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Developing Non-linear Dynamic Model to Estimate Value at Risk, Considering the Effects of Asymmetric News: Evidence from Tehran Stock Exchange

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KEYWORDS

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ABSTRACT

Empirical studies concluded the existence of stronger correlation among the major losses compared to the major profits in financial markets. This phenomenon makes symmetric distributions inefficient for modeling multivariate distributions and estimating portfolio's risk perfectly. Copula theory is an appropriate tool in order to model multivariate distributions which use marginal distribution and hires defined asset's correlation to describe complex correlation structure such as non-linear one. Therefore, this study calculated the risk of a portfolio including five-industry indexes in Tehran Stock Exchange Market with application of Value at Risk measure. In this regard, marginal distribution of each return series was estimated using GARCH and GJR models, and also correlation structure of assets was determined by implementation of DCC model. Subsequently, joint distribution of asset's portfolio is achieved, and finally VaR of equal weighted portfolio for each asset is calculated. The result of kupiec test illustrated that the proposed model calculated VaR efficiently, and also the amount of VaR calculated by t-Copula is less than Gaussian-Copula in 99 and 95 percent of the significant level.

1. Introduction

Modern portfolio investment theory was presented by Harry Markowitz in 1952. His model is also called mean-variance model due to the fact that it is based on expected returns (mean) and the standard deviation (variance) of the various portfolios. Subsequently, several tools were developed to measure portfolio's risk. Value at Risk (VaR) is a common measure used by financial analysts to determine the risk of individual assets or portfolios. Value at Risk (VaR) is a measure of the risk of investments. It estimates how much a set of investments might lose, given normal market conditions, in a set time period such as a day. Regarding the latest Basel committee issues, Value at Risk is the most general risk measure used in the investment banks and financial institutes widely. Investment banks commonly apply VaR modeling to firm-wide risk due to the potential of independent trading desks to expose the firm to highly correlated assets unintentionally. Employing a firm-wide VaR assessment allows for the determination of the cumulative risks from aggregated positions held by different trading desks and departments within the institution. Using the data provided by VaR modeling,

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financial institutions can determine whether they have sufficient capital reserves in place to cover losses or higher-than-acceptable risks require concentrated holdings to be reduced.

Unless most common methods estimate VaR under the hypothesis of joint distribution normality, it is proven that most of financial time series are skewed, fat tailed, and consequently non-normal. The use of linear correlation under the hypothesis of normality for the investment portfolio is also one of the other deficiencies to estimate VaR. On the other hand, calculating portfolio's risk measures is influenced by multivariate time series with non-linear correlation. Therefore, modeling accurate dependence structure is significantly important to estimate the risk of portfolio. Although Pearson correlation coefficient, Kendall's tau, and Spearman's rho are common indices which are used to determine the dependency of time series, they are not effective enough to determine the structure of dependency. Hence, the normal hypothesis is not well qualified to describe fat-tail distributions; instead, Copula theory is suggested to eliminate the whole deficiencies. In addition, Copula is able to explain and describe more complex multivariate dependency structures (such as non-linear dependence and tail series dependence) sufficiently. Empirical studies illustrate that asset's returns have a significant correlation during the more volatile or downswing periods of the market. Considering that the correlation of big losses is more striking than the major profits, using symmetric distributions is not enough qualified to model this asymmetric relations. Copula theory allows us to build a flexible multivariate distribution with marginal distributions and different dependence structures. Accordingly, joint distribution of investment portfolio puts non-normality and non-linear correlation of variables on notice. This study combines symmetric and asymmetric GARCH models, which are used for modeling the marginal distribution, and Copula functions to model joint distribution in order to develop an efficient alternative model to multivariate normal distribution or other traditional multivariate distributions. Also, this study performed DCC, dynamic conditional correlation, to consider time-varying assets correlation into developed Copula-GARCH model, which has not been used in previous studies.

2. Literature Review

Sklar (1959) proposed Copula theory to calculate nonlinear correlation between variables. After that, Copula functions were used in various fields including financial topics, such that the number of Copula applications articles on financial topics has increased in recent years and some of this research will be presented briefly as follows.

