International Journal of Industrial Eng. & Production Research (2008) pp. 57-67

Volume 19, Number 4, 2008

International Journal of Industrial Engineering & Production Research

Journal Website: http://ijieen.iust.ac.ir/

Applying Semi-Markov Models for forecasting the Triple Dimensions of Next Earthquake Occurrences: with Case Study in Iran Area

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Keywords

Semi-Markov Model, Probabilistic Forecasting Matrix, Forecasting Error, Karakaisis zoning

ABSTRACT

In this paper Semi-Markov models are used to forecast the triple dimensions of next earthquake occurrences. Each earthquake can be investigated in three dimensions including temporal, spatial and magnitude. Semi-Markov models can be used for earthquake forecasting in each arbitrary area and each area can be divided into several zones. In Semi-Markov models each zone can be considered as a state of proposed Semi-Markov model. At first proposed Semi-Markov model is explained to forecast the three mentioned dimensions of next earthquake occurrences. Next, a zoning method is introduced and several algorithms for the validation of the proposed method are also described to obtain the errors of this method.

1. Introduction

The Monrovian models can be used in analyzing many natural events. In each event, the states of Monrovian model can be defined accordingly. In this paper the error of proposed Semi-Monrovian model is analyzed where each zone is considered as a state. Generally, earthquake occurrences follow Monrovian models [1], [2]; hence a Semi-Markov model is used for earthquake forecasting. The earthquakes can be investigated as both mathematical and physical modeling [3]. In this paper, we consider a mathematical model of earthquakes.

The earthquake occurrences can be modeled by some probabilistic techniques [4] but applying Semi-Markov models in earthquake modeling is interest because Semi-Markov model is able to consider temporal dimension for earthquakes while simple Monrovian models cannot consider it easily. During the past few years, there have been some studies on earthquakes modeling using Monrovian models [5], [6], [7]. There is also some research on Semi-Monrovian models [1], [2], [8].

One of the advantages of our proposed model is that the model considers three dimensions such as time, space and magnitude, simultaneously. This could be considered as an advantage since the most of recent studies investigated only one or two dimensions. [9-12]. The validation of this proposed model can be evaluated by two methods and Nava et al. [13] also used another method for the validation of earthquake forecasting.

Some other advantages of Semi-Monrovian models in comparison to other models have been explained by [1]. In the following sections, the proposed model and its validation will be explained, and then the dimensions of the future earthquakes are forecasted using this model in a zoning method proposed by Karakaisis based on seismic points. The errors of this method are calculated. Iran is used as a case study in this paper.

2 Modeling

Semi-Markov model [14], [15] for forecasting the dimensions of the earthquakes has been examined in

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Paper first received Aug. 18, 2007, and in revised form Dec. 11, 2008.

[2] and [8], completely. One of the most important elements of the Semi-Markov processes is the interval transition probability matrix. The probability of a transition from state i to state j in the interval $[0,n]$ requires the process to make at least one transition during the interval. Interval transition probability matrix can be determined in the following matrix form:

$$F(n)=GW(n)+\sum_{m=0}^{n} G\otimes T(m)F(n-m)= \qquad (1)$$
$$GW(n)+\sum_{m=0}^{n} C(m)F(n-m); \quad n=0,1,2,\ldots$$

where $GW(n)$ is a diagonal matrix whose i th element is equal to $Gw_i(n)$, and $Gw_i(n)$ term is the probability that the waiting time in state i is greater than n. The interval transition probability $F(n)$ is obtained by a recursive procedure. Since $T(0)$ is equal to zero, $F(n)$ is obtained for the interval $1\le m\le n$. In case $n=0$, $F(n)$ is equal to the Kronecker Delta or identity matrix defined as follows:

$$F_{ij}(0)=\begin{cases}1 & i=j\\ 0 & i\neq j\end{cases}$$

In Eq. (1) G is a transition matrix that G_{ij} is the probability that last step is in state i and the next step is in state j. In other words, G_{ij} is the probability of motion from state i to state j. Also $T(m)$ is the holding time matrix and is defined as follows:

where n is the number of time intervals, $C(m)$ is the core matrix and is defined as follows:

$$C_{ij}(m)=G_{ij}T_{ij}(m); \quad i,j=1,\ldots,N,\ m=1,\ldots,n$$

where N is the total number of states in the system.

In the earthquake phenomenon, $F(n)$ can be used for studying earthquake hazards and evaluating seismic hazards risk.

Two interval transition probability matrices such as $FR(k)\ \forall k=1,\ldots,n$ for region to region transitions and $FM(k)\ \forall k=1,\ldots,n$ for magnitude to magnitude transitions can be determined by Eq. (1). If the last earthquake is occurred in region r_0 with magnitude m_0, then the matrix of probability forecasting after $k;\ \forall k=1,\ldots,n$ time periods ($FRM(k)\ \forall k=1,\ldots,n$) is obtained by the following formula:

$$FRM_{r_i m_j}(k)=FR_{r_0 r_i}(k)\times FM_{m_0 m_j}(k) \qquad (2)$$
$$\forall i=1,\ldots,r\ ;\ j=1,\ldots,m\ ;\ k=1,\ldots,n$$

where r is the number of considered zones of the supposed area and m is the number of considered partitions for all magnitudes.

