A Nadir Compromise Programming for Supplier Selection Problem under Uncertainty

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**Key Words**
Multi-objective programming,
Supplier selection,
Nadir compromise programming,
Stochastic programming.

**Abstract**
Supplier selection is one of the influential decisions for effectiveness of purchasing and manufacturing policies under competitive conditions of the market. Regarding the fact that decision-makers (DMs) consider conflicting criteria for selecting suppliers, multiple-criteria programming is a promising approach to solve the problem. This paper develops a nadir compromise programming (NCP) model for decision-making under uncertainty on the selection of suppliers within the framework of binary programming. Depending on the condition of uncertainty, three statuses are taken into consideration, and a solution approach is proposed for each status. A pure deterministic NCP model is presented for solving the problem in white condition (certainty of data), and a solution approach which is resulted from the combination of NCP and stochastic programming (SP) is introduced to solve the model in black (uncertainty of data) situation. The paper also proposes a NCP model under certainty and uncertainty for solving problem under grey (a combination of certainty and uncertainty of data) conditions. The proposed approaches are illustrated for a real problem in steel industry with multiple objectives. In addition, a simulation approach has been designed in order to examine the results obtained and verify capabilities of the proposed model.

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1. Introduction

The contemporary supply chain management is to maintain long-term partnership with suppliers and use fewer, yet reliable suppliers. Therefore, choosing the right suppliers involves much more than scanning a series of price lists, and choices will depend on a wide range of criteria [1]. Today, while keeping a good relation with each other, supply chains require a higher-level performance of their suppliers. Supply chains select their suppliers methodically; thus, the supplier selection becomes a very important strategic decision [2]. In general, supplier
selection is one of the most critical activities in supply chains because of the key role of supplier’s performance on cost, quality, delivery, and service in achieving the objectives of chains [3], such as automobile manufacturing [4], chemical industry [5], construction [6], hospitals [7], and telecommunications [8].

Supplier selection is a multiple criteria decision-making (MCDM) problem, which is affected by several conflicting factors [9, 10]. Consequently, a purchasing manager must analyze the trade-off between several criteria. Some researchers have tried to model and optimize multiple criteria supplier selection problem, most of which are based on deterministic approaches, e.g., see Pan [11], Chaudhry et al. [12], Crama et al. [5], Degraeve et al. [13], and Arunkumar et al. [14]. However, selection of suppliers in the global context under stochastic and non-deterministic fluctuations of data, such as quality and flexibility, has become critical and complex in today’s world. Hence, some of researches have been focused on modelling and solving the supplier selection problem under uncertainty conditions which are as follows:

Li et al. [15] studied the selection problem of contract suppliers in which the buying firm faces non-stationary stochastic price and demand. In addition, they developed a stochastic dynamic programming model to incorporate mixed strategies, purchasing commitments, and contract cancellations. Computational results of this study show that an increase in price (demand) uncertainty favors long-term (short-term) suppliers. Lam et al. [16] investigated a selection model based on fuzzy principal component analysis for solving the material supplier selection problem from the perspective of property developers. Hamdan and Cheaitou [17] presented a decision making tool to solve a multi-period green supplier selection and order allocation problem based on AHP, fuzzy TOPSIS, and multi-objective optimization. Awasthi et al. [18] considered supplier selection problem for a single manufacturer/retailer who faces a random demand. The objective was to find a low-cost assortment of suppliers, capable of satisfying the demand. Yang et al. [19] presented a stochastic demand multi-product supplier selection model with service level and budget constraints using a genetic algorithm. Results of this study show that the optimal value for the return on investment and the expected profit are obtained with a certain budget and service level constraint. Zhang and Zhang [20] proposed a supplier selection and purchase problem under stochastic demand, so that the objective was to select suppliers and allocate the ordering quantity properly among the selected suppliers to minimize the total cost, including selection, purchase, holding, and shortage costs.

The problem was modelled as a mixed integer programming and solved by branch-bound algorithm and a proposed algorithm. In another study, Torabi et al. [21] proposed a bi-objective mixed possibilistic, two-stage stochastic programming model to address supplier selection and order allocation problem to build the resilient supply based on operational and disruption risks so that a five-step method could be designed to solve the problem. Stochastic programming (SP) deals with a class of optimization models and algorithms, in which all or some of the parameters may be subject to significant uncertainty. The models of SP yield plans that are better and able to hedge against losses and catastrophic failures [22].

Ufuk Bilsel and Ravindran [23] presented a multi-objective stochastic sequential supplier allocation model to solve the supplier selection problem. Liao and Rittscher [24] extended a multi-objective supplier selection model under stochastic conditions. In this study, the stochastic supplier selection is determined with simultaneous consideration of the total cost, the quality rejection rate, the late delivery rate, and the flexibility rate. Hammami et al. [25] considered a buyer with multiple sites sourcing a product from heterogeneous suppliers and addressed both the supplier selection and purchased quantity decision. They modelled the problem using the mixed integer scenario-based stochastic programming method. In this model, the objective was to minimize the total system expected cost, including purchased price, inventory cost, transportation cost, and suppliers management cost. Guo and Li [26] investigated an integrated supplier selection and inventory control problems in supply chain management by developing a mixed integer nonlinear programming model for a multi-echelon system under stochastic conditions. In another study, Babbar and Hassanzadeh Amin [27] developed a novel mathematical model to select a set of suppliers and assign the order quantity. In this study, the proposed model comprises two phases: a two-stage QFD and a stochastic multi-objective mathematical model.

