A Mathematical Model for City Logistics Distribution Network Design with the Aim of Minimizing Response Time

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KEYWORDS
City logistics, Urban freight, Distribution, City distribution centers, Network design, Response time, Queuing theory.

ABSTRACT
The continuous increase in population in metropolises has caused major problems in delivering goods and urban services. City logistics models can be effective in solving these complexities. In addition to explaining concepts and definitions related to city logistics, this study presents a mathematical model for designing a city logistic distribution network with the aim of minimizing response time. This aim is optimal and may be efficient for emergency goods and services, especially at the time of crisis, as well as goods which require minimum delivery time. The network structure is so that the goods will be transferred through three levels: logistics centers around the city are the first level, second level is distribution centers within the city, and third level is sales terminals as demand areas. In fact, the goal is selecting a number of fixed sites for establishing city distribution centers. Demand is assumed to be probabilistic, and network modeling is conducted based on queueing theory. A numerical example is generated, and the results of solving using the BARON solver in GAMS software and model sensitivity analyses are explained. Results indicate the accuracy and validity of the proposed modeling. Finally, conclusions and suggestions for future studies are presented.

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1. Introduction
Recently, rapid urbanization has been observed around the world. Approximately one-half of the total world population of 7 billion people were living in urban areas in 2010 according to a survey by the United Nations and it is predicted to be over 60% by 2030. Problems relating to urban goods distribution are more difficult due to the concentration of population living in megacities. Further, the change in demographics in terms of increase in the number of the elderly generates the demand for more services to homes such as medical, nursing, and other daily services. To address these challenging issues, the concept of city logistics has been proposed. Several policy measures have been implemented and evaluated using mathematical models in a number of cities in the world [1]. City logistics is an integrated approach for urban freight transport
issues based on the systems approach. As urban freight transport issues are very complicated, city logistics solutions require an integrated approach, with contributions from a range of different disciplines such as systems engineering, transport planning, land use planning, information engineering, economics, geography, and social science. This issue can promote innovative schemes that reduce the total cost, including economic, social, and environmental cost for the movement of goods within cities [2].

City logistics plays an important role in creating efficient, environmental-friendly, and safe urban freight transport systems. A number of policy measures, including urban consolidation centers, regulations of access control to city centers, off-peak hour deliveries, and low emission zones, have been tested and implemented in urban areas of cities around the world. Modeling city logistics schemes is required for evaluating the effects of implementing these measures [3].

In the second section of this paper, concepts and definitions related to city logistics are presented. In the third section, literature and recent trends in the modeling of city logistics, especially in metropolitan areas, are briefly discussed. In the fourth section, the mathematical model to design city logistics distribution network is presented. The most important innovation in the proposed model is concerning response time as minimizing objective function and also use of queuing theory in modeling. A random numerical example is generated in the fifth section to solve the mathematical model proposed and, then, the analysis is presented. In the sixth section, the results and conclusions of this paper are presented and, finally, the references used in this paper are presented in the seventh section.

2. City Logistics Concepts

2-1. Definitions

Many terms and interpretations have been used interchangeably to address the transport of urban supplies. Therefore, CL must be considered in the broadest sense [4]. Examples are presented to illustrate the scope of the existing interpretations of CL:

- CL refers to freight transport in urban areas, specifically the freight flows associated with the supply of goods to city centers [5].
- CL “encompasses the routing and movement of freight across all transport modes as well as the associated activities such as warehousing and exchanging information for the management of freight at each end of its journey” [6].
- CL is a concept that tries to optimize urban freight transport systems by considering all stakeholders and movements in urban areas [7].
- “Any service provision contributes to the optimized management of the movement of goods in cities” [8].

2-2. Objectives

As shown in Figure (1), the objectives of city logistics can be divided into three economic, environmental, and social categories [9].