Also $FRM_{r_i m_j}(k)\ \forall i=1,\ldots,r\ ;\ j=1=1,\ldots,m\ ;\ k=1=1,\ldots,n$ is the probability that an earthquake occurs in region r_i with magnitude m_j after k time periods. Fig. 1 demonstrates the described comments more clearly:

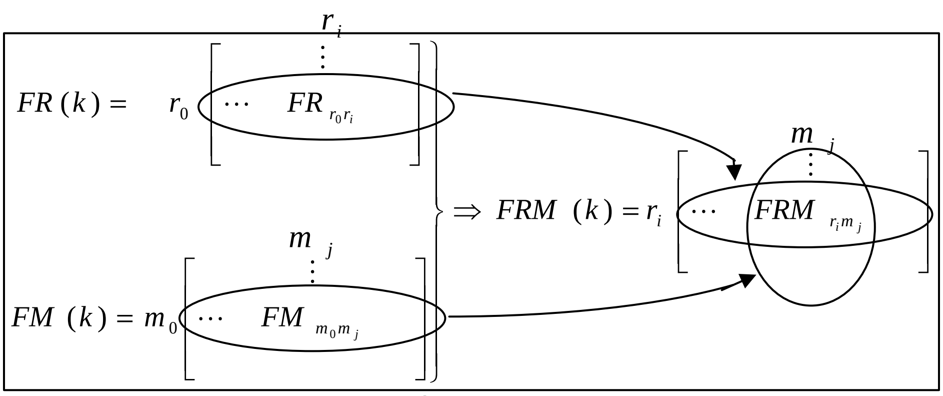

Fig. 1. The manner of determining FRM matrix

In this way, forecasting the dimensions of the following earthquakes is possible as by determining probability forecasting matrixes ($FRM(k)\ \forall k=1,\ldots,n$).

3. The Model Validation

In this section, we explain the validation procedure. Suppose $F\hat{R}M(k)$ is a probability forecasting matrix where its elements are the estimated probabilities of the next earthquake occurrences by our proposed model during the next k time periods, and $FRM(k)$ is a deterministic matrix defined as follows:

Here there are two fundamental questions:

The first question is to determine the likelihood estimation of the earthquake occurrences. In other words, can $F\hat{R}M$ forecast the next earthquake occurrences successfully.

$$FRM_{ij}(k) \equiv FRM_{r_i m_j}(k) = \begin{cases} 1 & \text{If an earthquake occurs in region } r_i \text{ with magnitude } m_j \text{ in } k \text{ th time period} \\ 0 & \text{Otherwise} \end{cases}$$

The next question is to see whether $F\hat{R}M$ can also be used for forecasting the next earthquake occurrences deterministically and not only probabilistically. The response for the first question can be found in methods 1, 2 and 3, and the response for the second question is explained in method 3.

Method 1: applying Mean Square Error (MSE) and Mean Absolute Deviation (MAD) [16]

Method 2: Mean Absolute Percent Error (MAPE) [17]

Method 3: an innovative plan for determining both probability forecasting error and deterministic forecasting (in this paper it is named zero and one method technically)

In methods 1 and 2, if the total history data is equal to n, the first n_1 data are used for forecasting the next n_2 data, and the next n_2 data are used to determine the forecasting error ($n = n_1 + n_2$). While all of the n data have occurred previously and are available, the forecasting errors can be calculated. In method 3, the first n_1 data are used for forecasting the next n_2 data along with benchmarking and the first $n_1 + n_2$ data are used for forecasting the next n_3 data along with determining the forecasting error ($n = n_1 + n_2 + n_3$), in case the whole n data have already occurred and are available; the forecasting errors can be calculated.

In these methods, calculation of the forecasting error can be performed in two forms: by using both subsequent data and random data within the set of whole data. In the first form, the first n_1 data are used for forecasting the next n_2 data, ($n = n_1 + n_2$), while in the second form the n_2 data within the main n data are eliminated randomly and then these n_2 data will be forecasted by their previous data.

3-1. Method 1

In this method equations based on Mean Square Error (*MSE*) and Mean Absolute Deviation (*MAD*) can be used. For this goal an algorithm is presented for *MSE*, which can be used with some changes for *MAD* and method 2.

Algorithm I:

This algorithm could be used to determine the forecasting error by *MSE* as follows:

Step 0: Begin

Step 1: $n_2(0) = 0$

Step 2: $i = 0$

Step 3: Use the first $n_1 + n_2(i)$ data to determine $F\hat{R}M$, FRM (n_1 is the number of first data which can be used for forecasting the next data and $n_2(i)$ is the number of data occurred during i time periods after the first n_1 data)

Step 4: Determine $F\hat{R}M$, FRM

Step 5:

$$MSE(i+1) = \frac{\sum_{i=1}^{r}\sum_{j=1}^{m}(FRM_{ij}(1) - F\hat{R}M_{ij}(1))^2}{r \times m} \tag{3}$$

Step 6: $i = i + 1$

Step 7: If $n_1 + \sum_{j=1}^{i} n_2(j) < n$ then go to step 3 else go to step 8 (n is the number of total data)

Step 8: $MSE = \dfrac{\sum_{k=1}^{i} MSE(k)}{i}$

Step 9: End

To calculate the forecasting error by *MAD*, use algorithm (I) and only substitute steps 5 and 8 in algorithm (I) by considering the steps as follow:

To calculate the forecasting error by *MAD*, use algorithm (I); just be careful to substitute steps 5 and 8 in algorithm (I) by considering the following steps:

Step 5:

$$MAD(i+1) = \frac{\sum_{i=1}^{r}\sum_{j=1}^{m}| FRM_{ij}(1) - F\hat{R}M_{ij}(1) |}{r \times m} \tag{4}$$

Step 8: $MAD = \dfrac{\sum_{k=1}^{i} MAD(k)}{i}$

For each forecasting if the past data are more complete, the forecasting results are more accurate. Therefore applying $F\hat{R}M(k_2)$ after $F\hat{R}M(k_1)$ gives more accurate information than $F\hat{R}M(k_1 + k_2)$. Accordingly, only $F\hat{R}M(1)$ and $FRM(1)$ are used in Eq. (3) and (4).

