Benders decomposition for Supply Chain Network Redesign with Capacity planning and Multi-Period Pricing

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Abstract

Demand fluctuations, network cost increase and proposing new services for customers make companies more interesting in network redesigning problems. These problems let the existing network to increase or decrease its capacity in order to meet changing customers demand. In this study, a linear mixed integer programming model is proposed to redesign a supply chain network which has price sensitive customers. In addition, because of the nature of the problem both strategic and tactical decisions are take in to account simultaneously. Strategic decisions consist of opening new facilities, closing existing ones and adding discrete capacity levels to all network facilities. Tactical facilities are pricing and determining flow between different network echelons. Pricing is an important fragment of Supply chain due to two reasons: first, it represents potential revenue of each product and second, based on supply-demand relations, enables Supply chain to provide demands by maki ng suitable changes in number of facilities and their capacities. Therefore, this model aims to consider both pricing and redesigning decisions simultaneously in order to maximize the network profit. Based on the solution time of designing/redesigning problems with CPLEX solver in GAMS, these problems are among the complex and challenging ones. To overcome this problem in this study, Benders decomposition approach was used to solve a multi-product, multi-echelon and multi-period supply chain network redesigning problem with price-sensitive customers. Results prove appropriate performance of the proposed algorithm for the model.

Keywords:
Supply Chain Network Redesign, Capacity Planning, Multi-Period Pricing, Benders Decomposition method

1. Introduction

Supply chain network design/redesign is a strategic planning with making decisions within entire structure. This would result a comprehensive and optimized system. Based on horizon length, decisions will be strategically, tactical and technical or operational. Designing a Supply chain network (SCN) consists of number and location of facilities, assign capacities, using technology, pricing and flows between different sectors within the whole system. All these actions should cause to meet every customer's demand with the lowest cost and the most profit. Based on current economic environment, revenue management grows as an important aspect of supply chain planning. Real world conditions make it possible for companies to plan for both designing and redesigning a supply chain. Redesigning would be a suitable decision if operational costs exceed expectations and customers are looking for new services. In these conditions, companies should develop their network or redesign it. As Ballou and et al. (1968) discussed, redesigning can reduce SCN costs from 5 to 15 percent[1]. In redesigning problems, there are several in action manufacturing and distribution centers in fixed locations. The main focus is developing or reducing the capacity of existing facilities, closing them, opening new centers and allocating capacity levels. SCN Tactical decisions consists of two categories: pricing and quantity decisions. Tallury and et al. (2006) mentioned that pricing determines the product price and their changes within the horizon[2]. Quantity decisions analyze amount of production of each item, the flow between different facilities, when to store them and the appropriate time for customer delivery. Considering both of these facts simultaneously cause a challenging issue in tactical level. Pricing itself determines the income per each kind of products and with supply-demand relations makes it possible to calculate necessary changes in facilities and their capacities to fulfill customers demand.

Studies by Wagner and et al. (1975), Hanjoul and et al. (1990), Ahmadi-Javid and et al. (2015), Fattahi and et al. (2015) and Nobari and et al. (2016) consider facilities location and profit in a SCN with price-sensitive customers [3-7]. Furthermore, combining issues like, rapid fluctuations in market, budget restrict, increase in facilities operational cost and considering possibility of redesign with pricing decision would be useful.

Pricing is only possible by having previous sale data and customers behavior. This model considers redesigning supply chain netwo rk with pricing. Objective functions parts and related constraints are described. Based on the basic model of this study, pricing is determined in tactical periods. One of the most serious challenging issues in the literature is proposing solution approaches for designing/redesigning networks with capacity planning. One of the contribution of this study is using Benders Decomposition Method for these kinds of problems. Comparing solution time and results of using this exact approach with literature proves its effectiveness and performance.

2. Literature review

In this paper, Literature review is divided to two parts. At first, dynamic supply chain literature with capacity planning and then pricing would be reviewed.

2.1 Dynamic supply chain network design

Since first studies of Geoffrion and et al. (1974) to design multi-product SCN, several optimization methods have been introduced[8]. Those efforts had remarkable impact on problem's model, algorithm and effectiveness of calculations.
Most of those models and papers like, Cordeau and et al. (2006), Olivares-Benitez and et al. (2012), and Sadjadi and et al. (2012) consider improvement issues and location problems within a

single time horizon [9-11]. Although, multi-period problems were managers and decisions makers crucial need. Based on the importance of this subject, recently many studies have considered their problem as a multi period one.

Also based on complexity and supply chain levels different papers have been proposed. For the first time, Ballou and et al. (1968) consider dynamic network location which included a two-echelon network[1]. Later Scott and et al. (1971) improved problem for several facilities[12]. In recent years, some studies mention multi-period design and redesign SCN with three and more echelons which cover location decisions. Decisions which these papers assumed in their models are represented in table 1.

Based on the nature and characteristics of designing supply chain networks with capacity planning problems, time intervals would be divided to strategic periods, tactical periods or a combination of them. Melo and et al. (2006) represent redesign multi-level network problem which makes changes possible in capacities through moving capacities from existed to new centers in a period[13]. Also, strategic decisions like: opening-closing and moving facilities with respect to budget restrict for each period is consi

dered. Also, Hinojosa and et al. (2000, 2008), Correia and et al. (2013) use strategic periods in their models [14-16]. Longinidis and et al. (2011), Georgiadis and et al. (2011) use tactical periods and for the first time, Salma and et al. (2010) combined strategic and tactical periods which recently used by Bashiri and et al. (2012)[17-20].

Badri and et al. (2013) proposed a three-level model with profit maximization objective which considered capacity improvement, opening new centers, transportation and inventory decision[21].

Existing papers within literature review, used two common function objectives. Some of them proposed their model in the format of multi objective problems. Most of the papers in the literature review mentioned cost reduction as objective function. A few studies used profit maximization as their objective function

Recently in capacity planning and location issues, different aspects gained attention and combined design and redesign problems. In location problems, assumptions like opening new facilities, closing existing facilities, and reopening closed facilities are considered and in capacity planning issues, capacity expansion, capacity reduction and transferring capacity levels seems to play great role. In table 1 differences within several papers are shown. Recently, M.J. Cortinhal (2015) considering redesigning supply chain network with opening new facilities, closing the existed ones and adding discrete capacity levels in strategic periods [22]. That study is the basic fundamental on this paper. To make it more realistic strategic periods consist of several tactical periods and also procing is added to the model.

In table 1 differences within several papers are shown.

Several different solving methods are used in this area. Papers like Canel and et al. (2001) and Aghezzaf and et al. (2005) use exact approach for solving SCN problems [23, 24]. Besides, Lagrangian-based heuristic methods are among the most important heuristic methods in this field. Hinojosa and et al. (2000, 2008) used such methods [14, 15]. In this model, the exact Bender decomposition method is used to solve SCN redesign problem.

2.2 Network design with pricing

Network design/redesign problems with maximization profit objectives divided in three categories. First, the competitive network design problems. In this category equilibrium price and decisions related to facility location like new centers and plants join the competitive environment. Aboolian and et al. (2007), Plastria and et al. (2009) consider equilibrium price and facility location decisions [25, 26]. Nagurney and et al. (2010) mentions capacity and equilibrium price[27]. Second approach deals with insensitive price customers.

Price plays role like a binary variable which determine whether a customer wants the provided services or not. Zhang and et al. (2001) and Shen and et al. (2006) reviewed such problems [28, 29].

Third approach include price-sensitive customers and required pricing and quantity decisions take simultaneously. In this category, linear sensitive demand function or logit function is used. These functions obtain based on previous sale's information and current competitors. Ahmadi-Javid and et al. (2014 and 2015), Fattahi and et al. (2015) and Nobari and et al. (2016) consider third category [5-7, 30].

Due to reviewed studies, recent researches concentrate on dynamic and comprehensive models for planning and designing SCN but there are few attempts to take in to account different aspects with dynamic network simultaneously. There are possible opportunities within pricing and price-sensitive customer's problems. Recent studies did not consider redesigning and pricing. According to functions that obtained based on previous sale's information, in this manner, we consider redesign problem and the goal is to improve current capacities and opening new centers and plants based on pricing and profit maximization. Besides, reducing capacity and closing current centers did not mentioned. In this study, reduction capacity is available because in real situation it is possible that high pricing in response to increase profit makes it logical to forfeit operational cost by closing or capacity limitation. Since decisions in redesigning problems are strategic ones and these decisions contains remarkable costs, using exact approaches seems necessary. In this model for the first time the exact Bender decomposition method is used to solve redesign SCN problem.

Table 1. literature review

Author	Model	Network Redesign	Multi Product	Opening, Closing, Reopening			Capacity Planning				Dynamic Pricing	Exact Solution	Solution Approach
				Open New	Close Existed	Reopen closed	Modular	Relocate	Expansion	Reduction			
Aghezzaf (2005)[24]	MILP-TSSP			✓					✓			✓	Lagrangian Relax Decomposition
Badri (2013)[21]	MILP		✓	✓			✓		✓				Lagrangian-based Heuristic
Bashiri (2012)[20]	MILP			✓			✓		✓				Commercial Solver
Canel (2001)[23]	MILP		✓	✓	✓	✓						✓	Lagrangian Relax Decomposition
Correia (2013)[16]	MILP			✓			✓		✓				Commercial Solver
Dias (2007)[31]	MILP		✓	✓	✓	✓							Prime-dual Heuristic
Georgiadis (2011)[18]	MILP-TSSP		✓										Commercial Solver
Hinojosa (2008)[14]	MILP		✓	✓	✓								Lagrangian-based Heuristic
Hinojosa (2000)[15]	MILP		✓	✓	✓								Lagrangian-based Heuristic
Longinidis (2011)[17]	MILP-TSSP		✓										Commercial Solver
Melo (2006)[13]	MILP		✓	✓	✓		✓	✓	✓	✓			Commercial Solver
Melo (2012)[32]	MILP	✓	✓	✓	✓			✓					Tabu search meta heuristic
Melo (2014)[33]	MILP	✓	✓	✓	✓			✓					Linear Relax-base Heuristic
Melachrinoudis (2000)[34]	MILP	✓		✓	✓			✓					Commercial Solver
Nickel (2012)[35]	MILP-TSSP		✓	✓	✓	✓							Commercial Solver
Pimentel (2013)[36]	MILP-TSSP			✓	✓	✓	✓		✓				Lagrangian-based Heuristic
Thanh (2008)[37]	MILP		✓	✓			✓		✓				Commercial Solver
Thanh (2010)[38]	MILP		✓	✓			✓		✓				Linear Relax-base Heuristic
Wilhelm (2013)[39]	MILP	✓	✓	✓	✓				✓	✓			Commercial Solver
M.J. Cortinhal (2015)[22]	MILP	✓	✓	✓	✓		✓		✓				Commercial Solver
M. Fattahi (2015)[6]	MILP		✓	✓			✓		✓		✓		SA & Linear Relax-base Heuristic
Correia (2017)[40]	MILP	✓	✓	✓	✓		✓		✓				Commercial Solver
This Study	MILP	✓	✓	✓	✓		✓		✓		✓	✓	Benders Decomposition

MILP:Mixed-integer linear programming, TSSP:Two stage stochastic programming

3. Model description and formulation

In this section, indices, parameters, variables would be defined. After that, determining price levels would be mentioned and finally, a mixed integer programming model would be proposed based on the structure of figure

1.

In this figure, existing facilities with dash line existed and new decisions like closing these centers or opening new facilities are presented respectively in dashed and full lines.

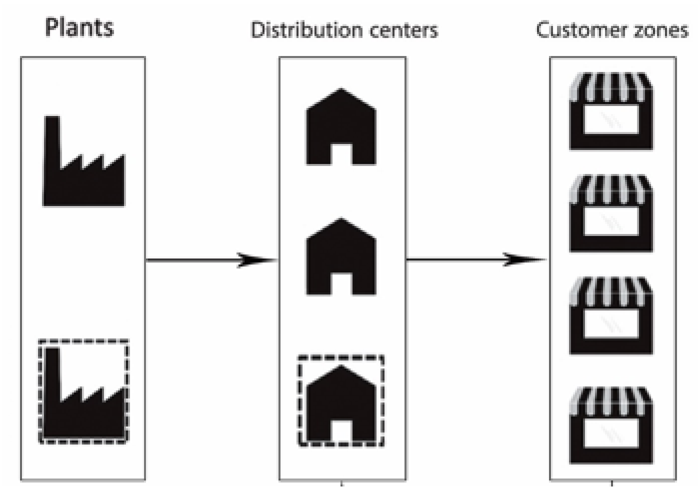

Figure 1 Network Structure

In this study, several assumptions considered. They are as follows:

- Supply chain network includes three layers: plants, distribution centers (DCs) and costumers' zones.
- Each strategic period (sp) includes tactical periods (tp).
- Customers demand in each tactical period depends on prices within that period.
- Objective is maximizing profit.
- It is possible to open new centers and close current center

s to reduce operational costs.

- Capacities are able to increase in discrete mode.
- Strategic and tactical decisions mentioned concurrently to enhance reality.
- Accelerating Benders decomposition method is used.
- Model include multi-products within multi-periods.

The notations in the paper are defined as follows:

Sets and indices:

Set	**Description**
T	Strategic periods

N	Tactical periods
C	Customer zones(Cz)
M	Products
OD	All origin-destination in network $OD=\{(p,m):p\in P,w\in W\}\cup\{(w,c):w\in W,c\in C\}$
L	Price levels
$Pe\cup We$	Existing plants and DCs
$Pn\cup Wn$	New plants and DCs
$P=Pe\cup Pn$	All plant locations
$W=We\cup Wn$	All DC locations
k_p, k_W	Discrete capacity levels

Parameters:

Parameters	Description
$D_{c,m,l,t,n}$	Demand of Cz in sp $t\in T/\{0\}$ and tp $n\in N$ for product $m\in M$ with price level $l\in L$
$PR_{c,m,l,t,n}$	Price of product $m\in M$ at price level $l\in L$ in sp $t\in T/\{0\}$ and tp $n\in N$
$FC_{j,t}$	Fixed cost of establishing new facility $j\in Pn\cup Wn$ in sp $t\in T/\{0\}$
$CC_{j,t}$	Fixed cost of closing an existing facility $j\in Pe\cup We$ in sp $t\in T/\{0\}$
$AC_{j,k,t}$	Cost of adding capacity k_j in location $j\in P\cup W$ in sp $t\in T/\{0\}$
$OC_{j,k,t}$	Fixed cost of operating $j\in P\cup W$ in sp $t\in T/\{0\}$
$CU_{j,t}$	Fixed cost of each facility $j\in P\cup W$ in sp $t\in T/\{0\}$
$TC_{O,D,m,t,n}$	Variable cost per unit of product $m\in M$ between O and D in sp $t\in T/\{0\}$ and tp $n\in N$
$MC_{p,m}$	Manufacturing cost per unit of $m\in M$ at plant $p\in P$ in sp $t\in T/\{0\}$ and tp $n\in N$
Q_j^e	Capacity of existing facility $j\in Pe\cup We$
$Q_{j,k}$	Capacity of level $k\in K_p\cup K_w$
Q_j^{max}	Maximum capacity of facility $j\in P\cup W$ in each sp
$\Upsilon_{p,m}, \Upsilon_{w,m}$	Coefficient for using storage capacity of each product $m\in M$ in each plant and DC
U_j^{max}	Maximum utilization rate of facility $j\in P\cup W$

Price-response related parameters:

Parameters	Description
$D_{c,m,t,n}^{max}$	Demand of Cz $c\in C$, for product $m\in M$ in sp $t\in T/\{0\}$ and tp $n\in N$

$P_{c,m,t,n}$	Price-sensitive parameter for customer $c \in C$, for product $m \in M$ in sp $t \in T / \{0\}$ and tp $n \in N$. If price of product is less than this value, the demand will be equal to $D^{max}_{c,m,t,n}$.
$P'_{c,m,t,n}$	Price-sensitive parameter for customer $c \in C$, for product $m \in M$ in sp $t \in T / \{0\}$ and tp $n \in N$. If price of product is more than this value, the demand will be equal to 0.

Variable:

Variables	Description
$Yn_{j,t}$	1 if new facility is established in location $j \in Pn \cup Wn$ in sp $t \in T / \{0\}$ and tp $n \in N$
$Ye_{j,t}$	1 if existing facility is closed in location $j \in Pe \cup We$ in sp $t \in T / \{0\}$ and tp $n \in N$
$U_{j,k,t}$	1 if capacity level $k \in K_p \cup K_w$ is installed in location $j \in P \cup W$ in sp $t \in T / \{0\}$ and tp $n \in N$
$x_{O,D,m,t,n}$	Quantity of product $m \in M$ shipped in sp $t \in T / \{0\}$ and tp $n \in N$ between O and D
$\Delta_{m,l,t,n}$	1 if price level $l \in L$ for per unit product $m \in M$ is selected in sp $t \in T / \{0\}$ and tp $n \in N$

3.1 Demand-price function modeling

In this study, the demand of each customer zone is sensitive to price. Figure 2 represents a logit price-demand function which shows relation between $D_{c,m,t,n}$ and $PR_{c,m,t,n}$ for customer zone c, product m, in strategic period t and tactical period n.

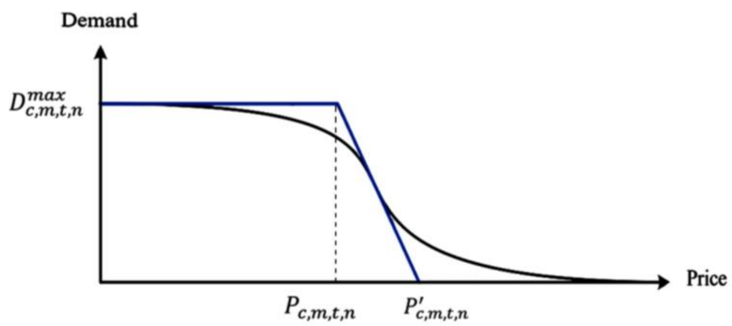

Figure 2- Demand-Price relation

Demand for costumer zone c, product m, within strategic period t and tactical period n is calculated by:

$$D_{c,m,t,n} = \begin{cases} D^{\max}_{c,m,t,n} & PR_{c,m,t,n} \le P_{c,m,t,n} \\ D^{\max}_{c,m,t,n}\left[\dfrac{P'_{c,m,t,n} - PR_{c,m,t,n}}{P'_{c,m,t,n} - P_{c,m,t,n}}\right] & P_{c,m,t,n} \prec PR_{c,m,t,n} \prec P'_{c,m,t,n} \\ 0 & PR_{c,m,t,n} \ge b_{c,m,t,n} \end{cases}$$

Demand-price function is a logit function. As Philip (2005) mentions, this kind of function is suitable for reflecting customers' behavior in response to changes in price[41]. Because a little change in price could make significant changes in demand. Figure 2 shows such function. In this function if price of $P_{c,m,t,n}$ was equal to zero and product price range was between $P'_{c,m,t,n}$ and $P_{c,m,t,n}$, the function change to a linear function.

In this study, assumed that optimal price for each product is in reach. $PR_{c,m,t,n}$ Shows the price of product m in zone c and is a non-negative continuous variable. Demand is obtained as follows:

$$D_{c,m,t,n} = \max\left\{\min\left\{D^{\max}_{c,m,t,n},\ D^{\max}_{c,m,t,n}\left[\frac{P'_{c,m,t,n} - PR_{c,m,t,n}}{P'_{c,m,t,n} - P_{c,m,t,n}}\right]\right\}, 0\right\}$$

Since using non-linear demand-price function leads to mixed non-linear programming, discrete price levels is used.

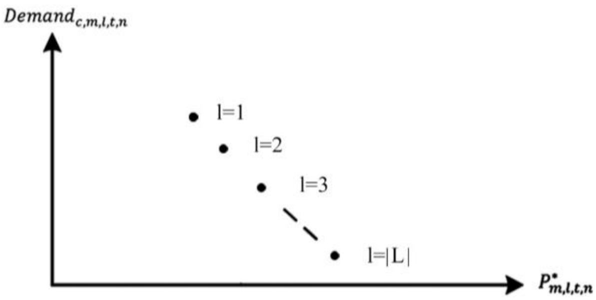

Figure 3- discrete Demand-Price function

Tallury and et al. (2006) use discrete price levels and mention that such approach is applicable in many real-world problems[2]. Ahmadi-Javid and et al. (2014, 2015) and Fattahi and et al. (2015) also used discrete pricing [5, 6, 30]. For each product, in each zone and within each tactical period the price is obtained by:

$$PR_{m,l,t,n} = \frac{\sum_c P_{c,m,t,n} D^{\max}_{c,m,t,n}}{\sum_c D^{\max}_{c,m,t,n}} + \frac{(l-1)}{(L-1)}\left(\frac{\sum_c P'_{c,m,t,n} D^{\max}_{i,m,t,n}}{\sum_c D^{\max}_{c,m,t,n}} - \frac{\sum_c P_{c,m,t,n} D^{\max}_{c,m,t,n}}{\sum_c D^{\max}_{c,m,t,n}}\right)$$

3.2 Mathematical Model

In this section, the SCN redesign model is presented. This model contains location and capacity planning for a three-echelon network.

3.2.1 Objective Function

Model's aim is to maximize supply chain network profit. First sentence is the obtained revenue and the rest is all types of cost in entire network. we supposed that to design/redesign the supply chain network in strategic period t+1, the needed capital should be expensed in strategic period t. Linguistic and symbolic form of objective function are as follows:

$$\text{Maximizing Profit} = \text{Revenue} - \text{Strategic costs} - \text{Tactical costs} - \text{Operational costs}$$

$$\begin{aligned} Max\ Z = &\sum\nolimits_{c,m,l,t/\{0\},n} PR_{c,m,l,t,n} Demand_{c,m,l,t,n} \Delta_{l,m,t,n} \\ &-\sum\nolimits_{w,c,m,t/\{0\},n} TC_{w,c,m,t} x_{w,c,m,t,n} - \sum\nolimits_{p,w,m,t,n} (MC_{p,m,t} + TC_{p,w,m,t}) x_{p,w,m,t,n} \\ &-\sum_{t/\{0\}} \Big(\sum_{j\in p\cup w} \sum_{k_j} \sum_{1}^{t} U_{j,k,\tau} OC_{j,k,\tau} \Big) - \sum_{t/\{0\}} \sum_{j\in Pn\cup Wn} Yn_{j,t} CU_{j,t} - \sum_{t/\{0\}} \sum_{j\in Pe\cup We} (1-Ye_{j,t}) CU_{j,t} \\ &-\sum_{t/\{T\}} \sum_{j\in Pn\cup Wn} (Yn_{j,t+1} - Yn_{j,t}) FO_{j,t+1} - \sum_{t/\{T\}} \sum_{j\in Pe\cup We} (Ye_{j,t} - Ye_{j,t}) CC_{j,t+1} \\ &-\sum_{t/\{T\}} \sum_{j\in p\cup w} U_{j,k,t+1} AC_{j,k,t+1} \end{aligned}$$

3.2.2 Equations

This section deals with constraints of proposed model and their explanations.

$$Yn_{j,t-1} \le Yn_{j,t} \qquad j \in Pn \cup Wn, t \in T\ /\{0\} \tag{1}$$

$$Ye_{j,t-1} \le Ye_{j,t} \qquad j \in Pe \cup We, t \in T\ /\{0\} \tag{2}$$

$$\sum_{k_j} U_{j,k,t} \le Yn_{j,t} \qquad j \in Pn \cup Wn, t \in T\ /\{0\} \tag{3}$$

$$\sum_{k_j} U_{j,k,t} \le 1 - Ye_{j,t} \qquad j \in Pe \cup We, t \in T\ /\{0\} \tag{4}$$

$$\sum_{1}^{t} \sum_{k_j} Q_{j,k} U_{j,k,\tau} \le Q_j^{\max} Yn_{j,t} \qquad j \in Pn \cup Wn, t \in T\ /\{0\} \tag{5}$$

$$\sum_{1}^{t} \sum_{k_j} Q_{j,k} U_{j,k,\tau} + Q_j^{e} (1 - Ye_{j,t}) \le Q_j^{\max} (1 - Ye_{j,t}) \qquad j \in Pe \cup We, t \in T\ /\{0\} \tag{6}$$

$$\sum_{m} \gamma_m^{p} \sum_{w} x_{j,w,m,t,n} \le U_j^{\max} \sum_{1}^{t} \sum_{k_j} Q_{j,k} U_{j,k,\tau} \qquad j \in Pn, t \in T\ /\{0\}, n \in N \tag{7}$$

$$\sum_{m} \gamma_m^{p} \sum_{w} x_{j,w,m,t,n} \le U_j^{\max} \sum_{1}^{t} \sum_{k_j} Q_{j,k} U_{j,k,\tau} + Q_j^{e} (1 - Ye_{j,t}) \qquad j \in Pe, t \in T\ /\{0\}, n \in N \tag{8}$$

$$\sum_{m}\gamma_{m}^{w}\sum_{w}x_{j,c,m,t,n}\leq U_{j}^{max}\sum_{1}^{t}\sum_{k}\sum_{j}Q_{j,k}U_{j,k,\tau}\quad j\in Wn,t\in T/\{0\},n\in N \tag{9}$$

$$\sum_{m}\gamma_{m}^{w}\sum_{w}x_{j,c,m,t,n}\leq U_{j}^{max}\sum_{1}^{t}\sum_{k}\sum_{j}Q_{j,k}U_{j,k,\tau}+Q_{j}^{e}(1-Ye_{j,t})\quad j\in We,t\in T/\{0\},n\in N \tag{10}$$

$$\sum_{l}\Delta_{m,l,t,n}=1\quad m\in M,t\in T/\{0\},n\in N \tag{11}$$

$$\sum_{w}x_{w,c,m,t,n}=\sum_{l}\Delta_{m,l,t,n}Demand_{c,m,l,t,n}\quad c\in C\ m\in M,t\in T/\{0\},n\in N \tag{12}$$

$$\sum_{c}x_{w,c,m,t,n}=\sum_{p}x_{p,w,m,t,n}\quad w\in W,m\in M,t\in T/\{0\},n\in N \tag{13}$$

$$x_{O,D,m,t,n}\geq 0$$

$$Yn_{j,t},Ye_{j,t},U_{j,k,t},\Delta_{m,l,t,n}\in\{0,1\}$$

First part of objective function determines the income from the sale. Other parts represent costs. First part includes costs related to distribution from distribution centers to retailer's zones. Second sentence is production costs and distribution costs from production plants. Third part represents operational cost of added capacity. Fourth and fifth parts represents operational cost of new and existing plants and distribution centers. The sixth and seventh part is the establishing cost of new and existing plants and distribution centers. The last part is fix cost of added capacity to them. Constraints 1-6 include conditions required for opening new facilities, closing existing facilities and assigning capacity to them. Constraint (1) insists that if a facility opens in period t could not be closed for the next period. Constraint (2) shows that a closing facility in period t should remain close for the next period. Constraints (3) and (4) represents how to assign capacity to facilities. New facilities have the same capacity during a strategic period. A closed facility does not have any capacity and a new facility will have capacity after establishment. Constraints (5) and (6) limit the improvement of facilities capacities within their total possible capacity. Constraints (7) and (8) guarantees that all produced products by a plant within a period is less than its capacity. Constraints (9) and (10) provide the same condition for distribution centers. According to constrain (11) for each product in a tactical period there is only one price level. Forcing to meet all demands in every customer zone represents in constraint (12). In 13th constrain the sum of sent products from a distribution center to customers must be equal to products sent to distribution center. Other constraints determine which variables are non-negative or binary.

4. Solution Approach

As mentioned in the previous part the model is mixed integer programing. Previous researches used heuristic, meta-heuristic and exact approaches to solve SCDN problems [42]. There are two reasons which make using exact approaches more appropriate for these kinds of problems. On the one hand, remarkable cost of strategic decisions in designing/redesigning of supply chain networks makes precise choosing within available decisions necessary. So, it seems reasonable to implement exact approaches to solve these problems. On the other hand, such problems are high-dimensional problems and by increasing the dimension the solution time even increases more. For example, by optimization software such as Gams, Lingo and CPLEX solution time is not reasonable as problem's size increases. Recently this exact method with significant performance has been used to solve network design problems [43, 44]. Dividable to smaller problems Capability of this model makes Benders decomposition method a useful exact approach with suitable productivity in high dimensional problems.

4.1 Benders decomposition method

In this part benders decomposition method is represented. Suppose the original problem is as follows: (model A)

$$\min = \sum_{i=1}^{n} C_i X_i + \sum_{j=1}^{m} D_j Y_j$$

s.t:

$$\sum_{i=1}^{n} A_{li} X_i + \sum_{j=1}^{m} E_{lj} Y_j \le B^{(l)}, l = 1,...,q \quad (14)$$

$$0 \le X_i \le X_i^{up}, i = 1,...,n \quad (15)$$

$$0 \le Y_j \le Y_j^{up}, j = 1,...,m \quad (16)$$

In this model, x variables are complicating variables which if fixed temporary decreases solution time remarkably[45]. Binary variables are complicating variables.

(Model B)

$$\min = \sum_{i=1}^{n} C_i X_i + \alpha$$

S.t:

$$\sum_{i=1}^{n} Y_i^{(k)} (X_i - X_i^{(k)}) + \sum_{j=1}^{m} D_j Y_j^{(k)} \le \alpha, k = 1,...,(rep-1) \quad (17)$$

$$0 \le X_i \le X_i^{up}, i = 1,...,n \quad (18)$$

$$\alpha^{down} \le \alpha \quad (19)$$

Model B is called master problem which includes only complicating variables. Constrain (17) is known as benders decomposition cut (BDC). For the first repetition, master problem is solved without BDCs and results will be used for Sup problem. In the next repeat, based on Sup problem results a new cut will be added. This process will continue till achievement to optimal result. As noted in master problem, *rep* is the number of repetition and α-down is a lower bound for α which is obtained from specialists. A suitable lower bound could reduce the time significantly.

(Model C)

$$\min = \sum_{i=1}^{n} C_i X_i + \sum_{j=1}^{m} D_j Y_j$$

S.t:

$$\sum_{i=1}^{n} A_{li} X_i + \sum_{j=1}^{m} E_{lj} Y_j \le B^{(l)}, l = 1,...,q \quad (20)$$

$$0 \le Y_j \le Y_j^{up}, j = 1,...,m \quad (21)$$

$$X_i = x_i^{(k)} : \gamma_i \; i = 1,...,n \quad (22)$$

Model C is called Sup problem and is the same as the model A but without complicating variables. γ_i Which used in BDC is equal to the optimal value of dual variable of constrain (22).

4.2 Benders decomposition method implementation

According to the Benders decomposition, the model in this paper can be implemented to the problem of this research as follows:

Master problem:

$$\begin{aligned} Max\,Z = &\sum\nolimits_{c,m,l,t/\{0\},n} PR_{c,m,l,t,n} Demand_{c,m,l,t,n} \Delta_{l,m,t,n} \\ &-\sum_{t/\{0\}} \Big(\sum_{j\in p\cup w} \sum_{k_j} \sum_{1}^{t} U_{j,k,\tau} OC_{j,k,\tau} \Big) - \sum_{t/\{0\}} \sum_{j\in P\cup W} Yn_{j,t} CU_{j,t} - \sum_{t/\{0\}} \sum_{j\in Pe\cup We} (1-Ye_{j,t}) CU_{j,t} \\ &-\sum_{t/\{0\}} \sum_{j\in Pn\cup Wn} (Yn_{j,t+1} - Yn_{j,t}) FO_{j,t+1} - \sum_{t/\{0\}} \sum_{j\in Pe\cup We} (Ye_{j,t} - Ye_{j,t}) CC_{j,t+1} \\ &-\sum_{t/\{0\}} \sum_{j\in p\cup w} U_{j,k,t+1} AC_{j,k,t+1} \end{aligned} \tag{23}$$

Equations

$$Yn_{j,t-1} \le Yn_{j,t} \qquad j \in Pn \cup Wn, t \in T\ /\{0\} \tag{24}$$

$$Ye_{j,t-1} \le Ye_{j,t} \qquad j \in Pe \cup We, t \in T\ /\{0\} \tag{25}$$

$$\sum_{k_j} U_{j,k,t} \le Yn_{j,t} \qquad j \in Pn \cup Wn, t \in T\ /\{0\} \tag{26}$$

$$\sum_{k_j} U_{j,k,t} \le 1 - Ye_{j,t} \qquad j \in Pe \cup We, t \in T\ /\{0\} \tag{27}$$

$$\sum_{1}^{t} \sum_{k_j} Q_{j,k} U_{j,k,\tau} \le Q_j^{max} Yn_{j,t} \qquad j \in Pn \cup Wn, t \in T\ /\{0\} \tag{28}$$

$$\sum_{1}^{t} \sum_{k_j} Q_{j,k} U_{j,k,\tau} + Q_j^{e}(1 - Ye_{j,t}) \le Q_j^{max}(1 - Ye_{j,t}) \qquad j \in Pe \cup We, t \in T\ /\{0\} \tag{29}$$

$$\sum_{l} \Delta_{m,l,t,n} = 1 \quad m \in M, t \in T\ /\{0\}, n \in N \tag{30}$$

$$Yn_{j,t}, Ye_{j,t}, U_{j,k,t}, \Delta_{m,l,t,n} \in \{0,1\} \tag{31}$$

The sub-problem can be defined as:

$$Max\,Z = -\sum\nolimits_{w,c,m,t/\{0\},n} TC_{w,c,m,t}\, x_{w,c,m,t,n} - \sum\nolimits_{p,w,m,t,n} (MC_{p,m,t} + TC_{p,w,m,t})\, x_{p,w,m,t,n} \tag{32}$$

S.t:

$$\sum_{m} \gamma_m^{p} \sum_{w} x_{j,w,m,t,n} \le U_j^{max} \sum_{1}^{t} \sum_{k_j} Q_{j,k} \bar{U}_{j,k,\tau} \qquad j \in Pn, t \in T\ /\{0\}, n \in N \tag{33}$$

$$\sum_{m} \gamma_m^{p} \sum_{w} x_{j,w,m,t,n} \le U_j^{max} \sum_{1}^{t} \sum_{k_j} Q_{j,k} \bar{U}_{j,k,\tau} + Q_j^{e}(1 - \bar{Y}e_{j,t}) \qquad j \in Pe, t \in T\ /\{0\}, n \in N \tag{34}$$

$$\sum_{m}\gamma^{w}_{m}\sum_{w}x_{j,c,m,t,n} \leq U^{\max}_{j}\sum_{1}^{t}\sum_{kj}Q_{j,k}\overline{U}_{j,k,\tau} \quad j \in Wn, t \in T/\{0\}, n \in N \tag{35}$$

$$\sum_{m}\gamma^{w}_{m}\sum_{w}x_{j,c,m,t,n} \leq U^{\max}_{j}\sum_{1}^{t}\sum_{kj}Q_{j,k}\overline{U}_{j,k,\tau} + Q^{e}_{j}(1-\overline{Ye}_{j,t}) \quad j \in We, t \in T/\{0\}, n \in N \tag{36}$$

$$\sum_{w}x_{w,c,m,t,n} = \sum_{l}\overline{\Delta}_{m,l,t,n} Demand_{c,m,l,t,n} \quad c \in C, m \in M, t \in T/\{0\}, n \in N \tag{37}$$

$$\sum_{c}x_{w,c,m,t,n} = \sum_{p}x_{p,w,m,t,n} \quad w \in W, m \in M, t \in T/\{0\}, n \in N \tag{38}$$

$$x_{O,D,m,t,n} \geq 0$$

Table 2 shows dual variables of sub-problem which obtained by solving sub-problem dual problem.

Table 2.Dual variables

Constraint	Dual variable
(31)	$A_{Pn,t,n}$
(32)	$A_{Pe,t,n}$
(33)	$A_{Wn,t,n}$
(34)	$A_{We,t,n}$
(35)	$\partial_{c,m,t,n}$
(35)	$\theta_{w,m,t,n}$

After define dual variables of Sub problem, Dual sub problem can define as follow:

$$\min = \sum_{j \in Pn \cup Wn}(U^{\max}_{j}\sum_{1}^{t}\sum_{kj}Q_{j,k}\overline{U}_{j,k,\tau})A_{j,t,n} + \sum_{j \in Pe \cup We}(U^{\max}_{j}\sum_{1}^{t}\sum_{kj}Q_{j,k}\overline{U}_{j,k,\tau} + Q^{e}_{j}(1-\overline{Ye}_{j,t}))A_{j,t,n} + \sum_{c,m,l,t,n} Demand_{c,m,l,t,n}\overline{\Delta}_{m,l,t,n}\partial_{c,m,t,n} \tag{39}$$

Equation

$$-\theta_{w,m,t,n} + \gamma^{j}_{m}A_{j,t,n} \geq -MC_{j,m,t} - TC_{j,w,m,t} \qquad j \in pn, w, m, t, n \tag{40}$$

$$-\theta_{w,m,t,n} + \gamma^{j}_{m}A_{j,t,n} \geq -MC_{j,m,t} - TC_{j,w,m,t} \qquad j \in pe, w, m, t, n \tag{41}$$

$$\theta_{j,m,t,n} + \gamma^{j}_{m}A_{j,t,n} + \partial_{c,m,t,n} \geq -TC_{j,c,m,t} \qquad j \in wn, c, m, t, n \tag{42}$$

$$\theta_{j,m,t,n} + \gamma^{j}_{m}A_{j,t,n} + \partial_{c,m,t,n} \geq -TC_{j,c,m,t} \qquad j \in we, c, m, t, n \tag{43}$$

$$A_{j,t,n} \geq 0$$

$$\theta_{j,m,t,n}, \partial_{c,m,t,n} \sim URS$$

Now based on sub-problem dual problem, master problem would determine an upper bound in each iteration for original problem. Optimal cuts and feasible cuts for this model are as follows:

$$\begin{aligned} UB \leq & \sum_{j \in Pn \cup Wn, t, n} \Big(U_j^{\max} \sum_{1}^{t} \sum_{k_j} Q_{j,k} \overline{U}_{j,k,\tau}\Big) A_{j,t,n} + \sum_{j \in Pe \cup We, t, n} \Big(U_j^{\max} \sum_{1}^{t} \sum_{k_j} Q_{j,k} \overline{U}_{j,k,\tau} + Q_j^{e}(1-\overline{Ye}_{j,t})\Big) A_{j,t,n} \\ & + \sum_{c,m,l,t,n} Demand_{c,m,l,t,n} \overline{\Delta}_{m,l,t,n} \sigma_{c,m,t,n} + \sum\nolimits_{c,m,l,t/\{0\},n} PR_{c,m,l,t,n} Demand_{c,m,l,t,n} \Delta_{l,m,t,n} \\ & - \sum_{t/\{0\}} \Big(\sum_{j \in p \cup w} \sum_{k_j} \sum_{1}^{t} U_{j,k,\tau} OC_{j,k,\tau} \Big) - \sum_{t/\{0\}} \sum_{j \in P \cup W} Yn_{j,t} CU_{j,t} - \sum_{t/\{0\}} \sum_{j \in Pe \cup We} (1-Ye_{j,t}) CU_{j,t} \\ & - \sum_{t/\{0\}} \sum_{j \in Pn \cup Wn} (Yn_{j,t+1} - Yn_{j,t}) FO_{j,t+1} - \sum_{t/\{0\}} \sum_{j \in Pe \cup We} (Ye_{j,t} - Ye_{j,t}) CC_{j,t+1} \end{aligned} \tag{44}$$

$$\begin{aligned} 0 \leq & \sum_{j \in Pn \cup Wn, t, n} \Big(U_j^{\max} \sum_{1}^{t} \sum_{k_j} Q_{j,k} \overline{U}_{j,k,\tau}\Big) A_{j,t,n} + \sum_{j \in Pe \cup We, t, n} \Big(U_j^{\max} \sum_{1}^{t} \sum_{k_j} Q_{j,k} \overline{U}_{j,k,\tau} + Q_j^{e}(1-\overline{Ye}_{j,t})\Big) A_{j,t,n} \\ & + \sum_{c,m,l,t,n} Demand_{c,m,l,t,n} \overline{E}_{m,l,t,n} \sigma'_{c,m,t,n} \end{aligned} \tag{45}$$

Now based on proposed description for benders decomposition method, its algorithm would be like this:

ABD algorithm

1. **LB= $-\infty$ and UB= $+\infty$**
2. **Consider the initial feasible solution**
3. **Iter = 1**
4. **While (UB-LB)>ε**
   Solve DSP with object function (39) subject to (40-43)

   If solved optimally

   - **add optimal cut**
   - **Update LB**
   - **Solve MP with object function (23) subject to (24-31) and (44)**

   Else

   - **add feasibility cut and solve MP with object function (23) subject to (24-31) and (45)**

5.iter = iter+1

Results

For analyzing efficiency of benders decomposition method 10 different samples is solved and the results gathered in tables 3 and 4. Size and solution time by benders decomposition and Gams and gap between objective values of CPLEX and BD are represented in tables 3 and 4. For solving the samples a PC is used with Intel® core i7-4790k, CPU @ 4.00 GHz and 16.0 GB RAM and commercial software GAMS 24.1.2 and CPLEX solver for MILP. It is obvious new method improves solution time for every problem remarkably.

For showing the effectiveness of benders decomposition method, 10 different problems in small, medium

and large scale are used. Table 3 represents the characteristics of these problems. To analyze the performance of the proposed algorithm samples which was used in Fattahi's [6] paper is solved.
As it is showed in table 4, small and medium scale problems are solved in less than 160 seconds with gap fewer than1 percent and large-scale ones solved with benders decomposition method less than 1000 seconds and gap less than 1.67 percent. Fattahi's large-scale results shows that the minimum solution time is around 6000 seconds. Therefore, the proposed benders decomposition algorithm would be suitable for redesigning a supply chain network with pricing.

Table 3-Sample' size

Size	num	PE	PN	WE	WN	m	c	l	k	t	n
small	T1	1	3	1	4	3	8	4	4	4	4
	T2	1	4	2	4	3	10	4	4	4	4
	T3	1	4	2	6	4	10	4	4	4	4
medium	T4	1	5	2	8	5	12	4	4	4	4
	T5	2	6	4	10	6	16	5	5	4	4
	T6	2	6	4	12	7	17	5	5	4	5
large	T7	2	7	4	14	7	18	5	5	4	5
	T8	2	10	4	15	10	30	5	5	4	5
	T9	2	12	4	17	11	40	5	5	4	5
	T10	2	15	4	20	15	50	5	5	4	5

Table 4- Results comparison table

Size	Cplex Time(second)	BD Time(second)	Iter	Gap
1	615	41	49	0
2	4032	26	30	0.4%
3	15105	64	66	0.18%
4	17706	89	48	0.02%
5	≥ 20000	94	49	0.68%
6	≥ 20000	154	43	0.61%
7	≥ 20000	186	33	1.3%
8	≥ 20000	467	39	1.67%
9	≥ 20000	683	29	1.17%
10	≥ 20000	917	31	1.76%

Conclusion

In this study a new model for redesigning a multi-period and multi-product supply chain network with considering pricing is proposed. In represented model, reducing price would lead to increase in demand and therefore need for adding more capacities and opening new facilities within SCN and increasing the price has opposite effects. This model helps choosing the best price in order to maximize the network revenue. There are several opportunities to develop the current paper. All the parameters in this paper are assumed to be determined which is unlikely to happen in real conditions. In this manner it would be appropriate to consider parameters such as transportation cost, stochastic. This paper only considers the effect of price on customer's behavior. But there are other characteristics like quality that might change customer's choice. Therefore, it would be appropriate to consider stochastic pricing. In this model fixed and unique prices are assigned to each customer zones. Assigning different prices to different zones would make the problem more interesting and a good chance to analyze network profit under this situation. In this study, benders decomposition method is applied which reduces solution time significantly. In this model all of the parameters are fixed and determined. It is also suggested to use accelerated benders algorithm methods to improve benders decomposition method such as valid inequalities, L-shape and Pareto optimal accelerator.

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