Retailer’s Replenishment and Pricing Decisions for Non-Instantaneous Deterioration and Price-Dependent Demand

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KEYWORDS
Inventory, Non-instantaneous Deterioration, Price-dependent demand, Variable holding cost, Dynamic pricing.

ABSTRACT
In this research, an integrated inventory model for non-instantaneous deteriorating items is analyzed when demand is sensitive to changes in price. The price used in this research is a time-dependent function of the initial selling price and discount rate. To control the deterioration rate of items at the storage facility, investment in preservation technology is incorporated. To provide a general framework to the model, an arbitrary holding cost rate is used. Toward the end of the paper, a numerical case is given to approve the model and the impacts of the key parameters of the model are studied by sensitivity analysis to deduce managerial insights.

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1. Introduction
The deterioration of items is a very common phenomenon in daily life. Items in inventory are subject to several risks such as breakage, evaporation, and pilferage. Items, such as food grains, blood, radioactive chemical, and medicines, deteriorate very rapidly over time. The first attempt to incorporate deterioration was made by Ghare and Schrader (1963). They used exponentially decaying inventory system for their study. Covert and Philip (1973) extended the work of Ghare and Schrader (1963) by using Weibull distribution and Gamma distribution. Shah and Shah (2000), Goyal and Giri (2001), and Bakker et al. (2012) gave a fine literature review of inventory models consisting deteriorating items.

These researchers assume that the items in inventory start decaying instantly from the time of their arrival. However, in reality, there are some items that maintain their original condition for some time and start to perish afterwards. This phenomenon is called non-instantaneous deterioration. Wu et al. (2006) developed an optimal replenishment policy for a non-instantaneous deteriorating item with stock-dependent demand and partial backlogging. Numerous researchers, e.g., Ouyang et al. (2006, 2008), Wu et al. (2009), Jaggi and Verma (2010), Soni and Patel (2012), Shah et al. (2013), and Tsao (2016), have utilized non-instantaneous deterioration for their study.

Preservation technology investment plays a vital role in reducing deterioration rate. Therefore, it has received good attention from researchers in recent years. Hsu et al. (2010) formulated an inventory model with a constant demand rate. They incorporated preservation technology investment to reduce the deterioration rate of
items. On this basis, Lee and Dye (2012) developed an inventory model with stock-dependent selling rate and preservation technology investment. In their model, they allowed shortages and partial backlogging. Considering an inventory system for a non-instantaneous deteriorating item, Dye (2013) studied the effect of preservation technology investment on inventory decisions when inventory system contains the non-instantaneous deteriorating item. In his study, he demonstrated that a higher optimal service rate can be obtained by higher preservation technology investment. Dye and Hsieh (2013) connected particle swarm optimization to figure the optimal production and preservation technology investment for minimizing the aggregate cost. Zhang et al. (2014) developed an algorithm to address the problem of pricing, preservation technology investment, and inventory control for deteriorating items. Liu et al. (2015) studied the joint dynamic pricing and preservation technology investment strategy. Many researchers, like Dye and Hsieh (2012), Singh and Gupta (2014), and Yang et al. (2015), also used preservation technology investment for deteriorating items in their model.

Setting the optimal sale price and procurement quantity for deteriorating items was studied by Sana (2010), Ghosh et al. (2011), their cited references, and many more. Sarkar and Sarkar (2013) formulated an economic manufacturing quantity model with probabilistic deterioration during a production system. These citations considered demand rate constant and deterministic. However, in the market, it is observed to be dependent on selling price, time, stock-displayed, advertisement, etc. Burwell et al. (1997), Wee (1997), Mondal et al. (2003), You (2005) studied the effects of various parameters on the demand.

Price is one of the important variables that can be manipulated by enterprises in order to generate more demand. In most inventory problems, the price of the product is kept fixed throughout the product’s market cycle. But, in practice, using appropriate pricing policies, enterprises can effectively control their losses caused by deterioration. Hence, dynamic pricing is an important strategy in today’s global market. Kincaid and Darling (1963) were the first to use dynamic pricing in their study. They used dynamic programming in development of their model. Cai et al. (2013) proposed an inventory model with dynamic pricing which modeled price as a function of time. The optimal policy was obtained by considering the feedback of price on demand per time unit. Wang et al. (2015), Dye and Yang (2016), Tsao and Sheen (2008) also considered dynamic pricing in their study.