For the remaining equations, $F\hat{R}M(1)$ and $FRM(1)$ are also used.

3-2. Method 2

One of the disadvantages of this method is that *MSE* and *MAD* neither have any mathematical interpretation, nor is there any benchmark for the goodness degree.
In method 2, one equation is used similar to algorithm (I). However we replace steps 5 and 8 in algorithm (I) as follows:
In this method, a similar equation to algorithm (I) is used, yet steps 5 and 8 in algorithm (I) are substituted by the following intended steps:

Step 5:

$$MAPE(i+1)=\frac{\sum_{i=1}^{r}\sum_{j=1}^{m}\frac{|FRM_{ij}(1)-F\hat{R}M_{ij}(1)|}{FRM_{ij}(1)}}{r.m}\times 100 \quad (5)$$

Step 8: $MAPE=\dfrac{\sum_{k=1}^{i}MAPE(k)}{i}$

where *MAPE* is Mean Absolute Percentage Error. *MAPE* is a useful equation since it specifies the error percentage, which is both comprehensible and interpretable for everyone. Therefore, this method will be concentrated upon in this study.

3-3. Method 3

Definition

i th order maximum:

The i th element in a sorted as well as decreased list, in which none of the elements are equal to each other, is named i th order maximum.
This method is an innovative plan which can be used for two goals.

a) Determining the Forecasting Error
b) Deterministic Forecasting of Earthquake Occurrences

a) Determining the Forecasting Error

In this section we present an algorithm to determine the forecasting error. This algorithm has two sections. The first section of the algorithm is devoted to benchmarking and the second section is assigned to determining the forecasting error. The algorithm is made of the following several steps:

Algorithm II:

Step 0: Begin

Step 1: $n_2(0)=0$

Step 2: $i=0$

Step 3: Use first $n_1+n_2(i)$ data for determining $F\hat{R}M, FRM$

Step 4: Determine $F\hat{R}M$, FRM

Step 5:
$F\hat{R}M1(i+1)=F\hat{R}M(1),\ FRM1(i+1)=FRM(1)$

Step 6: $i=i+1$

Step 7: If $n_1+\sum_{j=1}^{i}n_2(j)<n-n_3$ then go to step 3

else $k_1=i$ and go to step 8 (n_3 is the number of data used to determine the forecasting error)

Step 8: For $i=1$ to k_1 do

$F\hat{R}M2(i)=O$ (O is a zero matrix)

$$MAPE2(i)=\frac{\sum_{i=1}^{r}\sum_{j=1}^{m}\frac{|FRM1_{ij}(i)-F\hat{R}M2_{ij}(i)|}{FRM1_{ij}(i)}}{r.m}\times 100$$

Step 9: $MAPE(0)=\dfrac{\sum_{k=1}^{k_1}MAPE2(k)}{k_1}$

Step 10: $i=1$

Step 11: $M_{ij}=i$ th order maximum in $F\hat{R}M1(j),\ \forall j=1,\ldots,k_1$

Step 12: $F\hat{R}M2(j)=\left\lfloor\dfrac{F\hat{R}M1(j)}{M_{ij}}\right\rfloor,\forall j=1,\ldots,k_1$

($\lfloor a\rfloor$ obtains the greatest integer number smaller than the real number a)

Step 13: All of the elements greater than or equal to 1 in $F\hat{R}M2(j)\ \ \forall j=1,\ldots,k_1$, are replaced by 1.

Step 14:

$$MAPE2(j)=\frac{\sum_{p=1}^{r}\sum_{q=1}^{m}\frac{|FRM1_{pq}(j)-F\hat{R}M2_{pq}(j)|}{FRM1_{pq}(j)}}{r.m}\times 100,\ \forall j=1,\ldots,k_1$$

Step 15: $MAPE(i)=\dfrac{\sum_{j=1}^{k_1}MAPE2(j)}{k_1}$

Step 16: $i=i+1$

Step 17: If i th order maximum is available in $F\hat{R}M1(j),\ \forall j=1,\ldots,k_1$ then go to step 11

else go to step 18

Step 18: If $MAPE(t+1)$ is the first element greater than the obtained *MAPE* in method 2 the t th order maximum is the most proper one to be considered as a benchmark to calculate errors.

Step 19: $n_3(0)=0$

Step 20: $i=0$

Step 21: Use the first $n_1+n_2+n_3(i)$ data to determine $F\hat{R}M$, FRM (n_1 is the number of the first data which can be used to forecast the next n_2 data and n_2 is the number of data used in benchmarking section and $n_3(i)$ is the number of data occurred during i time periods after the first n_1+n_2 data)

Step 22: Determine $F\hat{R}M$, FRM

Step 23: $F\hat{R}M1(i+1)=F\hat{R}M(1),\ FRM1(i+1)=FRM(1)$

Step 24: $i=i+1$

Step 25: If $n_1+n_2+\sum_{j=1}^{i} n_3(j)<n$ then go to step 21

else $k_2=i$ and go to step 26

Step 26: $M_j=\{t$ th order maximum in $F\hat{R}M1(j),\ \forall j=1,\ldots,k_2\}$

Step 27: $F\hat{R}M2(j)=\left[\dfrac{F\hat{R}M1(j)}{M_j}\right],\forall j=1,\ldots,k_2$

([a] obtains the greatest integer number smaller than the real number a)

Step 28: All elements greater than or equal to 1 in $F\hat{R}M2(j),\ \forall j=1,\ldots,k_2$ are replaced by 1.