![City Logistics Objectives](image)

Fig. 1. City Logistics Objectives

From another perspective, city logistics has multiple objectives, relating to mobility, sustainability, livability, and resilience. Regarding mobility, a smooth and seamless flow of goods is required in operating urban goods movement systems. Regarding sustainability, the negative environmental impacts including air pollution, noise, and vibration from trucks should be minimized. Concerning livability, safety and security issues are the most important for regional communities because residents would like to have quiet and healthy living conditions. Recently, resilience has become more important in natural and manufactured disasters [10]. The relief supply distribution of water, food, and daily commodities to displaced people in shelters after catastrophic disasters requires efficient and agile operation of humanitarian logistics [11].
2-3. Stakeholders
As city logistics aims at total optimization, it needs to consider the objectives and behavior of several stakeholders who are involved. There are four main stakeholders in city logistics: shippers, freight carriers, administrators, and residents. Shippers hope to maximize their profits by choosing appropriate freight carriers and want to have reliable logistics services with less cost. Freight carriers try to provide better services by meeting the requests of shippers who usually set time windows for receiving their goods. Administrators in municipalities generally aim to promote economic development, better environment, and energy savings. Residents want their local area to be livable, safer, and attractive.

3. Literature Review
Modeling plays a vital role in facilitating city logistics. During the assessment stage of the urban freight transport management procedure, models are helpful for understanding urban freight transport and providing insights into the behavior of stakeholders as well as identifying the effects of policy measures [9]. The network design problem is very important in optimization problems, in which the multi commodity network design problem has many applications in different fields [12]. The excellent and complete review paper in this issue is given by Yaghini and Akhavan [13]. For a comprehensive review of design of a service network for freight transportation, see Wieberneit [14].

In general, two types of models are used in the area of urban freight transport: (1) optimization models and (2) simulation models. Optimization models aim to make the best use of available resources under certain constraints. For example, vehicle routing and scheduling problem with time windows (VRPTW) models have been widely studied and used in practice to minimize the costs of the vehicle operations of freight carriers. These models determine the allocation of vehicles to customers and the visiting order of vehicles at customers for delivering goods. Stochastic and dynamic VRPTW models provide very good guidance to improve vehicle operations in the cases of variable travel times on the road network. ITS provides a large amount of data on the variability of travel times on road networks.

Location models of logistics terminals are optimization models that determine the optimal size and location of logistics terminals usually using queuing theory and cost models. These models are useful for understanding the gap between the current level of efficiency in logistics operations and the optimal solutions. Normally and in practice, there are so many elements that are not considered in modeling, including the business customs of subcontracts with smaller freight carriers and irregular orders by shippers in order not to lose business. It is challenging to replicate the current situation of logistics operations in the real world. However, optimization models provide a benchmark for the level of costs as well as emissions of hazardous gases [9].

According to costly and difficult to reverse nature of facility location problems (FLPs) and its long time horizon impact, there is a critical management decision in the design of efficient logistics systems, which discuss about the choice of locations for distribution centers (DCs) to enhance operation efficiency and logistics performance. Applying queuing theory in FLPs, first was discussed by Larson (1974) where he analyzed problems of vehicle location-allocation and response district design in emergency response services that operate in the server to customer mode. Following Larson (1974) study, providing probabilistic models that consider queuing theory in FLPs has been developed by many researchers [15].

Location problems have mostly been developed in a single facility type. A current stream of location modeling is focused on a hierarchical approach. Hierarchical facility location models take into account the location of their interacting facilities within a multiple layer configuration. In a hierarchical system of facilities, the demand or customer sites is the base level and the underlying structure is assumed to be a network whose nodes represent facility and demand sites. Hierarchical facility location problems can be classified by four attributes: flow patterns (i.e.
flow features of services or goods on arcs between nodes of the network), service availability, spatial configuration of services, and objectives for locating facilities. These types of models have mainly been applied to health-care facility systems [3]. Location-routing problems are a class of problems usually used to determine both facility locations and routes around the facilities at the same single level. The potential application areas are waste management, production-distribution services, and postal services. In the case of production-distribution systems, when the distribution is handled through warehouses owned by a manufacturer, the structure is essentially a multi-level system of facilities. A common example is composed of retailers (and/or wholesalers), warehouses, and production plants. Products are first distributed from manufacturers’ plants to warehouses and, then, to retailers. In the case of the nested availability of services (i.e. direct delivery exists from production plants to retailers as well as from warehouses to retailers), the flow pattern is generally multi-flow [3].

Another important aspect of location modeling of urban logistics facilities is to incorporate the change into traffic flow on transport networks, such as using traffic assignment procedures. However, there have been few studies considering traffic assignment within location modeling [3].

