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RESEARCH PAPER

Expanded Fraction Defective Chart using Cornish-Fisher Terms with Adjusted Control Limits to Improve In-control Performance

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ABSTRACT

The number of nonconforming items in a sample is monitored using the fraction defective known as the np-chart. The performance of the np-chart in Phase II depends on the accuracy of the estimated parameter in Phase I. Although taking large sample sizes ensures the accuracy of the estimated parameter, it can be impractical for attributes in some cases. Recently, the traditional c-chart and the np-chart with some adjustments have been studied to guarantee the in-control performance. Due to technology progresses, researchers have faced high-quality processes with a very low rate of nonconformity, for which traditional control charts are inadequate. To ameliorate such inaccuracy, this study develops a new method for designing the np-chart, such that the in-control performance is guaranteed with a pre-defined probability. The proposed method uses Cornish-Fisher expansions and the bootstrap method to guarantee the desired conditional in-control average run length. Through a simulation study, this study shows that the proposed adjustments improve the np-charts' in-control performance.

KEYWORDS: *np-chart, Adjusted limits,Cornish-Fisher expansions,Bootstrap,Average run length (ARL).*

1. Introduction

Control charts are the most popular tools for monitoring a stable process against assignable cause(s). A common tool to monitor the attributes of a process is the attribute control chart (e.g., see [1] and [2]). Among them, the traditional *p*-chart is the standard attribute control chart. It monitors the fraction of nonconforming products in a process, and it is developed based on the Binomial distribution. The control limits are estimated using a Normal distribution [3].

In determining the chart's control limits, the nonconforming probability for the in-control process (p_0) should be given. If it is unknown, it should be estimated by m available in-control samples with size n. This is called Phase I. A good overview of Phase I activities and analyses can be found in [4]. In Phase II, the aim is to detect out-of-control situations as fast as possible. During Phase II, a usual measure of the performance is called the average run length (ARL). The in-control ARL (ARL_0) is the expected number of samples observed before a false alarm, i.e., when there is no shift in the process parameters. Thus, a larger value for ARL_0 is desirable. On the other hand, the out-of-control ARL (ARL_1) is the expected number of samples needed before the detection of a shift in the process parameter. The tendency toward the rapid detection of an assignable cause necessitates smaller values for ARL_1. Furthermore, the median of ARL (MDRL), the average of ARL (AARL), and the standard deviation of ARL (SDARL), which is also referred to as practitioner-to-practitioner variability, can be expressed as other performance measures for comparison purposes.

Due to different Phase I datasets (sampling distribution), the performance of control charts with various estimated parameters can be different. On the other hand, since the control limits are constructed using the estimated parameter and a pre-specified false alarm rate

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(FAR), the accuracy of the estimation plays a significant role in determining the performance values [5]. In the recent control chart literature, the impacts of parameter estimation on the performance measures have received the attention of several researchers. Most of the related research works have focused on the variable control charts and largely ignored the attribute charts (see e.g., [6] and [7]).

Mainly, there are two issues regarding the parameter's estimation: first, 1) there is no guarantee that the desired in-control performance is met and, thus, the SDARL plays an important role in this regard that should not be neglected (See, e.g., [8] and [9]) and 2) In some cases, the suggested Phase I sample sizes are impractical to have good protection against the variation in the in-control ARL between practitioners.

The overall conclusion indicates that the existence of an alternative method is necessary to overcome the above-mentioned challenges. Efron [10] introduced the concept of bootstrapping as an estimation method. It is an estimation method with fewer assumptions while giving accurate results. Therefore, the bootstrap method has turned into an attractive technique to use for a wide range of problems. Jones and Steiner [8] studied the effects of the estimation error on the performance of the risk-adjusted CUSUM charts. Gandy and Kvaløy [11] developed a bootstrap technique to guarantee the desired in-control conditional performance of control charts with a certain probability. The extensions of the proposed method for the S^2, EWMA, and *c* charts can be found in [12], [13], and [14], respectively. In order to avoid the errors of the bootstrap procedures, especially when the number of Phase I samples is small, Guo and Wang [15] developed an exact method to adjust the control limits of two-sided S^2 charts to achieve the desired conditional in-control performance. In the context of the S^2 chart, Aparisi et al. [16] proposed a new approach in the framework of an optimization problem that simultaneously treats the choice of the number of Phase I samples and the control limit adjustment by taking both the desired in-control and out-of-control performances as constraints. Vakilian et al. [17] applied the bootstrapping method to adjust the control and the warning limits of *c*-charts with adaptive sampling schemes, such as variable sample size, variable sampling intervals, and variable parameters. Diko et al. [18] studied the impact of practitioner to practitioner variability on the performance of the Phase II EWMA chart to give recommendations about the required number of Phase I subgroups to achieve nominal performance. For the *np*-charts, Faraz et al. [19] assessed alternative control limits and estimated parameters and applied the bootstrap method to adjust control limits' thresholds. All these researches indicated that the requirement of impractically large amounts of Phase I data is a challenge when a practitioner needs to have ARL_0 close to the desired value. In fact, the problem of reaching a desired ARL_0 with smaller amounts of Phase I data can be solved using a bootstrap method to adjust control limits.

On the other hand, because of the recent high-tech developments, high-quality processes with a very low rate of nonconformity are often detected in practice, where using the traditional chart leads to a high false alarm rate and, consequently, increases inspection costs. For these reasons, alternative methods have been proposed recently [20]. Among them, Winterbottom [21] introduced an improved *p*-chart with one correction term based on the Cornish-Fisher (C-F) expansion. According to their results, the improved *p*-chart a) shows false alarm risk much closer to the reference risk and b) allows working with smaller values for p_0. This correction was developed later in [22] for the *np*-chart, which is more convenient to monitor the number of nonconforming items in practice.