Step 29:

$$MAPE2(j)=\frac{\sum_{p=1}^{r}\sum_{q=1}^{m}\frac{|FRM1_{pq}(j)-F\hat{R}M2_{pq}(j)|}{FRM1_{pq}(j)}}{r.m}\times 100,\ \forall j=1,\ldots,k_2$$

Step 30: $MAPE=\dfrac{\sum_{j=1}^{k_2} MAPE2(j)}{k_2}$

Step 31: End

Note that in algorithm (II), *MAPE* can be substituted by either *MSE* or *MAD* but because of the aforementioned reasons, *MAPE* is preferred.

b) Deterministic Forecasting of Earthquake Occurrences

The section (a) of method 3 can also be used for deterministic forecasting of the earthquake occurrences by probability forecasting matrixes, such as $F\hat{R}M$. Considering algorithm (II) and using the following steps, it is possible to forecast the earthquake occurrences in the next several time periods deterministically (i.e. one and zero; so that one means that an earthquake has occurred and zero means that any earthquake has occurred).

Algorithm III:

Step 0: Begin

Step 1: Use the past n data to determine $F\hat{R}M$ (n is the number of total data)

Step 2: Determine $F\hat{R}M$

Step 3: $F\hat{R}M1(i)=F\hat{R}M(i)\ \forall i=1,\ldots,k$ (is the number of predictable time periods in future)

Step 4: $M_j=$ $\{t$ th order maximum in $F\hat{R}M1(j),\ \forall j=1,\ldots,k\}$

Step 5: $F\hat{R}M2(j)=\left[\dfrac{F\hat{R}M1(j)}{M_j}\right],\forall j=1,\ldots,k$

([a] obtains the greatest integer number smaller than the real number a)

Step 6: Replace any element(s) greater than or equal to 1 in $F\hat{R}M2(j),\ \forall j=1,\ldots,k$ by 1 and name the resulted matrices $FRMD(j),\ \forall j=1,\ldots,k$

Step 7: If $FRMD_{rm}(j)=1$ then an earthquake in region r with the magnitude m in j th time period will occur otherwise any earthquake in region r with the magnitude m in j th time period will not occur.

Step 8: End

4. Application

In this section, the proposed methods of this paper are studied using the actual data gathered in Iran. Next, the earthquake occurrences are forecasted deterministically and the forecasting error for the mentioned zoning method is determined. A zoning method of Iran area is considered consisting of Zoning by Karakaisis. Iran is selected as the area of investigation. This is bounded by longitudes $44.23°E$, $63.33°E$ and latitudes $25.05°N$, $39.78°N$. The data have been collected from United States Geology Sciences Center website[1]. After filtering and removing the unsuitable data according to Table 1 [18], 3179 data related to earthquakes occurred during 1973-2007 are used. The maximum time interval between the times of earthquake occurrences is 45 day, so by considering each of 10 days as one time unit, forecasting the next $\left[\frac{45}{10}\right]=5$ time units in each forecasting will be possible. Also each zone in each zoning method is considered as a state of a Semi-Markov model and the magnitude of total occurrences

[1] *Data taken from United States Geology Sciences Center website at* http://neic.usgs.gov/neis/epic/epic.html

is divided into 5 partitions, and each partition of magnitudes is considered as a state of a Semi-Markov model.

Tab. 1. A reference for distinguishing foreshocks and aftershocks from main-shocks

Magnitude (mb)	Space Distance (Km)	Time Interval (Days)
2.5	19.5	6
3	22.5	11.5
3.5	26	22
4	30	42
4.5	35	83
5	40	155
5.5	47	290
6	54	510
6.5	61	790
7	70	915
7.5	81	960
8	94	985

5. Zoning by Karaka Isis:

Karaka isis, a researcher in geosciences, in his work divided Iran area into 21 zones [19]. In this section his zoning method is applied. Karaka isis has not considered the center of Iran as a zone; hence we use the zoning to divide Iran into 22 zones, i.e. 21 zones by Karaka isis plus one central part, similar to Fig. 2.

Also, the magnitudes of past occurrences are divided into five classes as follows:

$$\begin{array}{ll} M_1: & mb \le 3.6 \\ M_2: & 3.6 < mb < 4.8 \\ M_3: & 4.8 < mb \le 5.4 \\ M_4: & 5.4 < mb \le 6.3 \\ M_5: & 6.3 < mb \end{array} \qquad (6)$$

These partitions have been obtained by Agglomerative Nesting (AGNES) method that is a technique for clustering data [2], [20]. The minimum of the considered magnitudes is 3.1 *mb* and their maximum is 7.1 *mb*. In the proposed model for each partition is considered as a state of a Semi-Markov model.
With respect to the Karakaisis zoning in Fig. 2 and Eq. (1) and (2) and (6) the transition probability matrix for both region to region and magnitude to magnitude transitions are obtained as Tables 1 and 2. By applying Tables 2 and 3 Interval transition probability matrices in both region to region and magnitude to magnitude transitions have been determined and by using these transition matrices and Eq. (1), probabilistic forecasting matrix for the next 5 time periods (i.e. next 50 days) after normalizing are determined.(see Table 4). The number of the total data is 3179. We have used 3000 data for the forecasting the next 179 earthquake occurrences and used 179 data for determining the forecasting error.