Recently, there have been logistics or supply chain-based models taking into account the interactions among various economic entities throughout the supply chain. Tavasszy, Ruijgrok, and Davydenko (2012) emphasized the importance of including logistics or supply chain elements into freight modeling [16]. This type of model typically simulates commodity flows in an urban area within the framework of supply chain management. Locations of manufacturers, distributors, and consumers are considered, whereas most models can only highlight a few industries where data are readily available, which is largely due to data availability. The stream of supply chain network modeling from Nagurney, Dong, and Zhang (2002) represents an important research direction using multilevel multi-tiered networks, which allows distinct flows to be captured, including logistical, informational, and financial flows within the same network system while retaining the spatial nature of the network of decision-makers [17]. In this context, Yamada, Imai, Nakamura, and Taniguchi (2011) proposed a supply chain-transport super-network equilibrium model, which is able to represent the mutual effects between the behavioral changes in supply chain networks and transport networks [18].

Risk management relating to natural and manmade disasters is a challenging topic in city logistics. As for natural disasters, humanitarian logistics is important subject associated with city logistics. After natural disasters including earthquakes, floods, tsunamis, hurricanes, tornadoes and bush fires, people who suffered from the disasters immediately need water, food, clothes and other daily commodities. Providing these required goods to displaced persons is urgent and critical for their lives and health. The objective of humanitarian logistics is minimizing the sufferings of affected people, while that of business logistics is minimizing the total costs. The constraints are also different in humanitarian logistics, since the commodities are not enough for all displaced persons and the capacity of trucks and drivers is not always enough for deliveries. Moreover, planning deliveries to displaced persons is very hard because their demands are uncertain and the road network is often damaged by disasters. Therefore, planning should be done quickly with very uncertain information [10].

Several existing studies have revealed key challenges for emergency logistics planning compared to business logistics. These involve uncertainties, communication and coordination, efficient and timely delivery and limited. Balci, et al. (2010) discussed coordination in humanitarian relief chain among various actors who are involved. Caunhye et al. (2011) showed that optimization models become powerful tools to tackle emergency logistics. They categorized various optimization models into two groups: (a) short-notice evacuation, facility location, and stock pre-positioning are drafted as the main pre-disaster operations, (b) relief distribution and casualty transportation are categorized as post-disaster operations. Liberatore, et al. (2011) studied the disruption caused by disasters focusing on the correlation effects between the facilities. They provided a tri-level formulation of the problem, and proposed an exact solution algorithm which makes use of a tree-search procedure to identify which facilities to protect [10]. Ahmadi et al. developed a multi-depot location-routing problem model to minimize total distribution time, penalty costs of unsatisfied demand and fixed costs of opening logistics.
distribution centers, and cost of unsatisfied demand. A real world case study in San Francisco district was carried out to demonstrate the capability of the formulation [19]. In facility location optimization for emergency logistics, maximal covering approach has typically been used in terms of number of facilities allocated to a demand point. A stochastic vehicle routing model was developed by Dessouky, Ordóñez, Jia & Shen (2006) for rapid distribution of medical supplies. Relief distribution and stock prepositioning are often combined into facility location models. The most common type of models for relief distribution and casualty transportation look at both resource allocation and commodity flow [3]. A key challenge in city logistics modeling is that many models that are good representations of reality are not only complex, but also very complicated, which make decision-makers avoid them. Rather, the true challenge is to develop models that are simultaneously more valid (scientifically rigorous) and intuitive [9].

4. Mathematical Modeling
4-1. Problem description
In a city, demands of chain stores or e-businesses are mostly met through city logistics distribution operators, which is usually undertaken by the third-party logistics providers (3PLs) responsible for goods delivery services from several large retailers. 3PLs collect goods from logistics centers (LCs) located in suburban areas of the city and, then, transport them to the selected distribution centers (DCs) for further processing procedures. Finally, these goods are distributed to widely distributed sales terminals (STs) of the distribution network. The proposed model makes decision on the problem of locating and allocating in city logistics distribution network. In fact, the objective is to choose a number of fixed sites to construct city distribution centers and to determine how the allocation of sales terminals to distribution centers and allocation of distribution centers to logistics centers should be. The ultimate objective is to minimize response time. This objective for goods and emergency services, especially in critical situations and also delivering the goods as soon as possible, is desirable and can have good performance. As shown in Figure (3), network structure is so that the goods will be transferred through three levels: logistics centers around the city are the first level, second level is distribution centers within the city, and third level is sales terminals as demand areas.