In traditional inventory models, holding cost is known and constant. However, in practice, holding cost may not always be constant. Researchers, like Naddor (1966), Weiss (1982), Goh (1994), Giri et al. (1996), Roy (2008), Pando et al. (2012), Shah et al. (2013), and Rabbani et al. (2015), used various functions describing holding cost to provide generalized framework to their model.

The paper is organized as follows: In section 2, basic notation and assumptions of the problem are given. Section 3 demonstrates the proposed mathematical model. In section 4, a numerical example is illustrated to support the proposed model. The sensitivity analysis is carried out in section 5 followed by conclusion in section 6.

2 Notation and Assumptions

2-1 Notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Ordering cost per order (in $)</td>
</tr>
<tr>
<td>C</td>
<td>Purchase cost per unit (in $)</td>
</tr>
<tr>
<td>h(t)</td>
<td>Holding cost per item per unit time ($/unit)</td>
</tr>
<tr>
<td>θ</td>
<td>Constant rate of deterioration (0 &lt; θ &lt; 1)</td>
</tr>
<tr>
<td>p₀</td>
<td>Initial selling price per unit (in $) (decision variable)</td>
</tr>
<tr>
<td>p(t)</td>
<td>The dynamic price of product per unit at time t</td>
</tr>
<tr>
<td>p'(t)</td>
<td>Changes in price per unit time</td>
</tr>
<tr>
<td>R(p(t))</td>
<td>Demand rate (t ≥ 0) units</td>
</tr>
<tr>
<td>T</td>
<td>Length of production inventory cycle (Taken as one year)</td>
</tr>
<tr>
<td>t_d</td>
<td>Point of time at which deterioration of items starts (t_d = βT, β &gt; 0) (years)</td>
</tr>
</tbody>
</table>
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\[ I_1(t) \] The Inventory level at time \( t \), \( 0 \leq t \leq t_d \) (units)

\[ I_2(t) \] The Inventory level at time \( t \), \( t_d \leq t \leq T \) (units)

\( Q \) The maximum inventory level

\( c_d \) Cost due to deterioration per unit

\( u \) Cost of preservation technology per unit time (decision variable)

\( \pi_1(p(t), u) \) Seller’s total profit when \( t_d \leq T \)

\( \pi_2(p(t)) \) Seller’s total profit when \( t_d \geq T \)

2-2. Assumptions

1. The inventory system has a single item.
2. The deterioration rate is considered as constant and there is no repair or replacement of deteriorated items during cycle time.
3. The replenishment is instantaneous and time horizon is infinite.
4. Lead time is negligible or zero and shortages are not allowed.
5. Dynamic selling price is considered as (as in Herbon et al. (2014))

\[
p(t) = \begin{cases} 
  p_0 & 0 \leq t \leq t_d \\
  p_0 \cdot e^{-r(t-t_d)} & t_d \leq t \leq T 
\end{cases}
\]

Therefore, the change in the selling price is given as follows:

\[
p'(t) = \begin{cases} 
  0 & 0 \leq t \leq t_d \\
  -p_0 \cdot r \cdot e^{-r(t-t_d)} & t_d \leq t \leq T 
\end{cases}
\]

6. Demand rate \( R(p(t)) \) is considered as

\[
R(p(t)) = a - bp(t) + cp'(t)
\]

where \( a > 0 \) is the scale demand, \( b \) and \( c \) are the parameters sensitive to price and change in price, respectively.

7. The holding cost considered in this study is the same as the one considered in Shah et al. (2013). The holding cost is considered as follows.

\[
h(t) = \begin{cases} 
  ch & 0 \leq t \leq T \\
  c_h + \delta(t-t_d) & t_d \leq t \leq T
\end{cases}
\]

where \( c_h, \delta > 0 \)

\[
I_2(t) = \frac{a}{\mu \theta} \left( e^{\mu \theta(T-t)} - 1 \right) - \frac{p_0(b-cr)}{(\mu \theta - r)} \left( e^{-r(t-T_d)} - e^{-r(t-t_d)} \right)
\]

8. \( \mu = 1 - f(u) \), where function \( f(u) = 1 - \frac{1}{1 + \lambda u} \) is continuous, increasing and concave function of \( u \) for \( \lambda > 0 \). Set \( f(0) = 0 \).