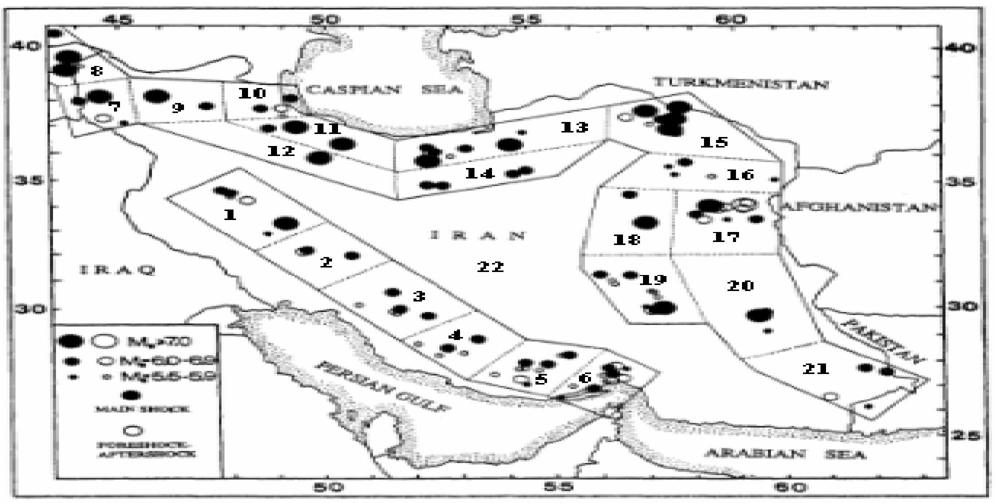

Fig. 2. The siesmogenic source areas of Iran proposed by Karaka isis

Tab. 2. Transition probability matrix of magnitude to magnitude transitions in zoning by Karaka isis

$G_M =$

	M_1	M_2	M_3	M_4	M_5
M_1	0.0482	0.9157	0.0241	0.0120	0.0000
M_2	0.0275	0.8368	0.1108	0.0222	0.0026
M_3	0.0112	0.8123	0.1317	0.0448	0.0000
M_4	0.0260	0.7532	0.1948	0.0130	0.0130
M_5	0.0000	1.0000	0.0000	0.0000	0.0000

In zoning by Karaka isis, the values of error of different algorithms have been calculated as follows:

MSE = 0.022
MAD = 0.05
MAPE = 5.00 %

Note that *MAPE* is suitable when $FRM_{ij}(1)$ is not equal to zero, while in the matrices that we have obtained there are also some elements which are equal to zero. In this case the total interval of errors replacing the elements can be used.

The total interval of errors is equal to 1; hence the denominator of Eq. (5) is equal to 1, therefore the equation is similar to the *MAD* equation. In method 3, all of data are divided into three clusters. The data ranging from 1 to 3000 are used for forecasting the next 104 data as benchmarking, according to algorithm (II) in validation section. The data ranging from 3001 to 3104, equal to 28 time periods (each time unit is equal to 10 days), then the data are used for deterministic forecasting of the data ranging from 3105 to 3179, which are the data later used for determining the forecasting error, of course in the case of the forecast to be deterministic.

In this way, according to algorithm (II) in the validation section *t* is equal to *5*. This value means that if the element(s) greater than the 5 th order maximum in forecasting matrixes are replaced by 1 and the other elements are replaced by 0, then the deterministic forecasting is the nearest forecasting to the real occurrences and its error is the least. However by considering $t = 5$ in this zoning method its *MAPE* gets equal to 2.398%.

6. Discussion

With respect to investigated zoning method, the percentages of earthquake occurrences in each zone are shown as Fig. 5. Also the percentages of earthquake occurrences in each class of magnitudes are as Fig. 6.

The transition probability matrices in magnitude to magnitude transitions (Table 2) show that the maximum and the minimum probabilities in transitions are related to the transitions from M_5 to M_2 (1.00) and from M_2 to M_5 (0.0026) (except to the non zero elements), respectively, which by considering Fig. 6 these results are clear

Tab. 3. Transition probability matrix of region to region transitions in zoning by Karaka isis

		R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8	R_9	R_{10}	R_{11}
	R_1	0.000	0.028	0.000	0.083	0.000	0.000	0.000	0.028	0.000	0.000	0.000
	R_2	0.000	0.000	0.045	0.091	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	R_3	0.069	0.000	0.034	0.034	0.000	0.000	0.034	0.000	0.000	0.000	0.138
	R_4	0.012	0.006	0.018	0.079	0.067	0.012	0.024	0.006	0.018	0.000	0.012
	R_5	0.000	0.000	0.000	0.126	0.264	0.000	0.011	0.000	0.000	0.011	0.011
	R_6	0.000	0.000	0.000	0.000	0.043	0.000	0.000	0.000	0.000	0.000	0.043
	R_7	0.009	0.009	0.000	0.018	0.053	0.000	0.035	0.000	0.035	0.018	0.026
	R_8	0.000	0.000	0.056	0.056	0.000	0.000	0.000	0.000	0.056	0.000	0.000
	R_9	0.015	0.000	0.000	0.090	0.000	0.015	0.030	0.015	0.045	0.000	0.030
	R_{10}	0.000	0.000	0.000	0.065	0.032	0.000	0.065	0.000	0.000	0.032	0.000
$G_R =$	R_{11}	0.000	0.000	0.010	0.040	0.010	0.010	0.040	0.000	0.030	0.010	0.280
	R_{12}	0.000	0.018	0.018	0.018	0.018	0.000	0.054	0.018	0.000	0.036	0.071
	R_{13}	0.051	0.010	0.010	0.051	0.031	0.010	0.051	0.000	0.010	0.010	0.020
	R_{14}	0.000	0.000	0.000	0.073	0.000	0.000	0.024	0.000	0.000	0.000	0.024
	R_{15}	0.000	0.000	0.000	0.048	0.048	0.000	0.000	0.000	0.000	0.048	0.000
	R_{16}	0.007	0.021	0.010	0.031	0.031	0.010	0.028	0.010	0.031	0.007	0.028
	R_{17}	0.016	0.000	0.012	0.053	0.016	0.000	0.041	0.016	0.016	0.016	0.012
	R_{18}	0.015	0.007	0.013	0.046	0.018	0.011	0.037	0.004	0.020	0.000	0.024
	R_{19}	0.013	0.003	0.006	0.054	0.006	0.006	0.035	0.003	0.028	0.013	0.016
	R_{20}	0.009	0.003	0.003	0.035	0.009	0.015	0.047	0.009	0.026	0.017	0.032
	R_{21}	0.004	0.007	0.004	0.067	0.026	0.007	0.049	0.000	0.011	0.007	0.022
	R_{22}	0.011	0.011	0.008	0.054	0.017	0.003	0.034	0.003	0.025	0.011	0.023