This paper introduces the C-F corrected *np*-chart, previously presented in the Ref. [22], to investigate its in-control performance for the first time such that it would exceed the desired value with a specified probability. Moreover, a bootstrap-based methodology is proposed to adjust the control limits of the C-F corrected *np*-charts. Then, the results of both methods are compared for any improvements in the performance metric. The rest of the paper is organized as follows: In Section 2, the improved *np*-chart using Cornish-Fisher expansion is reviewed. In the next section, bootstrap adjusted limits for the improved *np*-charts are described, and the procedure is presented in some steps. Section 4 includes some simulation experiments for investigating the sampling distribution of the in-control performance for the improved *np*-chart and, then, comparing the results with the bootstrap-based methodology. Finally, conclusions and future research are given in the last section.

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2. Improved Np-Charts Using Cornish-Fisher Expansion

A common approach to modeling the number of nonconforming items is to use the binomial distribution. Let us assume that $X_1, X_2, \ldots, X_m$ is an independent sequence of m initial samples with size n from a binomial distribution with parameter p_0 that is $X_i \sim Bin\ (n, p_0)$ for i=1,…,m. However, the value of p_0 is typically unknown in practice, and it should be estimated. This parameter can be estimated as follows [3]:

$$\overline{p} = \frac{\sum_{i=1}^{m} p_i}{m} = \frac{\sum_{i=1}^{m} X_i}{mn} \tag{1}$$

Given the estimated parameter, the upper control limit (UCL) and the lower control limit (LCL) of the *np*-chart can be estimated as follows:

$$\begin{cases} U\hat{C}L = [n\overline{p} + z_{1-\alpha/2}\sqrt{n\overline{p}(1-\overline{p})}\,] \\ L\hat{C}L = \max(0, [n\overline{p} - z_{1-\alpha/2}\sqrt{n\overline{p}(1-\overline{p})}\,]) \end{cases} \tag{2}$$

where $z_{1-\alpha/2}$ is the $(1-\alpha/2)^{th}$ percentile of the standard normal distribution. Note that when $LCL \leq 0$, the lower control limit holds no meaning and lies on zero; thus, the upper control limit is changed by substituting $z_{1-\alpha}$ instead of $z_{1-\alpha/2}$.

For high-quality processes with a very low rate of nonconformity, Winterbottom [21] proposed C-F corrected p-chart for being effective, accurate, and practical. The upper control limits of this expansion accommodated for *np*-chart are as follows [22]:

$$U\hat{C}L_1 = n\overline{p} + z_{1-\alpha/2}[n\overline{p}(1-\overline{p})]^{1/2} + \frac{(z_{1-\alpha/2}^2 - 1)(1-2\overline{p})}{6} \tag{3}$$

where $U\hat{C}L_1$ denotes upper control limits with one C-F correction term. When $z_{1-\alpha/2}$ is substituted by $z_{\alpha/2}=-z_{1-\alpha/2}$, a lower control limit can be obtained. By setting $z_{1-\alpha/2}=3$, the Shewhart *np*-chart can be corrected as follows:

$$\begin{cases} U\hat{C}L_1 = [n\overline{p} + z_{1-\alpha/2}\sqrt{n\overline{p}(1-\overline{p})} + \dfrac{4(1-2\overline{p})}{3}] \\ L\hat{C}L_1 = \max(0, [n\overline{p} - z_{1-\alpha/2}\sqrt{n\overline{p}(1-\overline{p})} + \dfrac{4(1-2\overline{p})}{3}]) \end{cases} \tag{4}$$

Authors [19] derived alternative control limits for the *np*-chart in order to 1) adjust the control limits so that ARL_0 can have a desired value at least, 2) avoid masking the problem of practitioner-to-practitioner variability using the three-sigma limits, and 3) split α as equally as possible between the two sides of the chart. The equations are as follows:

$$L\hat{C}L = F^{-1}(\alpha/2, n, \overline{p})$$
$$U\hat{C}L = \begin{cases} F^{-1}(1-\alpha/2, n, \overline{p}) & if\ L\hat{C}L \geq 1 \\ F^{-1}(1-\alpha, n, \overline{p}) & if\ L\hat{C}L = 0 \end{cases} \tag{5}$$

where $F^{-1}(\alpha, n, p_0)$ is an inverse cumulative distribution function of $Bin\ (n, p_0)$ at point α.

Despite the fact that the ARL_0 comes to be a random variable in Phase II, the conditional ARL_0 given the estimated nonconforming probability is obtained below:

$$ARL_0|\overline{p} = \frac{1}{\hat{\alpha}|\overline{p}}; \quad \hat{\alpha}|\overline{p} \neq 0 \tag{6}$$

Where

$$\hat{\alpha}|\overline{p} = \begin{cases} 1 - F(U\hat{C}L, n, p_0) + F(L\hat{C}L - 1, n, p_0) & if\ L\hat{C}L \geq 1 \\ 1 - F(U\hat{C}L, n, p_0) & if\ L\hat{C}L = 0 \end{cases} \tag{7}$$

where $F(x, n, p_0)$ is the cumulative distribution function of $Bin\ (n, p_0)$ at point x. Note that $L\hat{C}L$ and $U\hat{C}L$ are general terms, and their subscripts can vary according to Equation (3).

When the number of Phase I datasets (m) is not adequate enough, the probability of achieving the desired performance is small. In order to guarantee the desired in-control performance with a probability of $(1-\tau)100\%$, it is proposed to adjust limits using the bootstrap technique as described in the following section (τ is used to define percentiles).