Hotta and Palaro (2006) calculated Value at Risk of S&P500 and Nasdaq indices by using different Copulas and GARCH models to achieve marginal distribution. The results indicated the superiority of symmetrized Joe-Clayton Copula (SJC) [1]. Wang et al (2010) used GARCH EVT-Copula model to determine an optimal investment portfolio of China's four currency USD, EUR, JPY, and HKD. The results illustrate that t-Copula and Clayton Copula depict a better correlation structure than Gaussian Copula [2]. Deng et al. (2011) attempted to calculate the risk of portfolio consisting of four China's market indices by using CVaR measures through the Monte Carlo simulation method and determine optimal weights. In this study, they used EVT method to model tail returns series and used the pair Copula function to obtain the correlation structure. The results show that the pair Copula models worked better in showing the correlation structure and the pair Copula-GARCH-EVT-CvaR to have a better performance than the t-Copula-GARCH-EVT-CVaR [3]. Chen and Tu (2013) tried to estimate the Value at Risk of the hedge portfolio to demonstrate the potential risk of model for the inappropriate use of correlation coefficients and normal joint distribution between the variables. The results show that estimating the Value at Risk, in case of using conditional Copula and their hybrid models to form a joint distribution for calculating optimal hedge ratio, can be improved [4]. Ghorbel and Trabelsi (2014) attempted to obtain optimal investment portfolio of oil index and its derivatives by using Copula-EVT-FIGARCH-VaR model. To evaluate the efficiency of model, they compared the results with those of the classical methods for calculating VaR such as MGARCH-VaR. The results indicate the superiority of this method over the other methods [5]. Balibey and Turkyilmaz (2014) used VaR measure to calculate the risk of Turkish Stock Exchange Market. They apply FIGARCH (1,d,1) and FIAGARCH (1,d,1) to examine asymmetric and long memory existence. Also, they hired Kupiec test to assess estimated risk accuracy by the proposed models. The results indicated asymmetric and long memory existence in Turkish stock exchange market. Also, the result showed that FIAGARCH (1, d, 1) with skewed t-student distribution has better accuracy to calculate VaR. [6]. Tang et al. (2015) tried to composite daily, monthly, seasonally and annually portfolios of future natural gas in the Title Transfer Facility -TTF- by the use of GARCH-EVT-Copula model. This study, after modeling each return series by ARMA-GARCH, hired the EVT model to estimate the tail of each series and marginal distributions. Gaussian and t-copulas are used to model the correlation structure of portfolio. Then, with the application of VaR, the risk of equal weighted portfolio has estimated. The results show that although the amount of achieved VaR using t-Copula is greater than Gaussian-Copula, but the optimal weights of both methods are almost the same [7]. Herwartz and Raters (2015) used Copula-MGARCH by implementing BEKK (1,1) model to calculate the Value at Risk between the pair EUR/USD and USD/JPY. Results showed good flexibility of this model. Also, prediction accuracy of Value at Risk in this model is significant that leads to high performance of this model [8]. Messaoud and Aloui (2015) used VaR measure to form an optimal investment portfolio of stock market indices of Egypt, Malaysia, North Africa, and Turkey. In this study, implementing the effects of GJR-GARCH model of asymmetric shocks was considered, and it was achieved using the theory of extreme value fat tail distribution of return series. Then, by using Copula functions, joint distribution is calculated to achieve optimized weights for each series [9]. Khemawanita and Tansuchat (2016) investigated EVT-Copula-GARCH model, VaR and CVaR measures to find the optimal portfolio of the precious metals in their study. The results illustrated that ARMA-GARCH models with t-student are properly suitable to estimate marginal distribution. Also, Gold and Silver have the most share and Palladium and Platinum have the least share of optimal portfolio [10]. Razak and Ismail (2016) tried to calculate the risk of a combined portfolio consisting of Malaysia's stock market and S&P500 indexes with the application of VaR measure and Clayton Copula in order to specify the optimal weights. The marginal distribution of assets was estimated by ARMA-GARCH model, and the results concerning the VaR were compared to those of the traditional methods. Superiority of Copula-VaR Model has been proven based on the results significantly [11]. Ortiz et al. (2016), in their study, tried to evaluate the correlation of nine countries of Latin America. Then, calculation of the Value at Risk of portfolio was done which includes these country's stock market's index in different ways such as variance-covariance, historical simulation, and VaR-copula model. The results showed that the VaR-copula model is superior compared to the other models [12]. Berger (2016) compared the results of different Copula functions and the performance of these models in predicting Value at Risk. Monte Carlo simulation algorithm was used for estimating VaR. According to the results of prediction, t-copula model is more efficient than other models [13]. Karmakar (2017) implemented correlation structure and portfolio's risk of 5 currency pairs, comprising USD/INR, SF/INR, JPY/INR GBP/INR, and EURO/INR. In this study, AR-t-GARCH-EVT model was hired to achieve the correlation structure. Finally, by the use of risk measures VaR and CVaR, the risk of portfolio was calculated. The result indicated that, in the optimal state, the investment portfolio is absolutely tended to USD currency that illuminates the importance of this currency in the market [14]. In most previous studies in Iran concerning the field of investment management using Value at Risk, parametric methods were applied to estimate VaR; the main focus was related to modeling asset's volatility and forecasting conditional variance distribution; also, known asset's distributions are taken to estimate VaR. