3, and 0.5) and various degrees of importance for the objective functions. The value for levels of uncertainty for all model parameters in each stage of the implementation is considered constant, and this value is ρ = 0 for certain models. In addition to the impact of penalty coefficient (ϕ) on objective functions in both deterministic and non-deterministic conditions, sensitivity analysis was conducted; because of the required time for this analysis, it was conducted only on two problems, the results of which are shown in Table 5.

Tab. 9. Sensitivity analysis of (α) based on ϕ= 0.4

Test problem	Fuzzy possibilistic			
	(θ_1, θ_2, θ_3)	(z_1, μ_1)	(z_2, μ_2)	(z_3, μ_3)
	(0.3,0.3,0.4)	(426795.16,0.90)	(13.24, 0.71)	(4290.26, 0.88)
1	(0.3,0.4,0.3)	(454297.23,0.74)	(13.10, 0.72)	(4480.21, 0.66)
	(0.4,0.3,0.3)	(384616.9, 0.92)	(14.96, 0.48)	(4623.10, 0.69)
	(0.2,0.4,0.4)	(12.9 , 0.72)	(3397.4, 0.81)	(4099.8 , 0.72)
	(0.3,0.3,0.4)	(621713.4, 0.58)	(10.34, 0.49)	(2891.8 , 0.80)
2	(0.3,0.4,0.3)	(681282.9, 0.36)	(8.53 , 0.73)	(3341.51 , 0.72)
	(0.4,0.3,0.3)	(594781.7, 0.73)	(10.79, 0.64)	(3762.11, 0.58)
	(0.2,0.4,0.4)	(719156.2, 0.60)	(8.02 , 0.77)	(2678.05, 0.67)
	(0.3,0.3,0.4)	(355884.65,0.72)	(7.87 , 0.69)	(3941.5, 0.71)
3	(0.3,0.4,0.3)	(384946.50,0.56)	(6.51 , 0.74)	(4185.41,0.83)
	(0.4,0.3,0.3)	(323858.53,0.65)	(8.04 , 0.65)	(4518.2, 0.75)
	(0.2,0.4,0.4)	(465421.41,0.34)	(5.98, 0.70)	(3890.27,0.78)
	(0.3,0.3,0.4)	(235017.5, 0.82)	(4.37, 0.70)	(3889.2, 0.81)
4	(0.3,0.4,0.3)	(268916.31,0.58)	(3.56 , 0.86)	(4075.21, 0.75)
	(0.4,0.3,0.3)	(203864.19,0.64)	(5.29 , 0.64)	(4316.81, 0.70)
	(0.2,0.4,0.4)	(315261.75,0.45)	(3.15, 0.75)	(3463.85, 0.85)

According to the computational results presented in Tables 3 and 4, it can be seen that all uncertain problems have answers worse than certain problems. In addition, it can be concluded based on the results in Table 4 that TH method acquires unique solutions for every different degree of importance for the objective functions. In general, it can be said that TH is a good and eligible method for planning multi-objective problems, because it can achieve effective and efficient solutions.

Tab. 10. Results of sensitivity analysis on ϕ-value for problems based on α = 0.3 and θ = (0.3, 0.3, 0.4)

Test problem	Deterministic			
	ϕ	(z_1, μ_1)	(z_2, μ_2)	(z_3, μ_3)
	0.1	(325031.5,0.95)	(9.42, 0.82)	(3586.90,0.68)
	0.2-0.4	(346923.2 ,0.94)	(12.70,0.77)	(3404.43,0.73)
1	0.5-0.7	(377635.1,0.87)	(13.61,0.69)	(3249.68,0.86)
	0.8,0.9	(418583.5,0.75)	(14.27,0.62)	(3186.16,0.92)
	0.1-0.3	(499309.2,0.85)	(6.07,0.94)	(2270.9,0.79)
	0.4-0.6	(512639.7,0.79)	(2142.38,0.85)	(8.74,0.86)
2	0.5-0.8	(547687.5,0.72)	(9.80,0.75)	(1931.74,0.89)
	0.9	(590369.2,0.64)	(11.26,0.62)	(1845.31,0.95)

Tab. 11. Results of sensitivity analysis on ϕ-value for problems based on α = 0.3 and θ = (0.3, 0.3, 0.4)

Test problem	Fuzzy possibilistic			
	ϕ	(z_1, μ_1)	(z_2, μ_2)	(z_3, μ_3)
	0.1	(402687.5,0.93)	(10.92,0.79)	(4383.7,0.62)
	0.2-0.4	(426795.2,0.90)	(13.24,0.71)	(4290.2,0.68)
1	0.5-0.7	(459456.7,0.82)	(14.58,0.64)	(4133.6,0.79)
	0.8,0.9	(499614.7,0.71)	(15.03,0.55)	(4058.6,0.84)
	0.1-0.3	(589369.3,0.63)	(8.64, 0.56)	(2974.2,0.69)
	0.4-0.6	(621713.4,0.58)	(10.34,0.49)	(2891.8,0.75)
2	0.5-0.8	(653091.8,0.51)	(11.50,0.43)	(2704.5,0.82)
	0.9	(689769.9,0.46)	(12.44,0.38)	(2517.6,0.90)

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