3. Bootstrap Adjusted Limits for the Improved Np-Charts

In the previous studies, it has been revealed that the in-control performance is affected by the Phase I sampling variability or practitioner-to-practitioner variability. In fact, the average in-control performance value might be close to the target value, while a single in-control value can vary considerably. The bootstrap approach has been suggested to reduce the effect of sampling variability on the ARL performance with estimated parameters in recent literature. Similarly, in this section, a bootstrap-based methodology is applied to adjust the limits of the C-F corrected *np*-charts such that the conditional in-control performance meets or exceeds the

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desired value with a specified probability. The bootstrap algorithm is outlined below:

1. Estimate process parameter from a given Phase I dataset, called "training sample", using Equation (1).
2. Draw $y_i^* \sim Bin(mn, \overline{p})$ to estimate $p_i^* = y_i^* / mn$.
3. Calculate the improved *np*-chart limits using the C-F correction for each bootstrap estimated as follows:

$$L\hat{C}L_i^* = F^{-1}(\alpha/2, n, p_i^*)$$
$$U\hat{C}L_i^* = \begin{cases} F^{-1}(1-\alpha/2, n, p_i^*) & if\ L\hat{C}L_i \geq 1 \\ F^{-1}(1-\alpha, n, p_i^*) & if\ L\hat{C}L_i = 0 \end{cases} \tag{8}$$

4. Repeat Steps 2-3 many times, called bootstrap sample size, to obtain $i=1,...,B$ control limits:
$\{L\hat{C}L_1^*, L\hat{C}L_2^*, ..., L\hat{C}L_B^*\}$
$\{U\hat{C}L_1^*, U\hat{C}L_2^*, ..., U\hat{C}L_B^*\}$
5. Sort the B bootstrap control limits from Step 4 of increasing order as follows:
$\{L\hat{C}L_{(1)}^*, L\hat{C}L_{(2)}^*, ..., L\hat{C}L_{(B)}^*\}$
$\{U\hat{C}L_{(1)}^*, U\hat{C}L_{(2)}^*, ..., U\hat{C}L_{(B)}^*\}$
6. Find respectively $L\hat{C}L_\tau^*$ and $U\hat{C}L_\tau^*$ as the τ^{th} and $(1-\tau)^{th}$ percentiles to guarantee the in-control performance with probability $(1-\tau)100\%$. For cases with no lower control limit, store only the $(1-\tau)^{th}$ percentile.

Generally, it is expected that the adjusted control limits obtained based on the percentiles of bootstrap distribution result in widened control limits to counteract the effect of estimation errors caused by the unknown parameters.

4. Simulation Study

In this section, some simulation experiments are implemented with the aim of exploring the effect of parameter estimation on the behavior of the performance measure. It is obvious that when the process parameter is estimated, the calculated control limits and, as a result, the ARL_0 measure turn into functions of $\overline{p}$. Therefore, it is expected that the performance measure will become a random variable with a mean and the standard deviation.

In this study, r=10,000 different Phase I samples were simulated consisting of m samples each from $Bin(n, p_0)$. By choosing a desired false alarm rate, the control limits with C-F corrections were calculated for each Phase I dataset. For α=0.0027 and α=0.005, Tables 1 and 2 show the 10% and 25% percentiles, the median, the average, and the standard deviation of ARL_0 distribution following the computation of the control limits for various cases of the simulation (the case $m=\infty$ relates to the situation with a known process parameter, in which the performance measure becomes a constant value). Some conclusions about the effects of the number of Phase I datasets, the sample size, and the false alarm rate on the amount of guaranteed performance are summarized as follows:

- The effect of m: for example, when p_0=0.2, n=100, and α=0.0027, we have quartile $Q_{0.25}$=547 for m=25. That is, with 50 Phase I datasets, the desired ARL_0=370.4 can be guaranteed with a probability rate of 75%. Moreover, $Q_{0.10}$ goes beyond the desired value for the first time when $m\geq75$, which means that 75 samples are enough to guarantee $Pr(ARL_0>370.4)=90\%$.
- The effect of n: for the above-mentioned example, we have $SDARL_0$=179.03, while this value increases to 489.62 for n=50. That is, for larger sample sizes, the desired ARL_0 can be obtained with a smaller standard deviation and, thus, higher precision.
- The effect of α: the results are remarkable for α=0.005. In fact, the desired ARL_0 is guaranteed with a probability rate of 75% for m=50 and larger. However, in order to reach the desired ARL_0 with a probability of 90%, at least, m=125 samples are required. Nevertheless, there are some exceptions where m=25 samples are enough to have $Pr(ARL_0>200)=75\%$ and m=50 samples are enough to have $Pr(ARL_0>200)=90\%$.

Joekesa and Barbosa [23] suggested selecting the suitable chart as follows:

1. When $np(1-p)\geq5$ without correction,
2. When $np(1-p)\geq0.25$ one term of correction,
3. When $np(1-p)\geq0.08$ two terms of correction.

In this study, for n=50, $np(1-p)$ values are obtained as equal to 0.495, 0.98, 2.375, 4.5, and 8, respectively, for process parameters 0.01, 0.02,

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0.05, 0.10, and 0.20. For n=100, $np(1-p)$ values are obtained for the same process parameters as 0.99, 1.96, 4.75, 9, and 16, respectively. Faraz et al. [19] applied the np-chart without correction for the same simulation study. The comparison of their results in the Ref. [19] and the results of np-chart with one term of correction presented here indicates that the latter has superior performance for smaller p_0 values lower than 0.1. Such results correspond to the conclusion made in [21]. Moreover, according to the second rule suggested in [23], the np-chart with one term of correction is expected to have better performance for the smaller process parameters 0.01, 0.02, 0.05 by satisfying $np(1-p) \geq 0.25$.

Despite the fact that C-F corrected np-chart allows working with smaller parameters, there are cases in Tables 1 and 2 where no practical amount of Phase I datasets can guarantee the desired ARL_0 with a probability rate of 90%, and/or even with 75%. Therefore, the control limits should be adjusted. For this reason, a similar study was performed for the chart with the bootstrap adjusted limits by considering B=500.

Tab. 1. The distribution of ARL_0 for improved *np*-chart with one C-F correction term for different values of *n*, *m*, and p_0 when α=0.0027.

n		50					100				
		Lower Quartiles					*Lower Quartiles*				
p_0	m	$Q_{0.10}$	$Q_{0.25}$	$MARL_0$	$AARL_0$	$SDARL_0$	$Q_{0.10}$	$Q_{0.25}$	$MARL_0$	$AARL_0$	$SDARL_0$
0.01	25	72.37	626.50	626.50	1407.12	2168.71	291.35	291.35	291.35	957.80	1450.20
	50	626.50	626.50	626.50	906.25	1338.65	291.35	291.35	291.35	862.30	844.01
	75	626.50	626.50	626.50	795.66	1039.68	291.35	291.35	291.35	763.50	746.41
	100	626.50	626.50	626.50	690.51	639.96	291.35	291.35	291.35	747.13	715.77
	125	626.50	626.50	626.50	669.61	520.42	291.35	291.35	291.35	675.15	677.48
	150	626.50	626.50	626.50	650.20	383.81	291.35	291.35	291.35	661.88	669.35
	200	626.50	626.50	626.50	627.75	88.21	291.35	291.35	291.35	622.40	642.93
	∞	626.50	626.50	626.50	626.50	0.00	291.35	291.35	291.35	291.35	0.00
0.02	25	311.55	311.55	311.55	1037.12	1353.16	246.18	246.18	246.18	765.24	1085.86
	50	311.55	311.55	311.55	867.22	826.82	246.18	246.18	246.18	648.36	480.68
	75	311.55	311.55	311.55	769.35	778.29	246.18	246.18	246.18	610.51	410.72
	100	311.55	311.55	311.55	718.31	747.40	246.18	246.18	246.18	620.90	421.87
	125	311.55	311.55	311.55	657.14	704.03	246.18	246.18	246.18	623.02	411.89
	150	311.55	311.55	311.55	620.84	674.40	246.18	246.18	246.18	617.11	411.28
	200	311.55	311.55	311.55	592.72	649.14	246.18	246.18	246.18	603.88	409.69
	∞	311.55	311.55	311.55	311.55	0.00	246.18	246.18	246.18	246.18	0.00
0.05	25	313.64	313.64	313.64	701.05	811.40	233.96	233.96	682.90	616.30	664.36
	50	313.64	313.64	313.64	606.45	489.06	233.96	233.96	682.90	522.23	351.45
	75	313.64	313.64	313.64	561.98	443.64	233.96	233.96	682.90	507.71	266.34
	100	313.64	313.64	313.64	538.47	420.57	233.96	233.96	682.90	502.97	244.67
	125	313.64	313.64	313.64	520.98	407.98	233.96	233.96	682.90	508.53	236.30
	150	313.64	313.64	313.64	499.73	391.39	233.96	233.96	682.90	509.06	222.57
	200	313.64	313.64	313.64	471.07	366.21	233.96	233.96	682.90	512.15	219.27
	∞	313.64	313.64	313.64	313.64	0.00	682.90	682.90	682.90	682.90	0.00
0.10	25	106.90	310.57	310.57	606.41	614.22	218.30	218.30	505.42	551.65	511.19
	50	310.57	310.57	310.57	541.64	371.09	498.72	498.72	885.53	732.86	284.45
	75	310.57	310.57	310.57	502.82	318.64	498.72	498.72	885.53	726.97	245.33
	100	310.57	310.57	310.57	490.41	302.59	498.72	498.72	885.53	715.74	219.90
	125	310.57	310.57	310.57	481.26	296.64	466.73	498.72	885.53	714.40	212.07
	150	310.57	310.57	310.57	455.82	280.31	434.74	498.72	885.53	722.00	204.55
	200	310.57	310.57	310.57	439.41	267.74	434.74	498.72	885.53	717.39	201.57
	∞	310.57	310.57	310.57	310.57	0.00	885.53	885.53	885.53	885.53	0.00
0.20	25	369.84	395.96	888.80	830.92	489.62	293.54	547.22	547.22	549.62	179.03
	50	369.84	395.96	888.80	829.26	397.70	311.75	547.22	547.22	576.49	157.04
	75	369.84	395.96	888.80	798.18	315.23	547.22	547.22	547.22	585.26	141.85
	100	369.84	888.80	888.80	806.72	272.45	547.22	547.22	547.22	595.45	138.49
	125	369.84	888.80	888.80	808.32	232.70	547.22	547.22	547.22	596.94	134.07
	150	369.84	888.80	888.80	812.39	219.95	547.22	547.22	547.22	599.99	132.05
	200	369.84	888.80	888.80	820.70	184.32	547.22	547.22	547.22	607.83	141.12
	∞	888.80	888.80	888.80	888.80	0.00	547.22	547.22	547.22	547.22	0.00

Tab. 2. The distribution of ARL_0 for improved *np*-chart with one C-F correction term for different values of n, m, and p_0 when α=0.005.

n		50					100				
		Lower Quartiles					*Lower Quartiles*				
p_0	m	$Q_{0.10}$	$Q_{0.25}$	$MARL_0$	$AARL_0$	$SDARL_0$	$Q_{0.10}$	$Q_{0.25}$	$MARL_0$	$AARL_0$	$SDARL_0$
0.01	25	72.37	72.37	626.50	556.44	813.06	54.42	291.35	291.35	484.16	571.23
	50	72.37	626.50	626.50	541.86	387.18	291.35	291.35	291.35	362.04	367.31
	75	72.37	626.50	626.50	533.18	207.39	291.35	291.35	291.35	325.18	251.29
	100	72.37	626.50	626.50	563.99	175.31	291.35	291.35	291.35	303.81	161.74
	125	626.50	626.50	626.50	574.08	162.19	291.35	291.35	291.35	295.08	100.03
	150	626.50	626.50	626.50	591.92	134.05	291.35	291.35	291.35	292.28	61.61
	200	626.50	626.50	626.50	599.12	120.09	291.35	291.35	291.35	292.56	44.79
	∞	626.50	626.50	626.50	626.50	0.00	291.35	291.35	291.35	291.35	0.00
0.02	25	56.31	311.55	311.55	429.64	543.06	64.58	246.18	246.18	371.82	454.93
	50	311.55	311.55	311.55	369.88	380.37	246.18	246.18	246.18	303.64	245.26
	75	311.55	311.55	311.55	326.91	220.72	246.18	246.18	246.18	276.87	179.86
	100	311.55	311.55	311.55	314.94	125.63	246.18	246.18	246.18	268.94	148.45
	125	311.55	311.55	311.55	313.93	104.77	246.18	246.18	246.18	257.02	109.20
	150	311.55	311.55	311.55	312.16	64.74	246.18	246.18	246.18	251.90	79.17
	200	311.55	311.55	311.55	311.60	37.90	246.18	246.18	246.18	249.77	59.20
	∞	311.55	311.55	311.55	311.55	0.00	246.18	246.18	246.18	246.18	0.00
0.05	25	84.84	84.84	313.64	351.01	378.36	87.17	233.96	233.96	299.94	283.46
	50	84.84	313.64	313.64	298.17	201.34	87.17	233.96	233.96	266.72	176.89
	75	84.84	313.64	313.64	292.15	136.75	87.17	233.96	233.96	250.29	129.86
	100	84.84	313.64	313.64	292.91	101.55	233.96	233.96	233.96	242.86	102.95
	125	313.64	313.64	313.64	292.71	76.50	233.96	233.96	233.96	237.68	84.84
	150	313.64	313.64	313.64	297.77	64.26	233.96	233.96	233.96	237.59	74.16
	200	313.64	313.64	313.64	303.78	49.09	233.96	233.96	233.96	234.76	51.62
	∞	313.64	313.64	313.64	313.64	0.00	233.96	233.96	233.96	233.96	0.00
0.10	25	106.90	106.90	310.57	303.76	276.74	203.98	203.98	434.74	397.92	205.10
	50	106.90	310.57	310.57	266.87	160.12	203.98	203.98	434.74	414.31	187.14
	75	106.90	310.57	310.57	262.10	121.80	203.98	363.31	434.74	408.58	162.38
	100	106.90	310.57	310.57	264.23	104.02	203.98	434.74	434.74	407.80	141.06
	125	310.57	310.57	310.57	268.49	90.22	203.98	434.74	434.74	410.09	119.54
	150	310.57	310.57	310.57	272.69	83.73	203.98	434.74	434.74	409.97	110.58
	200	310.57	310.57	310.57	278.93	74.60	203.98	434.74	434.74	410.10	90.95
	∞	310.57	310.57	310.57	310.57	0.00	434.74	434.74	434.74	434.74	0.00
0.20	25	154.96	369.84	369.84	375.28	157.86	157.82	250.93	257.47	291.65	93.39
	50	154.96	369.84	369.84	386.50	143.65	250.93	250.93	257.47	306.89	85.73
	75	369.84	369.84	369.84	394.93	135.42	250.93	250.93	250.93	311.78	84.19
	100	369.84	369.84	369.84	397.26	127.11	250.93	250.93	250.93	314.67	82.52
	125	369.84	369.84	369.84	395.37	120.63	250.93	250.93	250.93	316.51	81.98
	150	369.84	369.84	369.84	396.62	116.34	250.93	250.93	250.93	316.89	81.44
	200	369.84	369.84	369.84	390.81	99.98	250.93	250.93	250.93	314.36	80.19
	∞	369.84	369.84	369.84	369.84	0.00	250.93	250.93	250.93	250.93	0.00

The results for α=0.0027 and α=0.005 are given in Tables 3 and 4, respectively. The results indicate that the values of the desired ARL_0 are satisfied with a probability degree of 90% using the bootstrap adjusted limits, while poor in-control performance is observed for the results of unadjusted limits shown in Tables 1 and 2.

Moreover, an increase in distributional values is significant. For example, when p_0=0.1, n=100, m=50, and α=0.0027, we obtain $Q_{0.10}$=1198.85 with bootstrap adjusted limits in Table 3, whereas the result of unadjusted limits is 498.72 in Table 1. For smaller parameter p_0=0.02, we obtain $Q_{0.10}$=1073.03 with bootstrap adjusted limits and $Q_{0.10}$=246.18 with unadjusted limits. The comparisons of ARL_0 distributions for the improved *np*-chart using one C-F correction term with adjusted and unadjusted limits are illustrated in Figure 1 for these cases.

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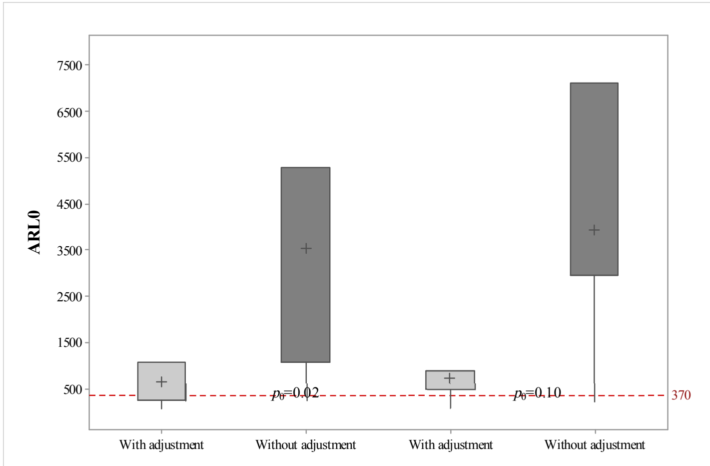

Fig. 1. The box plot of the distribution for in-control ARL with and without limits adjustment for m=50, n=100, and α=0.0027.

Such results indicate that the bootstrap adjusted limits have better in-control performance. In fact, when the amount of Phase I datasets (m) is not adequate enough for the C-F corrected np-chart to reach desired ARL_0, then the control limits should be adjusted by bootstrap. Otherwise, by a lower probability, it is expected to have good in-control performance.

Accordingly, once practitioners start using such limits which are of primary concern and when the parameter is unknown and/or practical amount of Phase I datasets is unavailable, it is desirable to estimate the thresholds so that they have a standard guide to achieving the guaranteed performance with a probability degree of 90%. The recommended upper and lower thresholds for various values of parameters are shown in Table 5.

Generally, the performance evaluation of attribute control charting is just messier than that of variable control charting. For the cases with low values of p_0, the quartiles are mostly the same for different values of m. This is a result of the discreteness of the data. Similar results can be found in the Ref. [19] such that the discreteness of the data affects the performance.

Tab. 3. The distribution of ARL_0 for improved np-chart with one C-F correction term and bootstrap adjusted limits for different values of n, m, and p_0 when α=0.0027.

n		50					100				
		Lower Quartiles					*Lower Quartiles*				
p_0	m	$Q_{0.10}$	$Q_{0.25}$	$MARL_0$	$AARL_0$	$SDARL_0$	$Q_{0.10}$	$Q_{0.25}$	$MARL_0$	$AARL_0$	$SDARL_0$
0.01	25	626.50	6863.92	6863.92	27852.6	85886.81	1870.79	1870.79	14067.93	15116.9	31737.79
	50	626.50	6863.92	6863.92	7805.65	13890.08	1870.79	1870.79	1870.79	5347.94	7348.27
	75	626.50	626.50	6863.92	5422.84	4392.98	1870.79	1870.79	1870.79	3043.17	4010.56
	100	626.50	626.50	6863.92	4546.10	3014.43	1870.79	1870.79	1870.79	2192.16	2105.01
	125	626.50	626.50	6863.92	4267.91	3074.91	1870.79	1870.79	1870.79	1926.37	1220.22
	150	626.50	626.50	626.50	3709.03	3118.81	1870.79	1870.79	1870.79	1858.38	811.24
	200	626.50	626.50	626.50	2864.49	2992.07	1870.79	1870.79	1870.79	1799.71	327.46
	∞	626.50	626.50	626.50	626.50	0.00	291.35	291.35	291.35	291.35	0.00
0.02	25	2091.10	2091.10	2091.10	14471.3	36464.69	1073.03	1073.03	5281.61	10187.8	25974.70
	50	2091.10	2091.10	2091.10	4913.81	6184.14	1073.03	1073.03	1073.03	3470.62	3806.59
	75	2091.10	2091.10	2091.10	2839.82	3369.07	1073.03	1073.03	1073.03	2351.40	2078.60
	100	2091.10	2091.10	2091.10	2259.28	1925.43	1073.03	1073.03	1073.03	1734.75	1544.34
	125	2091.10	2091.10	2091.10	2075.24	1123.47	1073.03	1073.03	1073.03	1457.84	1230.19
	150	2091.10	2091.10	2091.10	2001.24	542.47	1073.03	1073.03	1073.03	1247.04	856.16
	200	2091.10	2091.10	2091.10	1982.55	425.94	1073.03	1073.03	1073.03	1107.18	427.26
	∞	311.55	311.55	311.55	311.55	0.00	246.18	246.18	246.18	246.18	0.00
0.05	25	1322.78	1322.78	6306.63	8410.75	17144.00	682.90	2158.55	2158.55	6876.05	19312.10
	50	1322.78	1322.78	1322.78	3150.27	3540.35	682.90	682.90	2158.55	2449.62	2211.98

	75	1322.78	1322.78	1322.78	2127.96	1887.77	682.90	682.90	2158.55	1736.23	1197.64
	100	1322.78	1322.78	1322.78	1618.20	1261.82	682.90	682.90	682.90	1426.01	836.32
	125	1322.78	1322.78	1322.78	1409.60	818.30	682.90	682.90	682.90	1217.33	716.85
	150	1322.78	1322.78	1322.78	1318.51	555.88	682.90	682.90	682.90	1093.26	663.20
	200	1322.78	1322.78	1322.78	1258.35	291.17	682.90	682.90	682.90	903.55	528.09
	∞	313.64	313.64	313.64	313.64	0.00	682.90	682.90	682.90	682.90	0.00
0.10	25	995.40	995.40	3507.29	6479.33	14611.40	1238.28	2954.39	7106.43	7374.86	5853.09
	50	995.40	995.40	995.40	2464.27	2379.94	1198.85	2954.39	2954.39	3918.44	2529.30
	75	995.40	995.40	995.40	1699.36	1235.73	1198.85	1198.85	2954.39	2896.29	1579.85
	100	995.40	995.40	995.40	1355.52	919.78	1198.85	1198.85	2954.39	2460.90	1161.82
	125	995.40	995.40	995.40	1182.34	714.17	1198.85	1198.85	2954.39	2206.42	947.67
	150	995.40	995.40	995.40	1052.82	481.71	1198.85	1198.85	1198.85	2031.54	894.14
	200	995.40	995.40	995.40	979.29	267.98	1198.85	1198.85	1198.85	1749.22	810.62
	∞	310.57	310.57	310.57	310.57	0.00	885.53	885.53	885.53	885.53	0.00
0.20	25	2985.64	2985.64	8587.62	7639.77	6850.81	1323.52	2415.16	3488.32	4198.21	2518.86
	50	1056.29	2985.64	2985.64	3808.13	2453.75	1227.22	1631.12	2415.16	2411.90	993.77
	75	1056.29	1948.02	2985.64	2733.66	1276.00	1227.22	1227.22	1631.12	1842.18	605.73
	100	1056.29	1056.29	2985.64	2323.47	900.96	1227.22	1227.22	1631.12	1567.80	457.95
	125	1056.29	1056.29	1948.02	2107.72	853.90	986.32	1227.22	1227.22	1402.47	357.82
	150	1056.29	1056.29	1948.02	1878.55	797.21	986.32	986.32	1227.22	1293.17	318.88
	200	1056.29	1056.29	1056.29	1568.33	649.44	986.32	986.32	986.32	1151.39	278.56
	∞	888.80	888.80	888.80	888.80	0.00	547.22	547.22	547.22	547.22	0.00

Tab. 4. The distribution of ARL_0 for improved *np*-chart with one C-F correction term and bootstrap adjusted limits for different values of *n*, *m*, and p_0 when α=0.005.

n		50					100				
		Lower Quartiles					*Lower Quartiles*				
p_0	*m*	$Q_{0.10}$	$Q_{0.25}$	$MARL_0$	$AARL_0$	$SDARL_0$	$Q_{0.10}$	$Q_{0.25}$	$MARL_0$	$AARL_0$	$SDARL_0$
0.01	25	626.50	626.50	6863.92	8100.08	27025.23	291.35	1870.79	1870.79	4665.57	9552.14
	50	626.50	626.50	626.50	3001.25	3275.83	291.35	1870.79	1870.79	1839.05	2141.72
	75	626.50	626.50	626.50	1957.12	2556.06	291.35	291.35	1870.79	1378.70	933.44
	100	626.50	626.50	626.50	1217.58	1827.47	291.35	291.35	1870.79	1210.90	779.05
	125	626.50	626.50	626.50	940.64	1364.79	291.35	291.35	291.35	1040.95	788.78
	150	626.50	626.50	626.50	759.98	902.73	291.35	291.35	291.35	901.64	769.14
	200	626.50	626.50	626.50	642.72	317.67	291.35	291.35	291.35	700.11	691.82
	∞	626.50	626.50	626.50	626.50	0.00	291.35	291.35	291.35	291.35	0.00
0.02	25	311.55	2091.10	2091.10	4885.64	10199.63	246.18	1073.03	1073.03	3353.38	5636.39
	50	311.55	311.55	2091.10	1836.56	1926.45	246.18	1073.03	1073.03	1360.48	1360.27
	75	311.55	311.55	2091.10	1431.37	1057.68	246.18	1073.03	1073.03	967.81	663.72
	100	311.55	311.55	2091.10	1230.87	889.37	246.18	246.18	1073.03	864.87	433.77
	125	311.55	311.55	311.55	1042.23	875.52	246.18	246.18	1073.03	789.00	418.81
	150	311.55	311.55	311.55	871.04	826.29	246.18	246.18	1073.03	739.16	410.93
	200	311.55	311.55	311.55	641.84	691.93	246.18	246.18	246.18	619.42	411.51
	∞	311.55	311.55	311.55	311.55	0.00	246.18	246.18	246.18	246.18	0.00
0.05	25	313.64	1322.78	1322.78	2862.93	4409.44	682.90	682.90	2158.55	2837.83	7447.39
	50	313.64	313.64	1322.78	1266.27	1174.98	233.96	682.90	682.90	1030.90	841.03
	75	313.64	313.64	1322.78	937.23	569.41	233.96	682.90	682.90	747.67	480.98
	100	313.64	313.64	313.64	803.16	516.21	233.96	682.90	682.90	631.66	325.83
	125	313.64	313.64	313.64	696.82	489.91	233.96	233.96	682.90	578.40	232.76
	150	313.64	313.64	313.64	606.09	457.86	233.96	233.96	682.90	543.93	218.24
	200	313.64	313.64	313.64	479.95	374.43	233.96	233.96	682.90	498.39	220.90
	∞	313.64	313.64	313.64	313.64	0.00	233.96	233.96	233.96	233.96	0.00
0.10	25	310.57	995.40	995.40	2836.95	7717.00	1198.85	1198.85	2767.59	2812.45	2373.21
	50	310.57	310.57	995.40	1024.10	1127.13	498.72	1198.85	1198.85	1439.95	690.07
	75	310.57	310.57	995.40	769.34	463.68	498.72	885.53	1198.85	1075.65	373.00
	100	310.57	310.57	995.40	656.94	351.99	498.72	885.53	885.53	921.74	291.22
	125	310.57	310.57	310.57	567.39	331.74	498.72	885.53	885.53	841.03	247.96
	150	310.57	310.57	310.57	522.06	316.60	434.74	498.72	885.53	774.95	221.42
	200	310.57	310.57	310.57	419.10	250.27	434.74	434.74	885.53	711.74	216.65
	∞	310.57	310.57	310.57	310.57	0.00	434.74	434.74	434.74	434.74	0.00
0.20	25	1056.29	1056.29	2985.64	2540.64	1802.41	637.30	1227.22	1631.12	1691.28	849.06
	50	888.80	888.80	1056.29	1407.92	635.72	614.10	839.29	986.32	1057.60	378.41
	75	888.80	888.80	888.80	1127.15	485.07	547.22	547.22	986.32	859.04	263.02

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100	888.80	888.80	888.80	962.59	358.07	547.22	547.22	628.03	767.82	216.69
125	395.96	888.80	888.80	888.28	284.64	547.22	547.22	628.03	698.29	198.77
150	369.84	888.80	888.80	853.17	223.57	547.22	547.22	547.22	641.32	169.53
200	369.84	888.80	888.80	813.84	199.09	415.66	547.22	547.22	567.67	112.67
∞	369.84	369.84	369.84	369.84	0.00	250.93	250.93	250.93	250.93	0.00

Tab. 5. The bootstrap adjusted limits for the improved *np*-charts with one C-F correction term to guarantee Pr(ARL$_0$>B)=90%.

B		200				370.4			
n		50		100		50	100	50	100
p_0	*m*	LCL_τ	UCL_τ	LCL_τ	UCL_τ	LCL_τ	UCL_τ	LCL_τ	UCL_τ
0.01	25	0.00	4.14	0.00	5.56	0.00	4.56	0.00	6.07
	50	0.00	3.91	0.00	5.28	0.00	4.31	0.00	5.76
	75	0.00	3.80	0.00	5.14	0.00	4.18	0.00	5.59
	100	0.00	3.72	0.00	5.05	0.00	4.09	0.00	5.50
	125	0.00	3.67	0.00	4.99	0.00	4.04	0.00	5.43
	150	0.00	3.63	0.00	4.95	0.00	4.00	0.00	5.38
	200	0.00	3.58	0.00	4.89	0.00	3.94	0.00	5.31
	∞	0.00	3.23	0.00	4.48	0.00	3.56	0.00	4.87
0.02	25	0.00	5.52	0.00	7.80	0.00	5.98	0.00	8.38
	50	0.00	5.22	0.00	7.44	0.00	5.69	0.00	7.99
	75	0.00	5.08	0.00	7.27	0.00	5.53	0.00	7.82
	100	0.00	5.01	0.00	7.17	0.00	5.45	0.00	7.71
	125	0.00	4.95	0.00	7.10	0.00	5.38	0.00	7.63
	150	0.00	4.90	0.00	7.06	0.00	5.33	0.00	7.58
	200	0.00	4.85	0.00	6.98	0.00	5.27	0.00	7.50
	∞	0.00	4.45	0.00	6.51	0.00	4.83	0.00	6.97
0.05	25	0.00	8.61	0.00	13.12	0.00	9.21	0.00	13.87
	50	0.00	8.25	0.00	12.65	0.00	8.82	0.00	13.36
	75	0.00	8.09	0.00	12.43	0.00	8.65	0.00	13.14
	100	0.00	7.99	0.00	12.30	0.00	8.54	0.00	12.99
	125	0.00	7.92	0.00	12.21	0.00	8.46	0.00	12.88
	150	0.00	7.87	0.00	12.15	0.00	8.40	0.00	12.83
	200	0.00	7.80	0.00	12.06	0.00	8.32	0.00	12.72
	∞	0.00	7.31	0.00	11.46	0.00	7.80	0.00	12.07
0.10	25	0.00	12.75	1.61	21.35	0.00	13.46	1.04	22.25
	50	0.00	12.31	1.87	20.78	0.00	12.98	1.42	21.63
	75	0.00	12.12	1.97	20.52	0.00	12.78	1.54	21.35
	100	0.00	12.00	2.04	20.36	0.00	12.65	1.62	21.18
	125	0.00	11.91	2.09	20.27	0.00	12.57	1.66	21.07
	150	0.00	11.86	2.12	20.18	0.00	12.49	1.70	20.99
	200	0.00	11.77	2.18	20.08	0.00	12.40	1.74	20.86
	∞	0.00	11.22	2.50	19.34	0.00	11.80	2.07	20.07
0.20	25	1.87	20.38	7.97	34.25	1.35	21.16	7.27	35.34
	50	2.11	19.90	8.38	33.61	1.67	20.65	7.68	34.62
	75	2.22	19.67	8.57	33.31	1.78	20.40	7.87	34.28
	100	2.29	19.54	8.69	33.14	1.85	20.25	8.00	34.09
	125	2.33	19.44	8.77	33.01	1.90	20.16	8.08	33.96
	150	2.37	19.37	8.83	32.92	1.93	20.07	8.15	33.88
	200	2.42	19.27	8.91	32.79	1.98	19.98	8.23	33.72
	∞	2.75	18.63	9.46	31.92	2.31	19.29	8.80	32.80

5. Conclusion

In this study, with estimated process parameters, the in-control performance of the improved *np*-charts using Cornish-Fisher expansions was evaluated. Moreover, the bootstrap-based methodology was proposed to adjust the control limits for reducing the effect of sampling variability on the ARL$_0$ performance and achieving, or even exceeding, the desired performance value with a specified probability. Through a simulation study, it was shown that the proposed bootstrap-based adjustments improved the in-control performance of the C-F corrected np-chart.

This methodology can be extended to other types of control charts. For the C-F corrected *p*-chart, authors [23] suggested selecting the suitable chart as follows:

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1. When $np(1-p) \geq 5$ without correction,
2. When $np(1-p) \geq 0.25$ one term of correction,
3. When $np(1-p) \geq 0.08$ two terms of correction.

In this paper, the least value of $np(1-p)$ was 0.495 and, thus, the *np*-chart with one C-F correction term was studied to investigate only the in-control performance. It is suggested that a comprehensive study will be performed by considering all the above-mentioned conditions and investigating both in-control and out-of-control performance measures.
With fast and inexpensive computing developments, the bootstrap turned into an attractive technique. Nevertheless, its estimates are subject to bootstrap (statistical) error and a simulation (Monte Carlo) error. The first one depends on the number of source data and its accuracy cannot be eliminated using bootstrap. The second one can be reduced by increasing B due to inadequate randomness. Therefore, authors [12] proposed running the algorithm for a specified number of times, e.g., 1000 times, to obtain the results.
In practice, the value of B is left to the experimenter to choose. Note that the problem of selecting the best bootstrap sample size is also a potential topic, which was beyond the scope of this paper (refer to [24] for more information).
The design of control charts in statistical and/or economic terms was considered in the literature. For example, a multi-objective economic-statistical design (MOESD) of the improved *np*-chart using Cornish-Fisher expansions was presented and optimized [21]. Moreover, an economic-statistical design of the *CCC-r* chart for high yield processes was studied when an imperfect inspection was considered [25]. Salmasnia et al. [26] investigated the optimization of the expected total cost including quality, production and maintenance costs. Such optimization problems can be extended to bootstrap adjusted limits, too. The application of the Decision On Belief (DOB) approach to fault detection [27] and change point estimation of high-yield processes [28] are the other potential areas of study that can be extended for control charts by bootstrap adjusted limits.

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