![Image](https://via.placeholder.com/150)

**Fig. 3. City logistics distribution network**

In this model, the network nodes are server and goods are as clients. At the network nodes, the operations such as production, storage, packaging, barcode makers, cutting, mixing, combing, loading, unloading, sorting, processing, and delivery are done. These services are offered as customers. Terms of the problem are faced with some uncertainties; under such conditions, the demand for goods and serving time is considered possible. Finally, by solving the above model, the location of distribution centers and allocation will be selected so that the response time would be optimized in the network. Then, signs, parameters, and decision variables of the problem are discussed.

4-2. Indices and parameters
- \( i \): Index for LCs
- \( j \): Index for DCs to be set up candidate sites
- \( k \): Index for STs
- \( r \): Index for goods
- \( D^r_k \): Demands of goods type \( r \) for ST \( k \)
- \( d_{ij} \): Distance from LC \( i \) to DC \( j \)
- \( d_{jk} \): Distance from DC \( j \) to ST \( k \)
- \( f_i \): Unit construction cost for DC \( j \)
- \( B \): Total fixed cost at DCs
- \( W \): Number of DCs planned to construct
- \( \lambda \): Arrival rate of demands at STs
- \( \mu \): Service rate at the network nodes (\( \mu_1, \mu_2, \ldots, \mu_K \))
- \( \delta \): Parameter of negative exponential distribution
- \( c \): Lower bound of a uniformly distributed random variable that indicates the quantity of goods in a demand
- \( d \): Upper bound of a uniformly distributed

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random variable that indicates the quantity of goods in a demand

t^r_{ijk} \text{ Response time of the system for goods type } r \text{ from node } i \text{ to node } k, \text{ going through DC located at node } j

WT_r \text{ Sojourn time of goods type } r \text{ in the system}

WT^q_r \text{ Waiting time of goods type } r \text{ in the queue}

TR^r_{ij} \text{ Transportation time for goods type } r \text{ from node } i \text{ to node } j

TR^r_{jk} \text{ Transportation time for goods type } r \text{ from node } j \text{ to node } k

v^r_{ij} \text{ Transportation speed for goods type } r \text{ from node } i \text{ to node } j

v^r_{jk} \text{ Transportation speed for goods type } r \text{ from node } j \text{ to node } k

4-3. Decision variables

Z_j \begin{cases} 1 & \text{if DC } j \text{ is set up; } 0, \text{ otherwise} \\ x^r_{ij} \begin{cases} 1 & \text{if goods type } r \text{ is delivered from LC } i \text{ to DC } j; \ 0, \text{ otherwise} \\ y^r_{jk} \begin{cases} 1 & \text{if goods type } r \text{ is delivered from DC } j \text{ to ST } k; \ 0, \text{ otherwise} \\
\end{cases}
\end{cases}

4-4. Mathematical model

\begin{align*}
\text{Min } RT & = \sum_i \sum_j \sum_r t^r_{ijk} \\
& \quad \forall i \in I, j \in J, r \quad (1)
\end{align*}

\begin{align*}
x^r_{ij} & \leq Z_j \quad \forall i \in I, j \in J, r \quad (2)
\end{align*}

\begin{align*}
y^r_{jk} & \leq Z_j \quad \forall j \in J, k \in K, r \quad (3)
\end{align*}

\begin{align*}
\sum_j \sum_r x^r_{ij} & \geq 1 \quad \forall i \in I \quad (4)
\end{align*}

\begin{align*}
\sum_j \sum_r y^r_{jk} & \geq Z_j \quad \forall j \in J, k \in K, r \quad (5)
\end{align*}

\begin{align*}
\sum_{i,j} f_{ij} Z_j & \leq B \quad \forall i, j, r \quad (6)
\end{align*}

\begin{align*}
\sum_i Z_j & \leq W \quad (7)
\end{align*}

\begin{align*}
\sum_j x^r_{ij} & = Z_j \quad \forall j \in J, r \in R \quad (8)
\end{align*}

