Locating a Temporary Depot After an Earthquake Based on Robust Optimization

Masoud Rabbani*, Zahra Mousavi & Neda Manavizadeh

Masoud Rabbani, School of Industrial Engineering, College of Engineering, University of Tehran, Tehran, Iran.
Zahra Mousavi, School of Industrial Engineering, College of Engineering, University of Tehran, Tehran, Iran.
Neda Manavizadeh, Khatam University, Tehran, Iran

KEYWORDS
Humanitarian crisis management;
Multi-objective model;
Temporary depot location;
Robust optimization;
Multi-objective particle swarm optimization (MOPSO).

ABSTRACT
In today's world, natural disasters, such as earthquakes, and crises, such as terrorist attacks, threaten the lives of many people. Hence, this research presents an efficient mathematical model to locate temporary depot, equitable distribution of resources, and movement of injured people to health centers with the aim of developing a multi-objective model and considering multiple central depots, multiple temporary depots, and several types of relief items in the model. This paper is solved in certain and uncertain states, and three different levels of uncertainty are taken into account for effective parameters in robust optimization by considering the traditional and humanitarian objective functions simultaneously. The model is solved by a multi-objective Particle Swarm optimization algorithm (MOPSO). GAMS software is used to validate the proposed model. Some numerical examples are presented. In addition, sensitivity analyses of the model are presented, and the relationship among the number of temporary depot locations, the number of injured peoples to be transferred to health centers, and the number of uncovered damaged points is studied.

© 2019 IUST Publication, IJIEPR, Vol. 30, No. 1, All Rights Reserved

1. Introduction
In today's world, the local emergency response cannot deal with different kinds of disasters alone [1]. Transportation of required items is an important issue in critical situations. Considering various transportation modes in the area can improve the distribution of items between the nodes [2]. This paper has focused on the earthquake as a natural prevalent disaster. In this regard, the main issue of crisis management is about the distribution of goods to the areas affected by an earthquake. Therefore, special warehouses and distribution centers are to be established and to be divided into donation centers by the discretion of decision-makers. Different commodities, such as medicines, sanitation, food, water, and blankets, will be assigned to each center. These items should be shipped to the desired regions by existing vehicles [3, 4]. Table 1 presents different studies in recent years about the field of crisis.

* Corresponding author: Masoud Rabbani

Email: mrabani@ut.ac.ir

Received 05 May 2017; revised 05 February 2018; accepted 03 March 2019
Table 1. The comparative table of literature review in Crisis management

<table>
<thead>
<tr>
<th>Reference</th>
<th>Year</th>
<th>Number of objective function</th>
<th>Type of objective function</th>
<th>Type of modeling</th>
<th>solution methodology</th>
<th>optimization method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mete and Zabinsky [4]</td>
<td>2010</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Liu et al. [5]</td>
<td>2010</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Nolz et al. [6]</td>
<td>2011</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Lin et al. [7]</td>
<td>2011</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Widener and Horner [8]</td>
<td>2011</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Ozdamar and Demir [12]</td>
<td>2012</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Zhang et al. [14]</td>
<td>2012</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Doyen et al. [15]</td>
<td>2012</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Najafi et al. [16]</td>
<td>2013</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Ishii and lee [17]</td>
<td>2013</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Wex et al. [18]</td>
<td>2013</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Wang et al. [20]</td>
<td>2014</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Sheu and Pan [21]</td>
<td>2014</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Rennemo et al. [22]</td>
<td>2014</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Ye et al. [23]</td>
<td>2015</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Ahmadi et al. [24]</td>
<td>2015</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Huang et al. [25]</td>
<td>2015</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Kilici et al. [26]</td>
<td>2015</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Goli and Alinaghian [27]</td>
<td>2015</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>This study</td>
<td>2016</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

The main goal of this research is to provide an efficient plan to locate temporary depots, assign distribution of resources between nominated centers, and transfer the injured people from the damaged point to health centers. In addition, the proposed model is assessed under both certain and uncertain states. The proposed model is a multi-objective model that minimizes the distribution time, maximizes the number of people transported to health centers, and minimizes the injustice in transportation of the injured people from damaged points to health centers by considering capacity of health centers and the number of available vehicles in the central depots.

The rest of this paper is organized as follows. In Section 2, the problem description, mathematical modeling, and robust optimization of the model are discussed. The methodology of the problem is explained in Section 3. The numerical results and sensitivity analysis of the model are illustrated in Section 4. Finally, the conclusions described in Section 5.

2. Problem Description

The response centers should be prepared to control and minimize the destructive effects
caused by an earthquake. The logistical supports of response centers are divided into two main aims: the first goal is to transport the injured people from the affected areas to hospitals and health centers, while the second goal is about the fair distribution of relief goods to temporary depot areas. In these conditions, available resources are usually inadequate, thus providing an effective plan to locate the temporary depot centers and set a fair distribution of resources. This model aims to minimize the cost and time, maximize the number of injured people, and set a fair distribution scheme to achieve effective management of resources to reduce the damages of an earthquake [16].

Relief logistics network includes a central depot (CD), temporary depot (TD), the damaged point (DP), and health center (H). It is vital to set the best place for temporary depot as soon as possible due to time constraints in critical conditions. Perishable items, such as food (including canned food, milk, sterilized and pasteurized, milk, water, etc.), blankets, and other items, must be assigned to depots according to their capacity by vehicles. In the next step, the relief goods will be distributed by taking into account the demands of the damaged point and health center. Serum, blankets, and other conventional means of temporary depots should be assigned only to health centers. The main assumptions for the proposed model are described as follows:

* Central depot, the demand, damaged point, health centers, and the distance between them are known.
* Candidates’ temporary depot location is known like school, university, and mosque [7].
* Existing routes in the network and the number of damaged points is clear. It is assumed that the required model information and available transport infrastructure are possible by advanced crisis technologies such as satellite and GIS.
* Some of the available transport vehicles only transfer relief goods and products, while others only transfer the damaged people depending on the type of vehicle and their various capacities.
* After the earthquake, there are some possible damages such as the destruction of the health center; thus, the health center cannot be used. Therefore, the impacts of the destruction on the health centers have been considered in the model.
* Demand points include damaged points and health centers for required relief items and medicine.

2-1. Mathematical modeling

Indices

\[ N \] set of temporary depots; \( N = 1, \ldots, n \)

\[ M \] set of central depots; \( M = 1, \ldots, m \)

\[ I \] set of item types of relief goods; \( I = 1, \ldots, i \)

\[ J \] set of damaged points; \( J = 1, \ldots, j \)

\[ k \] set of vehicle types; \( K = 1, \ldots, k \)

\[ L \] Set of tours; \( L = 1, \ldots, l \)

\[ H \] set of health centers; \( H = 1, \ldots, h \)

Parameters

\[ S_t \] Available time of distribution

\[ d_{ij} \] number of demands for item \( I \) from damaged point \( j \)

\[ d_{hi} \] number of demands for item \( I \) from health center \( h \)

\[ s_{im} \] total number of items \( I \) in the central depot \( m \)

\[ s_k \] number of available vehicles \( k \)

\[ q_{in} \] capacity of temporary depot \( n \)
\( \varphi_{ik} \) capacity of vehicle \( k \) for carrying items

\( \varphi_{kp} \) capacity of vehicle \( k \) for carrying injured people

\( \varphi_h \) Capacity of health center \( h \) for injured people (if health center \( h \) destructs and becomes unusable; \( \varphi_h = 0 \))

\( q_j \) number of injured people in damaged point \( j \)

\( \tau_{ln} \) time of transportation from central depot \( m \) to temporary depot \( n \) using the tour of \( l \) (if the tours are destroyed and become unusable, \( \tau_{ln} = \infty \))

\( \tau_{hln} \) time of transportation from central depot \( m \) to health center \( h \) using the tour of \( l \) (if the tours are destroyed and become unusable, \( \tau_{hln} = \infty \))

\( \tau_{jln} \) time of transportation from temporary depot \( n \) to damaged point \( j \) using the tour of \( l \) (if the tours are destroyed and become unusable, \( \tau_{jln} = \infty \))

\( \tau_{hln} \) time of transportation from the temporary depot \( n \) to health center \( h \) using the tour of \( l \) (if the tours are destroyed and become unusable, \( \tau_{hln} = \infty \))

\( \tau_{hjl} \) time of transporting injured people from damaged point \( j \) to health centers \( h \) (if health center \( j \) is destroyed and becomes unusable or the tours destroyed; \( \tau_{hjl} = \infty \))

\( t_i \) time for unloading item \( i \)

\( t'_i \) time for loading item \( i \)

\( t_p \) Time for mounting and dismounting people injured people by vehicles

\( M \) a positive large number

**Set of decision variables**

\( Y_n \) binary variable that equals 1 if depot \( n \) is established, and 0 otherwise

\( X^n_j \) binary variable that equals 1 if demand point \( j \) is assigned to depot \( n \), and 0 otherwise

\( Z_j \) binary variable that equals 1 if demand point \( j \) is covered, and 0 otherwise

\( y_{ln} \) binary variable that equals 1 if tour \( l \) from central depot \( m \) starts, and vehicle type \( k \) is assigned, and 0 otherwise

\( y_{hln} \) binary variable that equals 1 if tour \( l \) from temporary depot \( n \) starts, and vehicle type \( k \) is assigned, and 0 otherwise

\( y_{hjl} \) binary variable that equals 1 if tour \( l \) from damaged point \( j \) starts, and vehicle type \( k \) is assigned, and 0 otherwise

\( x_{iln} \) number of items \( i \) from central depot \( m \) to temporary depot \( n \) by the tour of \( l \) and type of vehicle \( k \)

\( x_{ijk} \) number of items \( i \) from temporary depot \( n \) to damaged point \( j \) by the tour of \( l \) and type of vehicle \( k \)
Locating a Temporary Depot After an Earthquake Based on Robust Optimization