3. Mathematical Model

Based on the assumptions made in section 2, the inventory system evolves as follows. \( Q \) units of an item arrive in the inventory at the start of every cycle. Amid time interval \([0, t_d]\), the stock is exhausted just because of price-dependent demand rate. The inventory level reduces to zero due to demand and deterioration both during time interval \([t_d, T]\). The whole process is then repeated again. We analyze one cycle of length one year. Therefore, the differential equations that describe the inventory level in both cases are given by:

\[
\frac{dI_1(t)}{dt} = -R(p(t)), \quad 0 \leq t \leq t_d \tag{1}
\]

\[
\frac{dI_2(t)}{dt} = -R(p(t)) - \theta I_2(t)u, \quad t_d \leq t \leq T \tag{2}
\]

with boundary conditions \( I_1(0) = Q \) and \( I_2(T) = 0 \). Solving equations (1) and (2) yields:

\[
I_1(t) = (p_0b-a)T + Q \tag{3}
\]

From the continuity of \( I(t) \) at \( t = t_d \), we have \( I_1(t_d) = I_2(t_d) \). It follows equations (3) and (4), such that

\[
Q = I(0) = (a - bp_0)t_d + \frac{a}{\mu \theta} \left( e^{\mu \theta(T-t)} - 1 \right) - \frac{p_0(b-cr)}{(\mu \theta - r)} \left( e^{(\mu \theta - r)(T-t)} - 1 \right) \tag{5}
\]

By substituting equation (5) into equation (3), equation (3) yields:
\begin{align*}
I_1(t) &= (p_0b-a)(t-t_d) + \frac{a}{\mu \theta} \left(e^{\mu \theta (T-t_d)} - 1\right) - \frac{p_0(b-cr)}{\mu \theta - r} \left(e^{\frac{\mu \theta - r}{T-t_d}} - 1\right) \\
\end{align*}

Based on the values of \( T \) and \( t_d \), there may arise two cases viz. \( t_d \leq T \) and \( t_d \geq T \). The total profit of the inventory system is calculated in both cases.

**Case-1: \( t_d \leq T \)**

In this case, the inventory is consumed only due to demand rate over a time interval \([0, t_d]\) and the stock level is decreasing to zero inferable from demand and deterioration during time interval \([t_d, T]\). Therefore, the components of profit function of the inventory system are as follows:

\[
SR = \text{Net sales revenue} = \frac{1}{T} \int_0^T p(t) R(p(t)) dt \\
PC = \text{Purchase cost} = \frac{CQ}{T} \\
OC = \text{Ordering cost} = \frac{A}{T} \\
HC = \text{Holding cost} = \frac{1}{T} \left[ \int_0^{t_d} h(t) I_1(t) dt + \int_{t_d}^T h(t) I_2(t) dt \right] \\
DC = \text{Deterioration cost} = \frac{\theta \cdot c_d}{T} \int_{t_d}^T I_2(t) dt \\
PTC = \text{Preservation technology cost} = \frac{u \cdot T}{T}
\]

Therefore, the total profit per unit time \( \pi_1(p(t), u) \) is given by

\[
\pi_1(p(t), u) = \frac{1}{T} \left( SR - OC - PC - HC - DC - PTC \right) \tag{13}
\]

**Case-2: \( t_d \geq T \)**

For this situation, the model turns into the conventional inventory model. There is no deterioration of items in time interval \([0, T]\). The components of total profit, in this case, are as follows:

\[
SR = \text{Net sales revenue} = \frac{1}{T} \int_0^T p(t) R(p(t)) dt \tag{14}
\]

\[
PC = \text{Purchase cost} = \frac{CQ}{T} \tag{15}
\]

\[
OC = \text{Ordering cost} = \frac{A}{T} \tag{16}
\]

\[
HC = \text{Holding cost} = \frac{1}{T} \left[ \int_0^T h(t) I_1(t) dt \right] \tag{17}
\]

Therefore, the total profit per unit time \( \pi_2(p(t)) \) is given by

\[
\pi_2(p(t)) = \frac{1}{T} \left( SR - OC - PC - HC \right) \tag{18}
\]

Using the classical optimization technique, we calculate maximum profit in the respective case using the following flow chart.
4. Numerical Example

In order to illustrate the above solution procedure, we consider the following numerical examples. We have calculated maximum profit in each case using Maple 18 software for the inventory parameters.