\begin{align*}
\sum_{j \in J} y^r_{jk} & = 1 \quad \forall k \in K, r \in R \quad (9)
\end{align*}

\begin{align*}
t^r_{ijk} & = WT^r_i + WT^q_r + WT^{r'}_k + TR^r_{ij} \\
& \quad + TR^r_{jk} \quad (10)
\end{align*}

\begin{align*}
x^r_{ij}, y^r_{jk} & = 0,1 \quad \forall i \in I, j \in J, k \in K, r \in R \quad (11)
\end{align*}

\begin{align*}
Z_j & = 0,1 \quad \forall j \in J \quad (12)
\end{align*}

Equation (1) represents the objective function that minimizes total response time in the network. Equations (2) and (3) show a distribution center when it can join the distribution activities that are launched. Constraint (4) ensures that all logistics centers will join distribution activities. In fact, the objective of taking this constraint is that the potential of all goods is used to provide logistic centers.

While the capacity of logistic centers is to the extent that demand for distribution centers is supplied barely, this constraint is not required to be concerned and the model tries to apply all the capacity of these centers to supply demand.

Constraint (5) contributes an item delivered from the upper levels to the nearest nodes at lower levels. Equation (6) states that the fixed costs of distribution centers should be less than the amount of available funds. Constraint (7) specifies the total number of distribution centers should be built.

According to Constraints (8) and (9), if any distribution center is established, demand for goods of type r can only be supplied by a logistics center and any sale terminals can supply demand for goods of type r by a distribution center. Equation (10) suggests that the response time of the system is equal to the freight time in the network in addition to the time spent in the system (the nodes). Constraints (11) and (12) also express the nature of binary in decision-making variables that take the values of zero and one.

4-5. Queuing system

The studied queuing system is a series-parallel system and is composed of three levels. Each network node is assumed to be a queuing system in the form of M/M/1 and serving is done with exponential distribution using parameter $\mu$ and the system is FIFO. It is assumed that demand for goods in the sale terminal of $k$ follows exponential distribution with the parameter $\lambda_k$.

Because each of the distribution centers serves a group of demand areas, each demand in these distribution centers ($\lambda_j$) is equal to the total demand of its related clients. Therefore, the demand of distribution centers follows the following equation:

\begin{equation}
\lambda_j = \sum_{k \in K} \lambda_k y_{jk} \quad , \quad j \in J
\end{equation}

Also, demand of goods for logistic centers follows the exponential distribution and its
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Parameter is obtained from the following equation:

$$\lambda_i = \sum_{j \in I} \lambda_j x_{ij} = \sum_{j \in I} \sum_{k \in K} \lambda_k x_{ij} y_{jk} , i$$  \hspace{1cm} (14)

So, the figure of the network is schematically shown below:

![Diagram of the logistics distribution network](image)

**Fig. 4. Queuing system of city logistics distribution network**

Demand for goods is expressed by two indicators including request occurrence and value of request in each event. It is assumed the request occurrence of any good with random variable of $U$ has exponential distribution with density function $f_u(u)$ and the size of the request at any time with the random variable of $V$ has uniform distribution with density function $f_v(v)$. Since these two random variables are independent, the following equations are presented:

$$f_u(u) = \begin{cases} \delta e^{-\delta u} , & u \geq 0 \\ 0 , & u < 0 \end{cases}$$  \hspace{1cm} (15)

$$f_v(v) = \begin{cases} \frac{1}{d-c} , & c < v < d \\ 0 , & \text{otherwise} \end{cases}$$  \hspace{1cm} (16)

$$E(D) = E(UV) = E(U)E(V) = \frac{c + d}{2\delta}$$  \hspace{1cm} (17)

$$E(D_k^c) = \frac{c_k^c + d_k^c}{2\delta_k}$$  \hspace{1cm} (18)

The demand for sales terminals (demand areas) follows exponential distribution with parameter $\lambda$. $\rho$ is the rate of productivity and the following equation is established:

$$\lambda = \frac{1}{E(D)} = \frac{2\delta}{c + d}$$  \hspace{1cm} (19)

$$\rho = \frac{1}{\mu E(D)} = \frac{2\delta}{\mu(c + d)}$$  \hspace{1cm} (20)

Also, according to the models known in the context of queuing theory, queuing parameters for M/M/1 model are as follows [21]:

$$f_{WT}(t) = (\mu - \lambda)e^{-(\mu - \lambda)t}$$  \hspace{1cm} (21)

$$F_{WT}(t) = P(WT \leq t) = 1 - e^{-(\mu - \lambda)t}$$  \hspace{1cm} (22)

$$WT = E(WT) = \int_0^\infty f_{WT}(t)tdt = \frac{1}{\mu - \lambda}$$  \hspace{1cm} (23)

$$WT_q = WT - \frac{\lambda}{\mu}$$  \hspace{1cm} (24)

$$LR = \lambda WT = \frac{\lambda}{\mu - \lambda}$$  \hspace{1cm} (25)

$$LR_q = \lambda WT_q = \frac{\lambda^2}{\mu(\mu - \lambda)}$$  \hspace{1cm} (26)

According to the above equations, Equation (27) shows average time spent in the system at three stages of the network. Next equations respectively show the average waiting time in the queue, the average number of goods in the system, and the average number of goods in the queue.

$$WT_{sys} = \sum_{r \in R} \sum_{i \in I} \frac{1}{\mu_i - \sum_j \sum_k \left(\frac{1}{E(D_k)}\right) x_{ij}^r y_{jk}^r}$$

$$+ \sum_{r \in R} \sum_{j \in J} \sum_k \left(\frac{1}{E(D_k)}\right) y_{jk}^r$$

$$+ \sum_{r \in R} \sum_{k \in K} \mu_k \left(\frac{1}{E(D_k)}\right)$$  \hspace{1cm} (27)

$$WT_q = \sum_{r \in R} \sum_{i \in I} \left(\frac{1}{\mu_i - \sum_j \sum_k \left(\frac{1}{E(D_k)}\right) x_{ij}^r y_{jk}^r} \right)$$

$$+ \sum_{r \in R} \sum_{j \in J} \mu_j \left(\frac{1}{E(D_k)}\right) y_{jk}^r$$

$$+ \sum_{r \in R} \sum_{k \in K} \mu_k \left(\frac{1}{E(D_k)}\right)$$  \hspace{1cm} (28)
$$LR_{sys} = \sum_{r \in R} \sum_{i \in I} \left( \mu_i - \sum_{j \in J} \frac{1}{E(D_k^i)} x_{ij}^r y_{jk}^r \right) \sum_{k} \frac{1}{E(D_k)} y_{jk}^r + \sum_{r \in R} \sum_{j \in J} \left( \mu_j - \sum_{k} \frac{1}{E(D_k)} y_{jk}^r \right) + \sum_{r \in R} \sum_{k \in K} \mu_k - \frac{1}{E(D_k)}$$

(29)

$$LR_q = \sum_{r \in R} \sum_{i \in I} \mu_i \left( \frac{1}{E(D_k)} x_{ij}^r y_{jk}^r \right)^2 + \sum_{r \in R} \sum_{j \in J} \mu_j \left( \frac{1}{E(D_k)} y_{jk}^r \right)^2 + \sum_{r \in R} \sum_{k \in K} \mu_k \left( \frac{1}{E(D_k)} \right)^2$$

(30)

The objective function of the model is equal to the total response time and raised from combining two general parts; i.e. average time spent in the system and total transportation times in the network. So, it can be written in the form of Equation (31):

$$\min RT = WT_{sys} + \sum_{r \in R} \sum_{i \in I} \sum_{j \in J} TR_{ij}^r x_{ij}^r + \sum_{r \in R} \sum_{j \in J} \sum_{k \in K} TR_{jk}^r y_{jk}^r$$

(31)

5. Numerical Experiments

In this section, a problem is solved using a numerical example in order to verify the effectiveness of the proposed mathematical model. Also, sensitivity analysis will be performed. It is assumed that the relevant network consists of 8 logistics centers, 10 candidate sites to establish urban distribution centers and 15 sale terminals as demand areas. Also, the number of goods is equal to 3.

The distance of logistics centers from distribution centers and the distance of distribution centers from sales terminals are evenly distributed (uniform distribution) and assumed to be respectively between 30 and 80 km, and 1 to 10 km. Also, fixed costs to establish distribution centers are evenly between 5,000 and 6,000 monetary units. The maximum available budget to construct distribution centers will be 30,000 monetary units and 5 distribution centers will be constructed. The rate of serving at the first to third levels of network is respectively equal to 10, 5, and 3. Demand occurrence at sales terminals has exponential distribution with parameter $\delta$ and its values are between 1 and 5.

Also, lower and upper bounds of a uniform distribution function to determine the amount of demand is assumed to be 0 and 30. The average speed of freight transportation at the first level of network is 30 km/h and, at second level of network, it is 20 km/h. It is normal that transportation time between network levels is equal to the distance of division network areas by the average speed. Given the above values, the results of the model using BARON solver in Gams software are as in the following table.

<table>
<thead>
<tr>
<th>Decision variables</th>
<th>Optimum values</th>
</tr>
</thead>
<tbody>
<tr>
<td>RT</td>
<td>41.439</td>
</tr>
<tr>
<td>$Z_q$</td>
<td>3.5, 6</td>
</tr>
<tr>
<td>WT$_{sys}$</td>
<td>24.518</td>
</tr>
<tr>
<td>WT$_q$</td>
<td>17.166</td>
</tr>
<tr>
<td>LR$_q$</td>
<td>9.803</td>
</tr>
</tbody>
</table>

In sensitivity analysis of the model, parameters such as $c_k^r$, $d_k^r$, $\delta_k^r$, and $\mu$ are changed in the reduction range of 20 percent to increasing rate of 20 percent to specify their effect on objective function (RT) and queuing indexes (WT$_{sys}$, WT$_q$, LR$_q$). The results of these surveys are presented in the form of tables and graphs below.
As expected, by increasing the rate of serving ($\mu$), the objective function is reduced. This means that if goods are served faster, response time in the network is reduced. Change of other parameters has less effect on the objective function and changes of objective function in this case are not regular.

By increasing the rate of serving ($\mu$), the time spent in the system is reduced. Also, the time spent in the system is reduced in small-scale by changing the upper and lower bounds of demand. By increasing rate of demand, time spent in the system is also increased. Given the relationship between the average time spent in the system, changing pattern could be predicted.

As can be seen, by increasing the rate of serving, indices of queuing system, i.e. average waiting time in the queue and average number of the goods in the queue, are slightly increased. In the final step, the average number of goods in the queue is reduced. Also, by increasing the rate of demand occurrence, naturally, these indices are increased. Increased lower and upper bounds of demand affect the indices of the queue decreasingly. These changes could be predicted.
according to the mathematical equations of queuing parameters. Based on all the foregoing, the accuracy and validity of the model are proved.

6. Conclusion

The city logistics models presented so far have mostly focused on economical and ecological goals, and the concept of city logistics has rarely been connected with social goals, especially urban crises and delivery of emergency goods. The objective function of the model proposed in this paper, i.e. minimization of response time in the network, is mainly defined in order to fill this research gap, although it serves other purposes as well. In addition to clarifying the concept of city logistics, this study modeled a city logistics distribution network based on the queuing theory. General results indicate the acceptable potential and flexibility of the proposed model for designing a city logistics distribution network, especially at the time of crisis and for emergency goods. Sensitivity analyses also confirm the accuracy and validity of the model. The following managerial insights have been resulted from the findings:

1- Values of response time and mean sojourn time depend on service time and can be reduced by designing appropriate mechanisms and optimal time management in network nodes. This is highly important at the time of crisis and in the case of emergency goods.

2- Queueing theory can be used in analyzing the performance of time-dependent systems, using which we can optimize response time in a city logistics distribution network. Queue indices can be successfully incorporated as criteria for evaluating network status.

3- Fixing the intra-city flow of goods by establishing distribution centers is a well-known and useful method. Nevertheless, its possible effects on other city logistics parameters require further investigation.

4- Minimization of response time in the network indirectly considers economical and environmental goals as well, since the faster delivery of goods decreases transportation costs and urban pollutants.

In future studies, the queuing system in network nodes can be based on real data instead of the M/M/1 type. Moreover, the demand for each good can be supplied from multiple upstream nodes, in which case the demand rate of these nodes would differ from the value presented here.

References


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