Masoud Rabbani, Zahra Mousavi & Neda Manavizadeh

43

\[ x_{ikl}^n \] number of items i from temporary depot n to health center h by the tour of i and type of vehicle k

\[ x_{hiki}^m \] number of items i from central depot m to health center h by the tour of i and type of vehicle k

\[ p_{hijkl} \] number of injured people and transferred from damaged point j to health center h by the tour of i and type of vehicle k

\[ I_{in} \] Amount of item i stored in a temporary depot n

\[ b_{in} \] Shortage of item i in a temporary depot n

2-1-1. Mathematical formulation

\[
\begin{align*}
\min \ z_1 &= \sum_{h,j,k,i,m,n} \left[ (\tau_{hmn} + \tau_{hlm}) \cdot y_{kmn} + (\tau_{hjn} + \tau_{hjl}) \cdot y_{jlkn} + \tau_{hjl} \cdot y_{jklm} + t \cdot P_{hijkl} + \sum_t (t_j + t_l) (x_{nklm}^n + x_{hiki}^m + x_{jkl}^n + x_{hiki}^n) \right] \\
\min \ z_{21} &= \sum_{h,j,k,i,m,n} \left[ \frac{\sum_{i,j,l} p_{hijkl}}{q_j} - \frac{\sum_{j,k,l} \varphi_{p}}{q_j} \right] \forall j \in J \\
\min \ z_{22} &= \sum_{h,j,k,i,m,n} \left[ \frac{\sum_{i,j,l} p_{hijkl}}{q_j} - \frac{\sum_{j,k,l} \varphi_{p}}{q_j} \right] \forall j \in J \\
\max \ z_3 &= \sum_{h,j,k,i,m,n} p_{hijkl} \\
\end{align*}
\]

\[ s.t. \] \( \forall i \in I, n \in N \)

\[
\begin{align*}
\sum_{k,i,m,n} x_{iklm}^n + I_{in} &= \forall i \in I, n \in N \\
\sum_{j,k,i} x_{jkl}^n + \sum_{h,k,i} x_{hiki}^m &= \forall i \in I, n \in N \\
b_{in} &= \sum_d d_{ki} + \sum_j d_{kj} - \forall i \in I, n \in N \\
\left( \sum_{j,k,i} x_{jkl}^n + \sum_{h,k,i} x_{hiki}^m + I_{in} \right) &= \forall i \in I, n \in N \\
\sum_{k,i,n} x_{iklm}^n &\leq d_{ij} \forall i \in I, j \in J \\
\sum_{i,j,k} x_{ik}^n + \sum_{i,n} x_{ik}^n &\leq d_{ih} \forall i \in I, h \in H \\
\sum_{i,k,m} x_{iklm}^n + \sum_{i,j,k} x_{hiki}^m &\leq s_{im} \forall i \in I, m \in M \\
\left( x_{iklm}^n + I_{in} \right) &\leq \varphi_{in} \forall i \in I, n \in N \\
\sum_{j,k,i} x_{jkl}^n + \sum_{k,i} x_{hiki}^m &\leq \varphi_{in} \forall i \in I, n \in N \\
\sum_{i,j,k,m,n} x_{iklm}^n &\leq \varphi_{ik},s_k \forall k \in K \\
\sum_{i,j,k,m,n} x_{jkl}^n &\leq \varphi_{ik},s_k \forall k \in K \\
\sum_{i,j,k,m,n} x_{hiki}^m &\leq \varphi_{ik},s_k \forall k \in K \\
\sum_{h,j,k,m,n} x_{hiki}^m &\leq \varphi_{ik},s_k \forall k \in K \\
\sum_{h,j,k} P_{hijkl} &\leq \varphi_{ik},s_k \forall k \in K
\end{align*}
\]
\[
\sum_{h,k,l} p_{hkl} \leq q_j \quad \forall j \in J
\]
\[
\sum_{j,k,l} p_{jkl} \leq \varphi_h \quad \forall h \in H
\]
\[
\sum_{n} y_n = \mathcal{N}
\]
\[
y_n \leq 1 \quad \forall n \in \mathcal{N}
\]
\[
\sum_{n} x^j_n = z_j \quad \forall j \in J
\]
\[
\sum_{i,j,k,l} x^n_{jkl} \leq M \cdot y_n \quad \forall n \in \mathcal{N}
\]
\[
\sum_{i,j,k,l} x^m_{jkl} \leq M \cdot y_n \quad \forall n \in \mathcal{N}
\]
\[
\sum_{h,k,l} x^n_{hkl} \leq M \cdot y_n \quad \forall n \in \mathcal{N}
\]
\[
\sum_{i,j,k,l} x^m_{jkl} \leq M \cdot y_n \quad \forall n \in \mathcal{N}
\]
\[
\sum_{l} y_{kl} \leq M \cdot y_n \quad \forall n \in \mathcal{N}
\]
\[
\sum_{j,l} y_{jkl} \leq M \cdot y_n \quad \forall n \in \mathcal{N}
\]
\[
\sum_{h,k,l} y_{hkl} \leq M \cdot y_n \quad \forall n \in \mathcal{N}
\]
\[
\sum_{i,j,k,l} y^n_{jkl} \leq M \cdot y_n \quad \forall n \in \mathcal{N}
\]
\[
\sum_{i,j,k,l} y^m_{jkl} \leq M \cdot y_n \quad \forall n \in \mathcal{N}
\]
\[
\sum_{j,l} p_{jkl} \leq \sum_{j,l} M \cdot y_{jkl} \quad \forall l \in \mathcal{L}
\]
\[
\sum_{j,l} y_{jkl} + y_{jkl} \leq (\tau_{jkl} + \tau_{jkl})y_{jkl} + \tau_{jkl}y_{jkl} + t_{jkl}P_{jkl} + \sum_{l} \left( (1 + t_{jkl})x^n_{jkl} + x^m_{jkl} + x^n_{jkl} \right) \leq M
\]
\[
y_n, x^j_n, z_j, y_{jkl}, y_{jkl}, y_{jkl} \in \{0,1\}
\]
\[
x^n_{jkl}, x^m_{jkl}, y^n_{jkl}, y^m_{jkl}, P_{jkl}, I_{in}, b_{in} \geq 0
\]
\& integer

In the proposed model, Equation 1 shows the first objective function that minimizes the total transportation time including shipment time between central depot and temporary depot, central depot and health centers, temporary depot and damaged point, temporary depot and health centers, damaged points and health center, transferring time of injured people and the vehicle, and unloading/loading time of products. Equation 2 shows the second objective function that minimizes the injustice and differences in the transfer of injured people by considering the capacity and the number of existing vehicles for movement of injured people and the capacity of health centers. Equation 3 demonstrates the third objective function; this objective function maximizes the number of people brought to health centers among all of the affected people. Constraint 4 makes a balance between the flow of incoming and outgoing goods in the inventory of temporary depot. Constraint 5 calculates the shortage of each commodity and relief item in the temporary depot. Constraint 6 expresses that the total number of items sent to damage points is equal to or less than that of demand points. Constraint 7 ensures that the total number of items sent from the central depot and temporary depot to health center is equal to or less than the amount of health center demands. Constraint 8 shows that the total number of items sent from each central depot to temporary depot and health center is equal to or less than the inventory of center depot. Constraint 9 demonstrates the capacity constraints in a temporary depot.
Constraint 10 ensures that the total number of items sent from any temporary depot to the damaged point and health center is equal to or less than the capacity of a temporary depot. Constraint 11 ensures that the total number of items sent by each vehicle from the central depot to a temporary depot is equal to or less than the capacity of each vehicle carrying relief items and the number of vehicles. Constraint 12 shows that the total number of items sent by each vehicle from the temporary depot to damaged points is equal to or less than the capacity of each vehicle carrying relief items and the number of vehicles. Constraint 13 shows that the total number of items sent by each vehicle from the temporary depot to health centers is equal to or less than the capacity of each vehicle carrying relief items and the number of the vehicles. Constraint 14 expresses that the total number of items sent by vehicles from the central depot to a health center is equal to or less than the capacity of each vehicle to carry relief items and the number of the vehicles. Constraint 15 shows that the total number of people transferred by vehicles from damaged points to health center is equal to or less than the capacity of every vehicle and the number of vehicles. Constraint 16 demonstrates that the number of affected people moves to health centers from damaged point is equal to or less than the total number of affected people in every area. Constraint 17 indicates the limitation of capacity for reception of affected people in the health center. Constraint 18 sets the total number of temporary depots that should be located. Constraint 19 ensures that only one temporary depot can be open at a point. Constraint 20 indicates that every damaged point can be assigned to a temporary depot if that point is covered. Constraint 21 states that a damaged point can receive answers only from open depots. Constraint 22 states that relief goods and items can only be sent from the central depots to open temporary depots. Constraint 23 expresses that relief goods and items are permitted to be delivered from the active temporary depot to damaged points. Constraint 24 states that if a temporary depot is open, relief goods and items can be carried from that depot to health centers. Constraint 25 shows that the allocation of a tour from a temporary depot for movement of relief goods is possible, if a temporary depot is created. Constraint 26 states that the transportation of goods from a temporary depot to a damaged point is possible by a specific route, if that route is selected. Constraint 27 is similar to the previous equation related to health centers instead of damaged points. Constraint 28 expresses that transportation of goods from the central depot to a temporary depot is possible by a specific route, if that route is selected. Constraint 29 demonstrates that the transportation of goods from the central depot to health center is possible by a specific route, if that route is selected. Constraint 30 shows that the transportation of affected people from damaged points to health centers is possible by a specific route, if that route is selected. Constraint 31 shows time limitation to distribute relief items. Finally, binary and non-negative requirements are given by Constraints 32 and 33.

2-2. Robust optimization of the model

Here, the approach of Ben-Tal, A. and Nemirovsk [28, 29] is applied to deal with the uncertainty of the proposed model. The robust optimization approach is a common approach to dealing with the uncertainty of the supply chain [30]. Considering the uncertainty in the model increases the flexibility of the model and improves the efficiency of the proposed model [31]. In this approach, it is assumed that some data are uncertain. Therefore, changes in non-deterministic data in a specific range of uncertainties to select the optimum value of the objective function. The following deterministic model is considered:

$$\begin{align*}
\text{Min } c x + d \\
\text{s.t. } A x &\leq b \\
c, d, A, B &\in U
\end{align*}$$

(34)

where c, d, A, b define the uncertain sets within a range as follows:

$$U_{b_{k}} = \left\{ \mathbf{g} \in R^{n} : \left| g_{i} - \bar{g}_{i} \right| \leq \delta G_{i}, k = 1, \ldots, m \right\}$$

(35)

where $\bar{G}_{k}$ is the nominal value of $G_{k}$ as the $k^{th}$ parameter of vector $G$ n (n-dimension vector), the positive number $G_{i}$ represents the uncertainty scale, and $\delta \geq 0$ is the uncertainty level [34]. According to the description, Equation (36) is considered:

$$\begin{align*}
\forall & \mathbf{a}_{i} \leq x_{i} \leq b_{i}, & & \forall i \in \{1, \ldots, m_{x}\}, \\
\forall & \mathbf{b} \in U^{b}_{x} : & & \mathbf{b} \in R^{n} : \\
\left| b_{i} - \bar{b}_{i} \right| &\leq \delta G_{i}, & & i = 1, \ldots, m_{b}
\end{align*}$$

(36)

This equation can be transformed to Equation (37) [35].
46 Masoud Rabbani, Zahra Mousavi & Neda Manavizadeh

Equation (36) on equal terms can change as follows [34]:

\[ a_i x \geq \bar{b}_i - \delta_i G_i^+ , \quad \forall i \in \{1, \ldots, m_s\} \]  

(37)

According to the above description, the model should be linearized as in Equations 40 to 43:

\[ \frac{\sum p_{ijk}}{q_j} = p_{ij} \quad \forall j \in J \]  

(40)

\[ \frac{\sum p_{ij}}{q_j} = p_{ij} - p_2 \quad \forall j \in J \]  

(41)

\[ \frac{\sum p_{ijkl}}{q_j} = \phi_j \quad \forall j \in J \]  

(42)

\[ \frac{\sum p_{ijkl}}{q_j} = p_{ij} - p_2 \quad \forall j \in J \]  

(43)

Therefore, the linearization of the model could appear in Equations 44 to 47 as follows:

\[ | p_{ij} - p_2 | = p_{ij} + p_2 \quad \forall j \in J \]  

(44)

\[ | p_{ij} - p_2 | = p_{ij} + p_2 \quad \forall j \in J \]  

(45)

\[ p_{ij} - p_2 = p_{ij} - p_2 \quad \forall j \in J \]  

(46)

\[ p_{ij} - p_6 = p_{ij} - p_6 \quad \forall j \in J \]  

(47)

The model is rewritten as follows. Notably, some limitations are avoided because of repetition:

\[ \text{min} \quad z_2 = p_{ij} + p_2 \quad \forall j \in J \]  

(48)

\[ \text{min} \quad z_2 = p_{ij} + p_2 \quad \forall j \in J \]  

s.t.

\[ p_{ij} - p_2 = p_{ij} - p_2 \quad \forall j \in J \]  

(49)

\[ p_{ij} - p_6 = p_{ij} - p_6 \quad \forall j \in J \]  

(50)

\[ y_{1}^{+} + x_{1}^{+} + Z_{1}^{+} + y_{2}^{+} + y_{3}^{+} + y_{4}^{+} \geq 0 \quad \forall i \in I, j \in J, k \in K, \quad \forall \ell \in \{1, \ldots, \ell\} \]  

(51)

Notably, Restrictions (53), (55–64) have changed, and Restriction 54 is added to the model due to the limitations of the main model.

3. Methodology

3-1. Solving and optimization of model

In a multi-objective mathematical model, the objective functions are in conflict, and finding a single optimal solution is not possible [32]. The objective functions of the problem have different scales, and their meaning values have conflicting effects with each other. The weighted sum approach is a prevalent method to deal with multi-objective models. This method calculates the optimum value of each objective function; then, it considers weights for each objective.
function. The mathematical model is described as follows:

$$\min f = \sum_{i=1}^{m} w_i f_i(x)$$

$$\sum_{i=1}^{m} w_i = 1$$

where $w_i$ is the weight of the objective functions. Firstly, the optimum number of objective functions is determined; then, each objective function is divided into its optimum value. This feature allows for different types of objective functions to interact with each other simultaneously during the solving process of a multi-objective model, which is demonstrated as follows:

$$\min f = w_1(z_1 / z_1^*) + w_2(z_1 / z_2^*)$$

In this method, each $w_i$ shows the importance level of the related objective function ($\sum_{i=1}^{m} w_i = 1$). The extent to which an obtained solution is balanced depends on the value of $w_i$. The value of the relative importance of objective function depends on the judgment of decision-makers. Thus, we have assigned more amount of weights to the objectives related with human safety issues based on the decision-maker’s opinion. The weights of the objective functions are considered 0.1, 0.3, 0.3, and 0.3, respectively. However, variations in the value of the relative importance of objective function can change the value of objective functions and results [33].

3-2. Particle swarm optimization

Particle Swarm Optimization algorithm is a social search algorithm that optimizes the social behavior of groups of particles mass. The basic idea of the proposed method introduced by Eberhart and Kennedy [34] is to deal with multi-objective models in 1995. Multi-objective particle swarm optimization algorithm (MOPSO) was introduced in 2004 by Coello [35-36]. In fact, this is a generalization algorithm of particles Swarm Optimization algorithm used for solving multi-objective models. Particle Swarm algorithm is used to solve various engineering problems [38]. Particle Swarm algorithm is used to optimize time series forecasting [37]. Maldonado et al. [39] used this algorithm to solve fuzzy problems. Melin et al. [40] introduced a new method for implementing dynamic parameters and optimizing the fuzzy classification system. In addition, Uno et al. [41] presented an interactive fuzzy satisfying method with PSO for a multi-objective emergency facility location problem. In the MOPSO algorithm, a concept called external archive or repository has been added to the PSO algorithm, also known as the Hall of Fame. The external archive used to store the dominant responses so far has been produced.

If the external archive is empty, then the current responses are acceptable. When the new responses from the archive are beaten by someone, the call will be dropped. If none of the members of the foreign population was overcome, the new call would be archived. Speed and position of each particle in this algorithm are calculated and updated using Equations 67 and 68:

$$vel(i) = w \times vel(i) + c_1 r_1 (Pbest(i) - pop(i)) + c_2 r_2 (rep(h) - pop(i))$$

(67)

$$pop(i) = pop(i) + vel(i)$$

(68)

Therefore, the inertia weight is equal to 0.4 and random numbers are at the interval of [0, 1]. Choosing the best answer for each particle of the best personal recollection multi-objective optimization particle swarm algorithm is an important and essential step and. When the particles seek to work as a member of the tank leader, the leader must be a member of the tank. Pareto Front members represent the reservoir members and contain particles that are not dominated by others. To compare the best vector for personal memories, we use the following:

1. If a new position outperforms the best memory, then the new position is the best memory. A mathematical expression for Eq. 69 appears:

$$Pbest_i^{n+1} = X_i^{n+1}$$

(69)

2. If the best memory outperforms the new position, do nothing. A mathematical expression for Eq. 70 appears:

$$Pbest_i^n = Pbest_i^{n+1}$$

(70)

3. If none of them is better than the other one, the accident is considered as the leader of the best position.

3-3. Implementation of the algorithm (MOPSO) in model
The MOPSO algorithm is as follows:
1. Determine the parameters required for the implementation of multi-objective particle swarm algorithm (MOPSO): maximum repetitions for running the algorithm, population size, and the members of the store.
2. Create an initial population;
3. Assess each particle of the population;
4. Separate the non-dominated members and store them in an external archive;
5. Update the best personal memory of each particle;
6. Add non-dominated members of the current population;
7. Delete the members of the defeated archive;
8. If the number of members of the archive is more than the specified capacity, delete additional members (the size of the archive is limited);
9. If the agreement is terminated unfulfilled, return to Step 5. Otherwise, it will end.

4. Numerical Results
The goal of this section is explained as follows: first, to validate our model, a small-sized problem was solved through the GAMS and MATLAB software by a personal laptop processor 2 GHz and 4 GB of memory in a Windows environment. Data collection and parameter's determination of the model are implemented based on the crisis management expert's opinion. Second, the proposed robust optimization model on some random large-size test problems was tested to validate and compare the applicability of our meta-heuristic algorithm. The results are shown as follows:

4-1. Validation of the model
To validate our model, a small-sized problem is considered. In this problem, one central depot and two temporary depots are considered. Our results are presented by considering different levels of uncertainty as \( \delta = 0.1, 0.15, 0.25 \) to show their impact on the value of the objective functions. For all of the nondeterministic parameters, the scale of uncertainty is equal to the value of nominal parameters, which is equal to a case of example. The results of the experiments under various uncertain levels of the parameter are reported in Tables 2. In this table, the value of objective function \( (z_i) \), computational time, mean of objective function, and standard deviation are shown for the GAMS and MOPSO algorithm.

By comparing the results of solving the small-sized problems for both certain and uncertain states, it is realized that the quality of solutions using GAMS software is better and more efficient in a less amount of computational time. In addition, the output of the first problem including all details is presented in the appendix. Moreover, the result of the test problem in different sizes in certain and uncertain states is presented in Table 3. In this table, the value of objective function \( (z_i) \), computational time, mean of the objective function, and standard deviation are shown.

<p>| Tab. 2. Summary of test result in small size problem for certain and uncertain state |
|---------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|</p>
<table>
<thead>
<tr>
<th>Method</th>
<th>Size of problem ((m, n, j, h, l))</th>
<th>Objective function value ((z_i))</th>
<th>Computational time (sec.) ((Z_i))</th>
<th>Mean of objective function</th>
<th>SD of objective function</th>
</tr>
</thead>
<tbody>
<tr>
<td>GA</td>
<td>(3 \times 4 \times 2 \times (1,21))</td>
<td>407.0</td>
<td>102.0</td>
<td>414.0</td>
<td>0.0040</td>
</tr>
<tr>
<td></td>
<td>(3 \times 4 \times 2 \times (28))</td>
<td>415.0</td>
<td>122.0</td>
<td>414.0</td>
<td>0.0040</td>
</tr>
<tr>
<td></td>
<td>(4 \times 5 \times 3 \times (2,35))</td>
<td>421.0</td>
<td>131.0</td>
<td>414.0</td>
<td>0.0040</td>
</tr>
<tr>
<td></td>
<td>(1 \times 2 \times 1 \times (21))</td>
<td>417.0</td>
<td>111.0</td>
<td>414.0</td>
<td>0.0040</td>
</tr>
<tr>
<td>MOPSO</td>
<td>(3 \times 4 \times 2 \times (28))</td>
<td>425.0</td>
<td>132.0</td>
<td>424.0</td>
<td>0.0050</td>
</tr>
<tr>
<td></td>
<td>(4 \times 5 \times 3 \times (2,35))</td>
<td>432.0</td>
<td>138.0</td>
<td>424.0</td>
<td>0.0050</td>
</tr>
<tr>
<td></td>
<td>Level of uncertainty = 0.15</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

International Journal of Industrial Engineering & Production Research, March 2019, Vol. 30, No. 1
Tab. 3. Summary of test result in different problem for certain and uncertain state

<table>
<thead>
<tr>
<th>Method</th>
<th>Size of problem (m, n, j, h, l)</th>
<th>Objective function value (Z_i)</th>
<th>Computation al time (sec.) (Z_i)</th>
<th>Mean of objective function</th>
<th>SD of objective function</th>
</tr>
</thead>
<tbody>
<tr>
<td>GA</td>
<td>1×2×1×(1 21)</td>
<td>407.0</td>
<td>0.0 102.0 0.0 116</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MS</td>
<td>3×4×2×(1 28)</td>
<td>415.0</td>
<td>0.0 122.0 0.0 168</td>
<td>414.0 0.0 004.0 0.0 01</td>
<td></td>
</tr>
<tr>
<td>MO</td>
<td>3×4×2×(1 28)</td>
<td>525.0</td>
<td>0.0 142.0 0.0 163</td>
<td>424.0 0.0 005.0 0.0 012</td>
<td></td>
</tr>
<tr>
<td>PSO</td>
<td>4×5×3×(2 35)</td>
<td>532.0</td>
<td>0.0 149.0 0.0 182</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1×2×1×(1 21)</td>
<td>407.0</td>
<td>0.0 102.0 0.0 145</td>
<td></td>
<td></td>
</tr>
<tr>
<td>GA</td>
<td>3×4×2×(1 28)</td>
<td>415.0</td>
<td>0.0 122.0 0.0 174</td>
<td>414.0 0.0 004.0 0.0 013</td>
<td></td>
</tr>
<tr>
<td>MS</td>
<td>4×5×3×(2 35)</td>
<td>421.0</td>
<td>0.0 131.0 0.0 195</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MO</td>
<td>3×4×2×(1 28)</td>
<td>425.0</td>
<td>0.0 132.0 0.0 156</td>
<td>424.0 0.0 005.0 0.0 014</td>
<td></td>
</tr>
<tr>
<td>PSO</td>
<td>4×5×3×(2 35)</td>
<td>432.0</td>
<td>0.0 138.0 0.0 184</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Level of uncertainty = 0.1

<table>
<thead>
<tr>
<th>Method</th>
<th>Size of problem (m, n, j, h, l)</th>
<th>Objective function value (Z_i)</th>
<th>Computation al time (sec.) (Z_i)</th>
<th>Mean of objective function</th>
<th>SD of objective function</th>
</tr>
</thead>
<tbody>
<tr>
<td>MOP SO</td>
<td>4×8·100·38·15 (1498)</td>
<td>562</td>
<td>0.0 578 94.6 8 94</td>
<td>557.0 0.0 0119.0 0.0 009</td>
<td></td>
</tr>
<tr>
<td></td>
<td>38·100·38·16 (1512)</td>
<td>541</td>
<td>0.0 562 35.7 9 53</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1×2×1×(1 21)</td>
<td>569</td>
<td>0.0 581 62.8 10 32</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MOP SO</td>
<td>4×8·100·40·16 (1555)</td>
<td>562</td>
<td>0.0 586 94.6 9 54</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>38·100·38·16 (1512)</td>
<td>541</td>
<td>0.0 579 35.7 10 23</td>
<td>557.0 0.0 0119.0 0.0 0085</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1×2×1×(1 21)</td>
<td>569</td>
<td>0.0 593 62.8 11 62</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MOP SO</td>
<td>4×8·100·38·15 (1498)</td>
<td>562</td>
<td>0.0 597 94.6 12 78</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>38·100·38·16 (1512)</td>
<td>541</td>
<td>0.0 593 35.7 15 93</td>
<td>557.0 0.0 0119.0 0.0 0063</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1×2×1×(1 21)</td>
<td>569</td>
<td>0.0 608 62.8 14 89</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
4-2. Sensitivity analysis
In this section, the relationship among the number of temporary depots and distribution time, and transport and displacement in the model is examined. Figure 1 depicts the connection between the location of temporary depot and distribution time, and transport and displacement in the model. Based on Figure 1, reducing the number of temporary depots leads to higher transferring time of injured people from the central depot, damaged point, and health centers. In other words, in a critical situation, time is an important factor. The more we lose, the less chance we have to transfer relief items and injured people.

With sensitivity analysis on the number of temporary depots in the state of uncertainty at the level of \( \theta = 0.1 \), the relationship between the number of temporary storage locations and the number of damaged points that are not covered in the model is examined. Based on Figure 2, results show that with an increase in the number of temporary depots, the number of uncovered damaged points reduces and the number of covered damaged points increases. Therefore, we should increase the number of temporary depots to reduce the number of uncovered damaged points.

5. Conclusion
This study presented an efficient mathematical model to locate a temporary depot and set an equitable distribution of resources by considering different contradictory objective functions and multiple central depots, multiple temporary depots, and several type commodities. This paper was considered in two states of certainty and uncertainty; important parameters of the model were considered at three levels of uncertainty in the robust optimization approach. The model was solved with multi-objective Particle Swarm Optimization (MOPSO) method. Moreover, GAMS software was used to validate the model. The innovations of research include considering the possibility of destruction of the medical center, capacity constraints, and availability of vehicles with the humanitarian objective functions, including minimization of injustice and differences in the transportation of injured people from the damaged points to health centers, maximization of the number of people rescued and brought to health centers along with the traditional aims such as minimizing the transportation time. Further, the results showed that there was a reverse relationship among the number of temporary depot locations, the number of injured people moved to medical centers, and the number of uncovered damaged points. For future studies, developing other metaheuristics to solve the problem and considering other aspects of uncertainty in the model could be interesting ideas.

References


Locating a Temporary Depot After an Earthquake Based on Robust Optimization

Masoud Rabbani*, Zahra Mousavi & Neda Manavizadeh


[38] Pulido, M., Melin P. and O. Castillo, “Particle swarm optimization of ensemble neural networks with fuzzy aggregation for time series prediction of the Mexican


Appendix

Fig. 1. The logistical figure of an illustrative example

<table>
<thead>
<tr>
<th>Tab. 2. Parameters of an illustrative example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vehicle type</td>
</tr>
<tr>
<td>---------------</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>Item type</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>$q_j$</td>
</tr>
<tr>
<td>15</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Tab. 3. Parameters of an illustrative example</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_{ij}$</td>
</tr>
<tr>
<td>----------</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Tab. 4. Parameters of an illustrative example</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_{\alpha}$</td>
</tr>
<tr>
<td>------------------</td>
</tr>
<tr>
<td>1.10</td>
</tr>
<tr>
<td>1.11</td>
</tr>
<tr>
<td>1.12</td>
</tr>
</tbody>
</table>
Tab. 5. Result of decision variable of an illustrative example

<table>
<thead>
<tr>
<th>$x_j^n$</th>
<th>$b_{ln}$</th>
<th>$y_{jkj}$</th>
<th>$y_{jklm}$</th>
<th>$y_{kln}$</th>
<th>$y_{klm}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>8</td>
<td>11.2</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>2</td>
<td>7</td>
<td>15.2</td>
<td>1</td>
</tr>
</tbody>
</table>

Tab. 6. Result of decision variable of an illustrative example

<table>
<thead>
<tr>
<th>$x_{ikln}$</th>
<th>$x_{ikln}$</th>
<th>$p_{jkln}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1.3.1</td>
<td>3</td>
<td>1.21.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.2.11</td>
</tr>
</tbody>
</table>

Tab. 7 - Result of the objective function of an illustrative example

<table>
<thead>
<tr>
<th>Size of problem $(m, n, j, h, l)$</th>
<th>Objective function $z_1$</th>
<th>Objective function $z_{21}$</th>
<th>Objective function $z_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1, 1, 2, 1, 21)</td>
<td>682</td>
<td>1.25</td>
<td>0.407</td>
</tr>
</tbody>
</table>

Figures

Fig. 1. relationship between the number of temporary depots to locate and time of distribution of relief items and transferring of injured people
Fig. 2. Relationship between the number of temporary depots to locate and the number of uncovered damaged points

Follow This Article at The Following Site:

Rabbani M., Mousavi Z., Location of temporary depot after an earthquake based on robust optimization. IJIEPR. 2019; 30 (1) :39-55
URL: http://ijiepr.iust.ac.ir/article-1-746-en.html