The stochastic (scenario) approach helps manage the uncertainty in the order allocation process,
and trapezoidal fuzzy numbers are utilized to handle the vagueness in human thoughts.

Elahi et al. [28] proposed a fuzzy compromise programming for multi-objective supplier selection problem so that group decision-makers’ preferences could be taken into account, and the weight of each criterion was measured by forming pair-wise comparison matrices. Morovati Sharifabadi et al. [29] investigated the application of fuzzy Delphi in order to identify important factors in selecting a supplier in the steel industry and applied the interpretive structural modeling for supplier selection. Khalilzadeh et al. [30] proposed a fuzzy multi-objective model to allocate order to suppliers in uncertainty conditions. In this study, fuzzy TOPSIS was used to obtain supplier’s weights in the objective functions, and multi-objective imperial competitive optimization algorithm was applied to solve the model. Paydar et al. [31] proposed a MCDM approach to evaluate and select sustainable suppliers. They applied the failure mode and effects analysis (FMEA), as a risk analysis technique, to consider supplier’s risk in combination with the MCDM method. This study operates in two main stages. In the first stage, the score of the suppliers was obtained by integration fuzzy MOORA and FMEA. In the second stage, the output of the previous stage was used as input parameters in the developed mixed integer linear programming to select suppliers and order optimum quantity. Despite the applications of fuzzy logic to the decision-making process under uncertainty, due to subjective judgments of decision-makers (DMs) and use of linguistic expressions in the fuzzy programming, the final results may be error-prone.

Based on the literature reviewed and Ho et al. [1] surveys, multi-objective programming is one of the most important tools for solving the supplier selection problem. Multi-objective decision-making refers to determining, prioritizing, and optimizing a set of objectives under a solution space. Yet, multi-objective programming has many applications in fields, such as the internet, finance, biomedicine, management science, game theory, and engineering [32]. Over the years, some multi-objective decision-making methods have been proposed. The nadir compromise programming (NCP) [33] is one of the multi-objective mathematical programming models. This model allows, taking several objectives of a problem into account simultaneously, to choose the most satisfactory solution within a set of feasible solutions. More precisely, the NCP is designed to find a solution that maximizes the deviations between achievement level of the objectives and their nadir (anti ideal) values set. Based on Chai et al. [34] surveys, in 10.57% and 1.63% of researches related to the supplier selection, respectively, multi-objective programming and SP has been used for solving this problem. This shows a growing trend in the application of multi-objective programming techniques in solving supply selection problems and the lack of adequate studies on these problems in uncertain conditions. In another study, Peidro et al. [35] vastly surveyed a review of the literature related to supply chain-planning methods under uncertainty. The main objective of their work was to provide the reader with a starting point for modelling supply chain under uncertainty and applying quantitative approaches. They enumerated research on new methods for uncertainty modeling in supply chain problems as one of the areas for further investigation. Therefore, the present paper considers the selection supplier problem within the framework of binary programming (BP) under three decision-making assumptions of white (with certain data), black (with uncertain data), and grey (certain and uncertain data). NCP model is proposed for the problem of selecting supplier under white conditions, and a proposed approach with a combination of NCP and SP (that is called NCSP model) is presented for black conditions. In addition, for the problem of selecting supplier under grey conditions, models of NCP and NCSP are applied.

In short, the proposed approach of this paper has the following capabilities:

- Simplicity of modelling the supplier selection problem in a framework of BP and SP.
- Extension of the supplier selection problem under uncertainty conditions based on the SP.
- Conversion of the uncertainty problem into a certainty problem using NCSP model.
- Solving the supplier selection problem in a situation where simultaneously some data are deterministic and other some are non-deterministic.

This paper is organized as follows: Section 2 presents the NCP model. Section 3 expresses the selection problem of supplier in three statuses of white, black, and grey. The approaches to solving the problem under three assumptions are proposed on a separate basis in this section. In order to illustrate the proposed approach, a numerical example of supplier selection in stochastic environment is presented in Section 4. Finally, Section 5 presents the conclusions.
2. The NCP Model

Amiri et al. [33] proposed the NCP model. It consists of maximizing the distance between achievement levels and nadir values associated with each objective. If objective $k$ for $k = 1, \ldots, K$ be maximized, then nadir values ($f_{k*}$) can be obtained as follows:

$$\min f_k(x) \quad k = 1, \ldots, K,$$

subject to

$$x \in S,$$

where $x$ is a vector of decision variables, and $S$ is solution space. The final model of NCP by considering preference weights of objectives ($w_k$) is formulated as follows:

$$\max \left\{ \sum_{k=1}^K w_k \lambda_{k}^{P} \right\}^{\frac{1}{P}},$$

subject to

$$f_k(x) - \lambda_k = f_{k*}, \quad k = 1, \ldots, K,$$

$$x \in S,$$

where $P$ is the parameter of final utility function which can have values of metrics $\{1, 2, \ldots\} \cup \{\infty\}$. Also, if the objective $l$ for $l = 1, \ldots, L$ be minimized, then the nadir values ($f_{l*}$) can be obtained as follows:

$$\max f_l(x) \quad l = 1, \ldots, L,$$

subject to

$$x \in S.$$

The final model of NCP by considering preference weights of objectives ($w_l$) is formulated as follows:

$$\max \left\{ \sum_{l=1}^L w_l \lambda_{l}^{P} \right\}^{\frac{1}{P}},$$

subject to

$$f_l(x) + \tau_l = f_{l*}, \quad l = 1, \ldots, L,$$

$$x \in S.$$

Generally, if we maximize $K$ objective functions and minimize $L$ objectives, the final model of NCP can be written as follows [33]:

$$\max \left\{ \sum_{k=1}^K w_k \lambda_{k}^{P} + \sum_{l=1}^L w_l \tau_{l}^{P} \right\}^{\frac{1}{P}},$$

subject to

$$f_k(x) - \lambda_k = f_{k*}, \quad k = 1, \ldots, K,$$

$$f_l(x) + \tau_l = f_{l*}, \quad l = 1, \ldots, L,$$

$$x \in S.$$

where $\sum_{k=1}^K w_k + \sum_{l=1}^L w_l = 1$ ($w_k, w_l > 0$, for $k = 1, \ldots, K$ and $l = 1, \ldots, L$).

In the next section, the NCP model is used in the proposed approach under uncertainty.

3. Supplier Selection Problem

Consider a decision matrix for supplier selection problem as follows:

$$| A_1 | A_2 | \ldots | A_n |
\begin{array}{cccc}
C_1 & x_{11} & x_{12} & \ldots & x_{1n} \\
C_2 & x_{21} & x_{22} & \ldots & x_{2n} \\
& \vdots & \vdots & \ddots & \vdots \\
C_m & x_{m1} & x_{m2} & \ldots & x_{mn}
\end{array}
$$

Fig. 1. A matrix for supplier selection

In Fig. 1, the rows and columns show criteria and alternatives of the problem, respectively. Consider the alternatives independent, and the criteria as positive or negative. $x_{ij} \quad i = 1, \ldots, m, \quad j = 1, \ldots, n$ is given value assigned to alternative $i$, considering criterion $i$.

3.1. Supplier selection under white conditions

In this section, we suppose that the collected data for supplier selection are known. We name this case as ‘white’ that is suitable for and reliable on our decision, and we propose an algorithm to solve the supplier selection problem under white conditions as follows:

Step 1: Constructing the decision matrix

Consider decision matrix of Fig. 1.

Step 2: Constructing the Binary Multi-objective (BM) problem.

In this step, BM problem is constructed. For each criterion, one objective function is defined with binary variables, where coefficients of these variables in objectives are the row entries of decision matrix of Fig. 1.

Therefore, we can write the BM problem as follows:

$$\max/\min C_1 : \sum_{j=1}^n x_{1j} y_j,$$

$$\max/\min C_2 : \sum_{j=1}^n x_{2j} y_j,$$

$$\vdots$$

$$\max/\min C_m : \sum_{j=1}^n x_{mj} y_j,$$

(6)

where $y_j \in \{0, 1\}, \quad j = 1, \ldots, n$.

To select only one alternative in each time of solving the problem, one binary constraint is defined as follows:

$$\sum_{j=1}^n y_j = 1.$$

(7)

Step 3: Determining the nadir value of each criterion
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\[ C_i = \min C_i : \sum_{j=1}^{n} x_{ij} y_j, \quad i = 1, \ldots, m, \]
subject to system constraints. We assume that criterion \( i \) be positive:

\[ C_i = \min C_i : \sum_{j=1}^{n} x_{ij} y_j, \quad i = 1, \ldots, m, \]
subject to \( \sum_{j=1}^{n} y_j = 1, \)
\[ y_j \in \{0,1\} \quad j = 1, \ldots, n. \] (8)

Step 4: Solving the BM problem
Problems (6)-(7) are multi-objective linear programming ones that can be solved using Program (5). The BM problem is solved by \( n - 1 \) repetition; so far, all alternatives have been ranked. In each repetition, a binary variable related to the assigned alternative is removed, and nadir values are updated based on the remaining alternatives data.

3.2. Supplier selection under black conditions
In a real case, DMs do not have exact and complete information related to decision objectives and constraints. The collected data of supplier selection problems do not behave crisply and are typically uncertain in nature. Because of vague and complex nature of this situation, so far, scant researches have been dedicated to it. We name this case as ‘black’ and propose an algorithm for supplier selection problem under black conditions as follows:

Step 1: Constructing the Random Decision Matrix (RDM)
Construct a RDM, where its entries are normal random variables with known mean \( \mu_{ij} \) and variance \( \sigma_{ij}^2 \), i.e., \( \bar{x}_{ij} \sim N(\mu_{ij}, \sigma_{ij}^2) \). Fig. 2 shows a RDM for the supplier selection problem.

<table>
<thead>
<tr>
<th>( f_1 )</th>
<th>( A_1 )</th>
<th>( A_2 )</th>
<th>( \ldots )</th>
<th>( A_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_1 )</td>
<td>( \bar{x}_{11} )</td>
<td>( \bar{x}_{12} )</td>
<td>( \ldots )</td>
<td>( \bar{x}_{1n} )</td>
</tr>
<tr>
<td>( f_2 )</td>
<td>( \bar{x}_{21} )</td>
<td>( \bar{x}_{22} )</td>
<td>( \ldots )</td>
<td>( \bar{x}_{2n} )</td>
</tr>
<tr>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
</tr>
<tr>
<td>( f_m )</td>
<td>( \bar{x}_{m1} )</td>
<td>( \bar{x}_{m2} )</td>
<td>( \ldots )</td>
<td>( \bar{x}_{mn} )</td>
</tr>
</tbody>
</table>

Fig. 2. A RDM

Step 2: Constructing the Binary Multi-objective Stochastic (BMS) problem
For each criterion, one objective function is defined with binary variables, where coefficients of these variables in objectives are the row entries of RDM in Fig. 2.