$G_R =$

	R_{12}	R_{13}	R_{14}	R_{15}	R_{16}	R_{17}	R_{18}	R_{19}	R_{20}	R_{21}	R_{22}
R_1	0.000	0.111	0.000	0.000	0.083	0.111	0.222	0.083	0.083	0.056	0.111
R_2	0.000	0.000	0.000	0.000	0.045	0.091	0.091	0.091	0.227	0.136	0.182
R_3	0.000	0.103	0.034	0.000	0.034	0.069	0.103	0.069	0.103	0.034	0.138
R_4	0.018	0.024	0.012	0.012	0.091	0.061	0.140	0.110	0.073	0.061	0.140
R_5	0.000	0.023	0.023	0.000	0.080	0.080	0.069	0.092	0.057	0.080	0.069
R_6	0.000	0.000	0.000	0.000	0.043	0.043	0.174	0.000	0.217	0.217	0.217
R_7	0.035	0.061	0.000	0.009	0.044	0.070	0.167	0.088	0.114	0.088	0.123
R_8	0.000	0.000	0.000	0.000	0.167	0.056	0.333	0.167	0.111	0.000	0.000
R_9	0.015	0.060	0.000	0.000	0.075	0.045	0.134	0.134	0.119	0.104	0.075
R_{10}	0.032	0.000	0.032	0.000	0.065	0.097	0.065	0.161	0.194	0.000	0.161
R_{11}	0.020	0.010	0.010	0.010	0.110	0.030	0.080	0.050	0.080	0.050	0.120
R_{12}	0.179	0.036	0.000	0.000	0.018	0.125	0.125	0.000	0.089	0.018	0.161
R_{13}	0.031	0.163	0.031	0.010	0.102	0.061	0.082	0.020	0.092	0.071	0.082
R_{14}	0.000	0.000	0.098	0.000	0.122	0.049	0.171	0.049	0.073	0.146	0.171
R_{15}	0.000	0.095	0.000	0.000	0.190	0.048	0.048	0.190	0.095	0.095	0.095
R_{16}	0.010	0.017	0.014	0.007	0.140	0.101	0.143	0.119	0.084	0.056	0.091
R_{17}	0.024	0.020	0.012	0.008	0.093	0.093	0.167	0.077	0.114	0.085	0.106
R_{18}	0.015	0.020	0.018	0.007	0.094	0.074	0.182	0.098	0.098	0.092	0.107
R_{19}	0.009	0.044	0.013	0.006	0.070	0.089	0.130	0.120	0.073	0.108	0.155
R_{20}	0.015	0.017	0.009	0.003	0.076	0.084	0.160	0.102	0.172	0.087	0.073
R_{21}	0.015	0.015	0.011	0.004	0.086	0.086	0.109	0.101	0.120	0.116	0.131
R_{22}	0.011	0.028	0.006	0.014	0.099	0.056	0.152	0.127	0.124	0.076	0.107

Tab. 4. Probabilistic forecasting matrixes during time periods 1 to 5 in zoning by Karakaisis

	M_1	M_2	M_3	M_4	M_5
R_1	0.0007	0.0536	0.0190	0.0034	0.0000
R_2	0.0007	0.0536	0.0190	0.0034	0.0000
R_3	0.0007	0.0400	0.0142	0.0027	0.0000
R_4	0.0027	0.2266	0.0807	0.0142	0.0000
R_5	0.0007	0.0801	0.0285	0.0054	0.0000
R_6	0.0000	0.0136	0.0047	0.0007	0.0000
R_7	0.0014	0.1201	0.0427	0.0075	0.0000
R_8	0.0000	0.0136	0.0047	0.0007	0.0000
R_9	0.0014	0.1065	0.0380	0.0068	0.0000
R_{10}	0.0007	0.0536	0.0190	0.0034	0.0000
R_{11}	0.0014	0.1065	0.0380	0.0068	0.0000
R_{12}	0.0007	0.0400	0.0142	0.0027	0.0000
R_{13}	0.0014	0.1201	0.0427	0.0075	0.0000
R_{14}	0.0000	0.0265	0.0095	0.0020	0.0000
R_{15}	0.0007	0.0665	0.0237	0.0041	0.0000
R_{16}	0.0047	0.4132	0.1479	0.0265	0.0000
R_{17}	0.0027	0.2266	0.0807	0.0142	0.0000
R_{18}	0.0075	0.6398	0.2286	0.0414	0.0000
R_{19}	0.0061	0.5197	0.1859	0.0332	0.0000
R_{20}	0.0061	0.4796	0.1716	0.0305	0.0000
R_{21}	0.0041	0.3331	0.1194	0.0217	0.0000
R_{22}	0.0122	1.0000	0.3575	0.0645	0.0000

$F\hat{R}M(1)$

	M_1	M_2	M_3	M_4	M_5
R_1	0.0018	0.0668	0.0136	0.0018	0.0000
R_2	0.0018	0.0533	0.0108	0.0018	0.0000
R_3	0.0018	0.0515	0.0108	0.0018	0.0000
R_4	0.0108	0.3631	0.0732	0.0117	0.0009
R_5	0.0045	0.1536	0.0307	0.0045	0.0000
R_6	0.0009	0.0434	0.0090	0.0018	0.0000
R_7	0.0081	0.2773	0.0560	0.0090	0.0009
R_8	0.0009	0.0361	0.0072	0.0009	0.0000
R_9	0.0045	0.1617	0.0325	0.0054	0.0000
R_{10}	0.0018	0.0650	0.0126	0.0018	0.0000
R_{11}	0.0054	0.1798	0.0361	0.0054	0.0000
R_{12}	0.0036	0.1174	0.0235	0.0036	0.0000
R_{13}	0.0072	0.2240	0.0452	0.0072	0.0009
R_{14}	0.0018	0.0696	0.0145	0.0018	0.0000
R_{15}	0.0018	0.0524	0.0108	0.0018	0.0000
R_{16}	0.0199	0.6631	0.1328	0.0208	0.0018
R_{17}	0.0154	0.5068	0.1021	0.0163	0.0009
R_{18}	0.0307	1.0000	0.2014	0.0316	0.0027
R_{19}	0.0226	0.7525	0.1518	0.0235	0.0018
R_{20}	0.0253	0.8347	0.1680	0.0262	0.0018
R_{21}	0.0181	0.5827	0.1174	0.0181	0.0018
R_{22}	0.0271	0.8862	0.1780	0.0280	0.0018

$F\hat{R}M(2)$

	M_1	M_2	M_3	M_4	M_5
R_1	0.0025	0.0712	0.0109	0.0017	0.0000
R_2	0.0017	0.0469	0.0067	0.0017	0.0000
R_3	0.0017	0.0528	0.0084	0.0017	0.0000
R_4	0.0109	0.3554	0.0536	0.0101	0.0008
R_5	0.0050	0.1660	0.0251	0.0050	0.0008
R_6	0.0017	0.0453	0.0067	0.0017	0.0000
R_7	0.0075	0.2490	0.0377	0.0075	0.0008
R_8	0.0008	0.0402	0.0059	0.0008	0.0000
R_9	0.0042	0.1475	0.0226	0.0042	0.0000
R_{10}	0.0017	0.0645	0.0101	0.0017	0.0000
R_{11}	0.0059	0.1953	0.0293	0.0059	0.0008
R_{12}	0.0034	0.1157	0.0176	0.0034	0.0000
R_{13}	0.0067	0.2205	0.0335	0.0067	0.0008
R_{14}	0.0025	0.0788	0.0117	0.0025	0.0000
R_{15}	0.0017	0.0436	0.0067	0.0017	0.0000
R_{16}	0.0193	0.6320	0.0964	0.0184	0.0017
R_{17}	0.0168	0.5608	0.0855	0.0168	0.0017
R_{18}	0.0302	1.0000	0.1517	0.0293	0.0025
R_{19}	0.0201	0.6731	0.1023	0.0193	0.0017
R_{20}	0.0226	0.7653	0.1165	0.0226	0.0017
R_{21}	0.0176	0.5809	0.0880	0.0168	0.0017
R_{22}	0.0243	0.8013	0.1215	0.0235	0.0017

$F\hat{R}M(3)$

	M_1	M_2	M_3	M_4	M_5
R_1	0.0025	0.0696	0.0098	0.0016	0.0000
R_2	0.0016	0.0475	0.0066	0.0016	0.0000
R_3	0.0016	0.0581	0.0082	0.0016	0.0000
R_4	0.0106	0.3571	0.0524	0.0106	0.0008
R_5	0.0049	0.1679	0.0246	0.0049	0.0008
R_6	0.0016	0.0442	0.0066	0.0016	0.0000
R_7	0.0074	0.2457	0.0360	0.0074	0.0008
R_8	0.0008	0.0401	0.0057	0.0008	0.0000
R_9	0.0041	0.1433	0.0205	0.0041	0.0000
R_{10}	0.0016	0.0631	0.0090	0.0016	0.0000
R_{11}	0.0057	0.1982	0.0287	0.0057	0.0008
R_{12}	0.0033	0.1155	0.0172	0.0033	0.0000
R_{13}	0.0066	0.2162	0.0311	0.0066	0.0008
R_{14}	0.0025	0.0803	0.0115	0.0025	0.0000
R_{15}	0.0016	0.0459	0.0066	0.0016	0.0000
R_{16}	0.0188	0.6200	0.0901	0.0180	0.0016
R_{17}	0.0164	0.5545	0.0811	0.0156	0.0016
R_{18}	0.0303	1.0000	0.1458	0.0287	0.0025
R_{19}	0.0197	0.6437	0.0942	0.0188	0.0016
R_{20}	0.0221	0.7355	0.1073	0.0213	0.0016
R_{21}	0.0172	0.5758	0.0835	0.0164	0.0016
R_{22}	0.0229	0.7633	0.1114	0.0221	0.0016

$F\hat{R}M(4)$

	M_1	M_2	M_3	M_4	M_5
R_1	0.0025	0.0712	0.0099	0.0025	0.0000
R_2	0.0017	0.0480	0.0066	0.0017	0.0000
R_3	0.0017	0.0613	0.0083	0.0017	0.0000
R_4	0.0108	0.3634	0.0522	0.0108	0.0008
R_5	0.0050	0.1722	0.0248	0.0050	0.0008
R_6	0.0017	0.0464	0.0066	0.0017	0.0000
R_7	0.0075	0.2500	0.0356	0.0075	0.0008
R_8	0.0008	0.0406	0.0058	0.0008	0.0000
R_9	0.0041	0.1440	0.0207	0.0041	0.0000
R_{10}	0.0017	0.0637	0.0091	0.0017	0.0000
R_{11}	0.0058	0.2028	0.0290	0.0058	0.0008
R_{12}	0.0033	0.1175	0.0166	0.0033	0.0000
R_{13}	0.0066	0.2185	0.0315	0.0066	0.0008
R_{14}	0.0025	0.0836	0.0116	0.0025	0.0000
R_{15}	0.0017	0.0464	0.0066	0.0017	0.0000
R_{16}	0.0190	0.6283	0.0894	0.0182	0.0017
R_{17}	0.0166	0.5629	0.0803	0.0166	0.0017
R_{18}	0.0298	1.0000	0.1424	0.0298	0.0025
R_{19}	0.0199	0.6523	0.0927	0.0190	0.0017
R_{20}	0.0224	0.7434	0.1060	0.0215	0.0017
R_{21}	0.0174	0.5828	0.0828	0.0174	0.0017
R_{22}	0.0232	0.7740	0.1101	0.0232	0.0025