**Example 1:** We consider an inventory system with the following parameters in appropriate units:
- \( a = 500 \), \( b = 41 \), \( c = 90 \), \( \theta = 10\% \), \( \lambda = 0.9 \), \( C = \$8 \), \( A = \$50 \),
- \( c_b = \$0.1 \), \( \delta = 50\% \), \( r = 80\% \), \( c_d = \$1 \),
- \( T = 1 \) year, and \( \beta = 0.6 \). In this example, we assume that deterioration starts before the inventory is depleted to zero.

In this case, the optimal solution is \( (p_0^*, u^*) = (18.83, 7.239) \). That means by setting initial selling price as \$18.83 and investing \$7.239 in preservation technology to reduce deterioration rate, the decision-maker attains maximum profit of \( \pi_1(p_0^*, u^*) = \$1398.70 \). The concavity of the profit function is shown in figure 2.

**Example 2:** Again, the data are same as in Example 1 except that \( \beta = 1.2 \). Also, \( \theta, \lambda \), and \( c_d \) are not used here as there is no deterioration during interval \([0,T]\).

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**Fig. 1:** Flow chart used for solution process

**Fig. 2.** Concavity of profit function in case 1
In this case, the profit function is concave with respect to $p_0$ and the optimal value of initial selling price is $p_0^* = 10.22$. The maximum profit obtained in this case is $\pi_2(p_0^*) = 149.53$. Figure 3 projects the concavity of the profit function in case-2.

![Figure 3. Concavity of profit function in case 2](image)

5. Sensitivity Analysis and Managerial Insights

In this section, we extend some managerial insights based on the sensitivity analysis of the inventory parameters. The sensitivity analysis is performed by changing each value of the parameter by -20%, -10%, 10%, and 20% while keeping remaining parameters unchanged. The computational results are shown in Figures 4 and 5.

From figure 4, we can observe that

- Selling price is highly sensitive to parameters $\beta$ and markup of selling price $b$. It decreases drastically when parameters $\beta$ and $b$ increase. Parameters $r$, $c$ and scale demand $a$ moderately increase selling price. Other parameters have minor effect on selling price.
- Preservation technology cost $u$ reduces heavily when parameter $\beta$ is increased. Preservation technology cost $u$ observes significant decrease when parameter $\lambda$ and markup of selling price $b$ increase. Preservation technology cost $u$ is very less sensitive to parameter deterioration cost $c_d$. All other parameters bring a significant increase in preservation technology cost.

Scale demand $a$ has a large impact on profit function. It is obvious that when holding cost $c_h$, purchase cost $C$ and ordering cost $A$ increase, then total profit decreases. Profit function is highly sensitive to parameter $\beta$ and markup of selling price $b$. Profit increases significantly when parameters $r$ and $c$ increase. Profit decreases when replenishment time $T$ is increased. Other parameters have minor effect on profit.
Fig. 4. Sensitivity analysis for case-1
From figure 5, we can observe that
- Selling price is highly sensitive to scale-demand $a$, markup of selling price $b$, and purchase cost $C$. It increases drastically when scale demand $a$, as well as purchase cost $C$ increase. A heavy decrease is observed in selling price when the markup of selling price $b$ is decreased. Other parameters have minor effect on selling price.
- Similar to selling price, total profit is also very sensitive to scale demand $a$, markup of selling price $b$, and purchase cost $C$. Profit shows a heavy rise when scale-demand $a$, increases which is an obvious observation. Profit reduces drastically when markup of selling price $b$ and purchase cost $C$ increases. Other parameters have minor effect on profit.

In the second case, if the scale demand is increased by 80%, profit is increased by nearly 15 times. Inventory depletes faster before deterioration starts. In addition, if markup of selling price $b$ decreases by 50%, profit is higher when a player depletes stock before deterioration starts. It suggests to the player that increase in
scale demand $a$, mark up of selling price $b$ and $c$ are beneficial if items are getting sold off before they start deteriorating.

6. Conclusion
In this research, we have developed an inventory model for a non-instantaneous deteriorating item. To enhance the dynamic feature of the problem, we have considered demand function dependent on dynamic price and changes in price. We have also incorporated preservation technology investment to reduce deterioration and maximize seller’s total profit. Apart from the above features, we have also considered time-dependent holding cost which generalizes the developed model. A solution procedure is developed for the proposed model to attain maximum profit per unit item. The study concludes with a couple of numerical examples and sensitivity analysis which helps provide some important managerial insights.

This research can be extended to permissible delay in payment, allowed shortages, variable deterioration, etc.

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DOI: 10.22068/ijiepr.28.2.101