Therefore, the BMS problem can be written as follows:

\[ \max / \min f_i : \sum_{j=1}^{n} \bar{x}_{ij} y_j, \] (9)
subject to \( \sum_{j=1}^{n} y_j = 1, \)
\[ y_j \in \{0,1\} \quad j = 1, \ldots, n. \]

Step 3: Determining the nadir value of each criterion
If \( \bar{x}_{ij} \sim N(\mu_{ij}, \sigma_{ij}^2) \) and \( \Phi(\pm 3.49) = \text{Prob} (Z \leq \pm 3.49) = 1 \) (Fig. 3), then

\[ \frac{\max[\bar{x}_{ij}] - \mu_{ij}}{\sigma_{ij}} = +3.49 \Rightarrow x_{ij}^+ = \max \{\bar{x}_{ij}\} \] (10)
\[ \mu_{ij} + 3.49\sigma_{ij}, \]
and

\[ \frac{\min[\bar{x}_{ij}] - \mu_{ij}}{\sigma_{ij}} = -3.49 \Rightarrow x_{ij}^- = \min \{\bar{x}_{ij}\} = \] (11)
\[ \mu_{ij} - 3.49\sigma_{ij}. \]

Fig. 3. A standard normal distribution \( Z \sim N(0,1) \)

\( f_i, i = 1, \ldots, m \) is the worst solution of the objective function \( \sum_{j=1}^{n} \bar{x}_{ij} y_j, \quad i = 1, \ldots, m, \) subject to system constraints. We assume that \( f_i = 1, \ldots, m \) must be maximized:

\[ f_i = \min f_i : \sum_{j=1}^{n} \bar{x}_{ij} y_j, \quad i = 1, \ldots, m, \] (12)
subject to \( \sum_{j=1}^{n} y_j = 1, \)
\[ y_j \in \{0,1\} \quad j = 1, \ldots, n. \]

Step 4: Solving the BMS problem
In this step, the BMS problem is solved using the proposed approach. This approach, which is hybrid of NCP and SP (NCSP), is modelled as follows:

3.2.1. Maximizing random objectives
\( \bar{x}_{ij} \) represents random and normally distributed parameters. We assume that \( r \) objectives must be maximized. It can be said:
\[ \sum_{j=1}^{n} \bar{x}_{ij}y_j \geq f_{i*}, \quad i = 1, ..., r. \] (13)

Based on SP, the objective is maximizing \( \lambda_i \) subject to:

\[ \text{Prob}(\sum_{j=1}^{n} \bar{x}_{ij}y_j \leq f_{i*} + \lambda_i) \geq 1 - \gamma_i, \quad i = 1, ..., r, \] (14)

where \( \gamma_i = 1, ..., r \) is the threshold value of the \( i \) th objective.

\[ \text{Prob}(\sum_{j=1}^{n} \bar{x}_{ij}y_j - f_{i*} \leq \lambda_i) \geq 1 - \gamma_i, \quad i = 1, ..., r, \] (15)

Let \( \tilde{A}(\mathbf{x}) = \sum_{j=1}^{n} \bar{x}_{ij}y_j - f_{i*}, \) \( \tilde{A}(\mathbf{x}) \) be normally distributed and \( E(\tilde{A}(\mathbf{x})) \) and \( \text{Var}(\tilde{A}(\mathbf{x})) \) be the mean and variance, respectively. Thus, we have:

\[ \text{Prob}(\tilde{A}(\mathbf{x}) \leq \lambda_i) \geq 1 - \gamma_i, \quad i = 1, ..., r, \] (16)

\[ \text{Prob}(\frac{\tilde{A}(\mathbf{x}) - E(\tilde{A}(\mathbf{x}))}{\sqrt{\text{Var}(\tilde{A}(\mathbf{x}))}} \leq \frac{\lambda_i - E(\tilde{A}(\mathbf{x}))}{\sqrt{\text{Var}(\tilde{A}(\mathbf{x}))}}) \geq 1 - \gamma_i, \quad i = 1, ..., r, \] (17)

\[ E(\sum_{j=1}^{n} \bar{x}_{ij}y_j) + \Phi^{-1}(1 - \gamma_i) \sqrt{\sum_{j=1}^{n} \text{Var}(\bar{x}_{ij}y_j) - \lambda_i + \delta_i} = f_{i*}, \quad i = 1, ..., r. \] (21)

3-2-2. Minimizing random objectives

We assume that \((m-r)\) objectives must be minimized. Therefore, it can be said:

\[ \sum_{j=1}^{n} \bar{x}_{ij}y_j \leq f_{i*}, \quad i = r + 1, ..., m. \] (22)

Based on SP, the objective is maximizing \( \tau_i \) subject to:

\[ \text{Prob}(\sum_{j=1}^{n} \bar{x}_{ij}y_j \geq f_{i} - \tau_i) \geq 1 - \gamma_i, \quad i = r + 1, ..., m, \] (23)

where \( \gamma_i = r + 1, ..., m \) is the threshold value of objective \( i \).

\[ \text{Prob}(f_{i*} - \sum_{j=1}^{n} \bar{x}_{ij}y_j \leq \tau_i) \geq 1 - \gamma_i, \quad i = r + 1, ..., m. \] (24)

Let \( \tilde{B}(\mathbf{x}) = f_{i*} - \sum_{j=1}^{n} \bar{x}_{ij}y_j, \) \( \tilde{B}(\mathbf{x}) \) be normally distributed, and \( E(\tilde{B}(\mathbf{x})) \) and \( \text{Var}(\tilde{B}(\mathbf{x})) \) be respectively the mean and the variance. Thus, we have:

\[ \text{Prob}(\tilde{B}(\mathbf{x}) \leq \tau_i) \geq 1 - \gamma_i, \quad i = r + 1, ..., m, \] (25)

\[ \text{Prob}(\frac{\tilde{B}(\mathbf{x}) - E(\tilde{B}(\mathbf{x}))}{\sqrt{\text{Var}(\tilde{B}(\mathbf{x}))}} \leq \frac{\tau_i - E(\tilde{B}(\mathbf{x}))}{\sqrt{\text{Var}(\tilde{B}(\mathbf{x}))}}) \geq 1 - \gamma_i, \quad i = r + 1, ..., m, \] (26)

\[ \frac{\tau_i - E(\tilde{B}(\mathbf{x}))}{\sqrt{\text{Var}(\tilde{B}(\mathbf{x}))}} \geq \Phi^{-1}(1 - \gamma_i), \quad i = r + 1, ..., m, \] (27)

\[ \tau_i \geq E(\tilde{B}(\mathbf{x})) + \Phi^{-1}(1 - \gamma_i) \sqrt{\text{Var}(\tilde{B}(\mathbf{x}))}, \quad i = r + 1, ..., m, \] (28)

\[ -E(f_{i*} - \sum_{j=1}^{n} \bar{x}_{ij}y_j) - \Phi^{-1}(1 - \gamma_i) \sqrt{\text{Var}(f_{i*} - \sum_{j=1}^{n} \bar{x}_{ij}y_j) + \tau_i} \geq 0, \quad i = r + 1, ..., m. \] (29)

If \( \text{Var}(f_{i*} - \sum_{j=1}^{n} \bar{x}_{ij}y_j) = \sum_{j=1}^{n} \text{Var}(\bar{x}_{ij}y_j) \) and \( \delta_i = r + 1, ..., m \) be surplus variable \( i \), then the standard form of Eq. (29) is:

\[ E(\sum_{j=1}^{n} \bar{x}_{ij}y_j) - \Phi^{-1}(1 - \gamma_i) \sqrt{\sum_{j=1}^{n} \text{Var}(\bar{x}_{ij}y_j) + \tau_i - \delta_i} = f_{i*}, \quad i = r + 1, ..., m. \] (30)
Generally, if we maximize \( r \) objective functions and minimize \((m-r)\) objectives, then the final model for solving Program (9) can be stated as follows:

\[
\begin{align*}
\text{max} & \{ \sum_{i=1}^{r} w_i (\lambda_i - \delta^-_i)^{p} + \sum_{i=r+1}^{m} w_i (\tau_i - \delta^+_i)^{p} \}^{\frac{1}{p}}, \\
\text{subject to} & \\
& \sum_{j=1}^{n} Var(\tilde{x}_{ij})y_j - \lambda_i + \delta^-_i = f_i^-, \quad i = 1, \ldots, r, \\
& \sum_{j=1}^{n} Var(\tilde{x}_{ij})y_j + \tau_i - \delta^+_i = f_i^+, \quad i = r + 1, \ldots, m, \\
& \sum_{j=1}^{n} y_j = 1, \\
& y_j \in \{0,1\} \quad j = 1, \ldots, n.
\end{align*}
\]

For each threshold, Problem (31) is solved with \( n - 1 \) repetition; so far, all alternatives have been ranked. In each repetition, the binary variable related to alternative assigned is removed, and nadir random variables are updated based on the remaining alternatives data.

### 3-3. Supplier selection under grey conditions

In this section, we suppose that the collected data have incomplete nature. Therefore, we name this case as ‘grey’ where the aim is selecting and ranking suppliers based on the criteria defined. Our proposed algorithm to solve the supplier selection problem under grey conditions is as follows:

**Step 1:** Constructing the Combinatorial Matrix (CM)

Fig. 4 shows a CM with certainty and uncertainty data, where \( \tilde{x}_{ij} \sim N(\mu_{ij}, \sigma_{ij}^2) \) for \( i = t + 1, \ldots, m, j = 1, \ldots, n \).

\[
\begin{align*}
\text{max/min} \ C_1 : & \sum_{j=1}^{n} x_{1j}y_j, \\
& \vdots \\
\text{max/min} \ C_t : & \sum_{j=1}^{n} x_{tj}y_j, \\
\text{max/min} \ f_{t+1} : & \sum_{j=1}^{n} \tilde{x}_{t+1,j}y_j, \\
& \vdots \\
\text{max/min} \ f_m : & \sum_{j=1}^{n} \tilde{x}_{mj}y_j, \\
\text{subject to} & \sum_{j=1}^{n} y_j = 1, \\
& y_j \in \{0,1\} \quad j = 1, \ldots, n.
\end{align*}
\]

**Step 3:** Determining the nadir values

According to white and black conditions introduced in sections 3.1 and 3.2, the nadir values of each criterion of above CM can be determined.