$F\hat{R}M(5)$

With respect to zoning by Karakaisis in Table 3, it is clear that the maximum and the minimum probabilities in this table are related to $R_8 \rightarrow R_{18}$ (0.333) and $R_{22} \rightarrow R_6, R_{22} \rightarrow R_8$ (0.0028), respectively.

Forecasting probabilities for the future five time periods in zoning by Karaka isis are shown in Table 4. The mentioned tables show that in zoning by Karaka isis the maximum probability of earthquake occurrences is related to $R_{22}M_2$ in time period 1 and $R_{18}M_2$ in time periods 2, 3, 4 and 5. By considering the errors by method 3, it is obvious that the minimum error is related to $t=5$ in zoning by Karaka isis and. In other words, for obtaining the minimum error, it is sufficient to replace all of the first *t* maximum elements by 1 and the other elements by 0 in all probabilistic forecasting matrices. In this manner the forecasting error is the least and the deterministic forecasting is available.

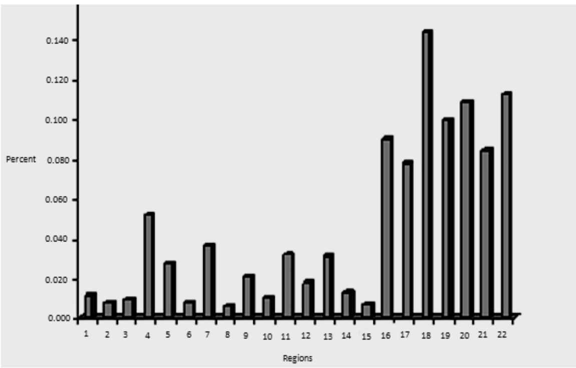

Fig. 3. The percentages of occurrences in each region of zoning by Karaka isis

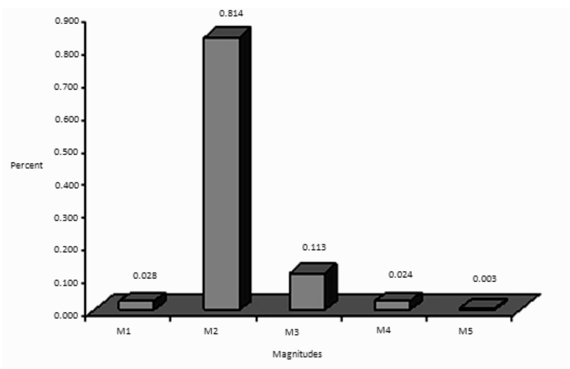

Fig. 4. The percentages of occurrences in each class of magnitudes

7. Conclusion

In this paper in addition to explaining a forecasting method by Semi-Markov models, the zoning method by Karaka isis and several methods for calculating the forecasting errors were introduced.

The first, Semi-Markov models for the forecasting earthquake occurrences in their three dimensions in Iran were used. Iran was divided into 22 zones and each zone was considered as a state of the proposed Semi-Markov model.

Next, a zoning method by Karaka isis was introduced and then forecasting errors of the proposed method were calculated by several algorithms. The obtained errors show that the *MAPE* in Karaka isis zoning is equal to 5%.

With respect to method 3 in calculating the forecasting errors deterministically, this method can forecast the next earthquake occurrences deterministically. The last earthquake considered in this paper occurred in March 26th, 2007 with magnitude 4.9 mb, M_3, and in region R_{22} in Karakas isis zoning. In this manner, deterministic forecasting during the future five time periods, equal to the future 50 days, for Karaka isis zoning are determined as shown in Table 5:

Tab. 5. Deterministic forecasting matrix during 1 to 5 time periods in zoning by Karaka isis

Periods	Karaka isis zoning method
1 to 5	$R_{16}M_2$, $R_{18}M_2$, $R_{19}M_2$, $R_{20}M_2$, $R_{22}M_2$

After 50 days, it has been specified that the occurred earthquakes in zoning method by Karaka isis are as follow:

Tab. 6. Real occurrences during next 5 time periods after last earthquake occurrences

Periods	Karaka isis zoning method
1	$R_{16}M_2$, $R_{20}M_2$, $R_{21}M_2$
2	R_2M_2, $R_{13}M_2$, $R_{14}M_2$
3	R_7M_2, $R_{17}M_2$, $R_{18}M_2$, $R_{19}M_2$, $R_{22}M_2$
4	$R_{12}M_2$, $R_{18}M_1$, $R_{18}M_2$, $R_{19}M_2$, $R_{20}M_3$
5	$R_{20}M_2$, $R_{21}M_2$, $R_{22}M_3$

These obtained results show that in zoning by Karaka isis 42% of earthquakes have truly been forecasted. The places of 16% of earthquakes have correctly been forecasted but not their magnitudes. 16% of the forecasted earthquakes occurred in their neighborhood but their magnitudes were right and also 26% of earthquakes were never forecasted.

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