**Step 4:** Solving the BCM problem

Problem (32) is a multi-objective linear programming problem that has some of random and known parameters.

Generally, if we maximize \( r \) objective functions and minimize \((t-r)\) objective under white conditions and also maximize \((s-t)\) objective functions and minimize \((m-s)\) objective under black conditions, the final model solves the BCM problem, formulated based on Programs (5) and (31) as follows:
\[
\max \left( \frac{\sum_{i=1}^{n} w_i \lambda_i^p + \sum_{i=r+1}^{m} w_i \tau_i^p}{\sum_{i=1}^{r+1} w_i (\lambda_i - \delta_i^-)^p + \sum_{i=r+1}^{m} w_i (\tau_i - \delta_i^+)^p} \right)^{\frac{1}{p}},
\]
subject to
\[
\sum_{j=1}^{n} x_{ij} y_j - \lambda_i = f_{i*}, \quad i = 1, \ldots, r,
\]
\[
\sum_{j=1}^{n} x_{ij} y_j + \tau_i = f_{i*}, \quad i = r + 1, \ldots, t,
\]
\[
E\left( \sum_{i=1}^{n} \tilde{x}_{ij} y_j \right) + \Phi^{-1}(1 - \gamma_i) \sqrt{\sum_{j=1}^{n} \text{Var}(\tilde{x}_{ij}) y_j}
\]
\[
-\lambda_i + \delta_i^- = f_{i*}, \quad i = t + 1, \ldots, s,
\]
\[
E(\sum_{j=1}^{n} \tilde{x}_{ij} y_j) - \Phi^{-1}(1 - \gamma_i) \sqrt{\sum_{j=1}^{n} \text{Var}(\tilde{x}_{ij}) y_j}
\]
\[
+\tau_i - \delta_i^+ = f_{i*}, \quad i = s + 1, \ldots, m,
\]
\[
\sum_{j=1}^{n} y_j = 1,
\]
\[
y_j \in \{0,1\}, \quad j = 1, \ldots, n.
\]

Program (33) is solved with \( n - 1 \) repetition; so far, all alternatives have been ranked. In each repetition, the binary variable related to an alternative assigned is removed, and nadir values are updated based on the remaining alternatives data.

Flowchart of Fig. 5 shows the proposed methodology in this section.

### 4. Case Study

When there are multiple suppliers available, one of the decision tasks for any purchasing firm is to find a reliable process to implement a multiple source policy [2]. One of the decision-making methods is to make use of several criteria. Based on a survey of 170 purchasing managers, Dickson [36] identified 23 different criteria evaluated in supplier selection. Among these criteria, price, delivery performance, and quality of the buyer are deemed particularly important in evaluating the suppliers. Weber et al. [37] reviewed 74 articles on supplier evaluation, and concluded that quality is of the highest importance, followed by delivery performance and cost. They also suggested that the supplier selection decision must not be based exclusively on the least cost criterion; other critical factors, such as quality and delivery performance, should be incorporated into the evaluation and selection processes. Contained in the various evaluation methods proposed in the available literature, price, delivery performance, and quality are the most common criteria [38].

Fulad Mehr Sahand company is now one of the largest pipe and profile manufacturers in the northwest of Iran and is located in 30 km of northwest of Shahid Salimi industrial zone in Tabriz. The company executive operation began in 2008 and has been operating since May, 2009. Over the past years, nominal capacity during a 3-part amendment has reached 160 thousand tons of steel pipe and tubes production. Quality and customer satisfaction are of the highest priority of company’s activities, and market acceptance of products is the best reason of this claims. Now, the company requires that some of the active suppliers in the market supply its needed raw materials to produce new products. In this regard, the company will rank five suppliers based on its considered criteria. The purchasing criteria are reliability, flexibility, quality, on-time delivery, waste percentage, and price. Because the grey status involves two white and black statuses and is more compatible with this problem, we, therefore, consider the purchasing system under grey conditions. CM related to data of suppliers is shown in Table 1. Data of Table 1 have been obtained based on the output of distributed questionnaires among customers of the suppliers.
Tab. 1. CM related to numerical example

<table>
<thead>
<tr>
<th>Criterion</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
<th>S5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reliability</td>
<td>+ N(0.8, 0.05)</td>
<td>N(0.85, 0.02)</td>
<td>N(0.8, 0.03)</td>
<td>N(0.85, 0.05)</td>
<td>N(0.8, 0.02)</td>
</tr>
<tr>
<td>Flexibility</td>
<td>+ N(0.7, 0.08)</td>
<td>N(0.8, 0.05)</td>
<td>N(0.8, 0.06)</td>
<td>N(0.7, 0.05)</td>
<td>N(0.75, 0.02)</td>
</tr>
<tr>
<td>Quality</td>
<td>+ N(0.9, 0.03)</td>
<td>N(0.85, 0.03)</td>
<td>N(0.9, 0.04)</td>
<td>N(0.85, 0.04)</td>
<td>N(0.8, 0.01)</td>
</tr>
<tr>
<td>On-time delivery</td>
<td>+ 0.8</td>
<td>0.7</td>
<td>0.9</td>
<td>0.8</td>
<td>0.9</td>
</tr>
<tr>
<td>Waste percentage</td>
<td>– 0.03</td>
<td>0.05</td>
<td>0.04</td>
<td>0.05</td>
<td>0.03</td>
</tr>
<tr>
<td>Price ($)</td>
<td>– 8</td>
<td>7</td>
<td>10</td>
<td>9</td>
<td>8</td>
</tr>
</tbody>
</table>

In Table 1, four criteria are positive and two criteria are negative. In addition, data related to reliability, flexibility, and quality criteria are normal random variables with known mean and variance; other data have been collected with certainty. For example, in Table 1, $\bar{x}_{22} \sim N(0.8,0.05)$ and $x_{51} = 0.03$.

To solve this problem, steps of the proposed approach are as follows:

**Step 1:** Constructing the BCM problem

According to Program (32), BCM problem is as follows:

\[
\begin{align*}
\text{max}_{\text{Reliability}} & \sum_{j=1}^{5} \bar{x}_{1j}y_j, \\
\text{max}_{\text{Flexibility}} & \sum_{j=1}^{5} \bar{x}_{2j}y_j, \\
\text{max}_{\text{Quality}} & \sum_{j=1}^{5} \bar{x}_{3j}y_j,
\end{align*}
\]
Step 2: Determining the values of nadir
Table 2 presents nadir values of the problem criteria at the first repetition of solution.

**Tab. 2. Nadir value of each criterion at the first repetition (for \( i = 1, \ldots, 6 \))**

<table>
<thead>
<tr>
<th>Criterion (i)</th>
<th>Reliability</th>
<th>Flexibility</th>
<th>Quality</th>
<th>On-time delivery</th>
<th>Waste percentage</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nadir value ( (f_i) )</td>
<td>0.6255</td>
<td>0.4208</td>
<td>0.7104</td>
<td>0.7</td>
<td>0.05</td>
<td>10</td>
</tr>
</tbody>
</table>

**Step 3: Solving Program (34)**

Program (34) (with assumption \( P = 1 \)) can be converted into a single-objective problem by using Program (33) as follows:

\[
\begin{align*}
\max & \quad (w_1(\lambda_1 - \delta_1^-) + w_2(\lambda_2 - \delta_2^-) + w_3(\lambda_3 - \delta_3^-) + w_4\lambda_4 + w_5\tau_5 + w_6\tau_6), \\
\text{subject to} & \\
E \left( \sum_{j=1}^{5} \tilde{x}_{ij}y_j \right) + \Phi^{-1}(1 - \gamma_i) \left( \sum_{j=1}^{5} \text{Var} (\tilde{x}_{ij}) y_j - \lambda_i + \delta_i^- \right) = f_i^w, \\
\sum_{j=1}^{5} x_{ij}y_j - \lambda_i = f_i^a, \\
\sum_{i=1}^{5} x_{ij}y_j + \tau_i = f_i^a, & \quad i = 5,6, \\
\sum_{j=1}^{5} y_j = 1, \\
y_j \in \{0,1\} & \quad j = 1, \ldots, 5.
\end{align*}
\]  

where parameters of this program are the data related to Tables 1 and 2. Also, in Program (35), \( \gamma_i = 1,2,3 \) is equal to 0.05.

Program (35) was solved four times by Lingo software package, where one supplier was selected in each time. To optimize Program (35), different preference weights have been considered for objectives. Results obtained are presented in Table 3.

**Tab. 3. Ranking suppliers based on a set of preference weights of objectives**

<table>
<thead>
<tr>
<th>Preference weights</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1/6, 1/6, 1/6, 1/6, 1/6, 1/6)</td>
<td>S2→S1→S5→S3→S4</td>
</tr>
<tr>
<td>(0.4,0.1,0.1,0.2,0.1,0.1)</td>
<td>S2→S1→S5→S3→S4</td>
</tr>
<tr>
<td>(0.2,0.4,0.1,0.1,0.1,0.1)</td>
<td>S2→S1→S3→S5→S4</td>
</tr>
<tr>
<td>(0.1,0.1,0.4,0.1,0.1,0.1)</td>
<td>S2→S1→S3→S5→S4</td>
</tr>
<tr>
<td>(0.1,0.2,0.1,0.4,0.1,0.1)</td>
<td>S2→S1→S5→S3→S4</td>
</tr>
<tr>
<td>(0.1,0.1,0.1,0.2,0.4,0.1)</td>
<td>S2→S1→S5→S3→S4</td>
</tr>
<tr>
<td>(0.1,0.1,0.2,0.1,0.1,0.4)</td>
<td>S2→S1→S5→S3→S4</td>
</tr>
</tbody>
</table>

Table 3 presents ranking suppliers by considering 7 sets of preference weights, where the best choice is S2 and the worst is S4 in the general case. The results of the ranking in Table 3 indicate that S1, S2, and S4 are not sensitive to preference weight changes of objectives, and S3 and S5 are sensitive to these changes, so their rank varies. This can be due to the primal data proximity of S3 and S5, whose ranks change in the ranking process with a small change in the preference weights.
4-1. Simulation

In this section, a simulation method is used for solving the problem presented in Section 4 in order to study the efficiency of the proposed model; and the results of both methods are analyzed and evaluated.

Execution steps of simulation are as follows:

**Step 1:** Generating the random data for random parameters of Program (34) in such a way that:

$$(R_{nd})_{ij} \in \left\{ \mu_{ij} \pm 3.49\sigma_{ij} \right\}, \quad i = 1, 2, 3 \quad \text{and} \quad j = 1, \ldots, 5,$$

where $(R_{nd})_{ij}$ is random data generated for random parameter $ij$ (entry of row $i$ and column $j$) from RDM, selected from the interval of $\{\mu_{ij} \pm 3.49\sigma_{ij}\}$.

**Step 2:** Program (34) is converted into Program (37) based on Program (5):

$$\max \left( \sum_{i=1}^{5} w_{i} \lambda_{i} + \sum_{i=6}^{8} w_{i} \tau_{i} \right),$$

subject to

$$\sum_{j=1}^{5} (R_{nd})_{ij} y_{j} - \lambda_{i} = f_{i}, \quad i = 1, 2, 3,$$

$$\sum_{j=1}^{5} x_{4j} y_{j} - \lambda_{4} = f_{4},$$

$$\sum_{j=1}^{5} x_{5j} y_{j} + \tau_{i} = f_{i}, \quad i = 5, 6,$$

$$\sum_{j=1}^{5} y_{j} = 1,$$

$$y_{j} \in \{0, 1\}, \quad j = 1, \ldots, 5. \quad (37)$$

**Step 3:** With each round of new random data generation, the suppliers’ selection and ranking process continue in such a way that, with each supplier selection, the related variable is removed from Program (37), and all information is updated. Ranking process continues insofar as all suppliers are selected and ranked. Table 4 presents the results obtained within 300 repetitions of simulation given the equality of the objectives’ importance weights. Meanwhile, Table 4 presents the results obtained from solving the proposed model.

### Tab. 4. Results of simulation method and the proposed model

<table>
<thead>
<tr>
<th>Suppliers</th>
<th>Rank 1</th>
<th>Rank 2</th>
<th>Rank 3</th>
<th>Rank 4</th>
<th>Rank 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>42</td>
<td>89</td>
<td>102</td>
<td>57</td>
<td>10</td>
</tr>
<tr>
<td>S2</td>
<td>225</td>
<td>56</td>
<td>18</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>S3</td>
<td>8</td>
<td>32</td>
<td>57</td>
<td>164</td>
<td>39</td>
</tr>
<tr>
<td>S4</td>
<td>0</td>
<td>0</td>
<td>8</td>
<td>45</td>
<td>247</td>
</tr>
<tr>
<td>S5</td>
<td>25</td>
<td>123</td>
<td>115</td>
<td>33</td>
<td>4</td>
</tr>
<tr>
<td>Best by</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>simulation</td>
<td>S2</td>
<td>S5</td>
<td>S5</td>
<td>S3</td>
<td>S4</td>
</tr>
<tr>
<td>Best by</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>the</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>proposed</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>model</td>
<td>S2</td>
<td>S1</td>
<td>S5</td>
<td>S3</td>
<td>S4</td>
</tr>
</tbody>
</table>

In the simulation method, each supplier that has the maximum number of repetitions in each column is considered as the best selection of that column. The results of Table 4 imply that, in both proposed and simulation methods, S2 is the best alternative and S4 is the worst alternative, and in other ranks, the results obtained are similar with the least difference. In addition, based on the information of Table 4 and the simulation results, there is no place for S1.

The efficiency of the results of the (40) proposed model compared to that of simulation method can be defined as below:

$$\text{Efficiency} = \frac{\text{Number of ranks, which assign similar alternatives}}{\text{Number of alternatives}} \times 100\%.$$  

Considering Eq. (40), the efficiency obtained is equal to $\frac{4}{5} = 80\%$, which implies the appropriate performance of the proposed model. In other words, $80\%$ of the results of the proposed model in one repetition are similar to the results of the simulation method.
obtained by the simulation method in 300 repetitions.

5. Conclusions
Nowadays, due to the lack of enough facilities and sources, companies require to establish relations with foreign suppliers, through which they can meet their necessary requirements. Hence, supplier selection is always one of the most important strategies of companies. Generally, supplier selection is not an easy process. The suitable decision in this process involves precise determinations including understanding the suppliers’ multiple-criteria problem, in which buying companies try to select the most cost effective suppliers considering such criteria as quality, service level, price, etc. Some suppliers’ variable performances can result in a decision-making problem under uncertainty conditions. Integration of variable conditions of suppliers and decision-making criteria of buying companies is, therefore, deemed necessary. This paper presented a combined approach of the NCP model and SP under white, black, and grey conditions for integrating criteria of purchasing companies and uncertain conditions of suppliers. Based upon NCP concept, models were developed which have been presented in three situations of white, black, and gray environments corresponding to suppliers’ vague data. For illustrating the proposed approach, a six-objective stochastic problem of selecting suppliers under grey conditions was introduced, and the best and worst choices were determined. A simulation method has been developed in order to study the efficiency of the proposed model. The real case was simulated 300 times, so that parameters can be randomly changed and the final results be compared with the NCP model. The results of the proposed model and also a simulation were similar in 80% of the cases.

It was shown that the proposed approach is capable to help DM(s) optimize multiple criteria problems under uncertainty environment.

References


A Nadir Compromise Programming for Supplier Selection Problem under Uncertainty


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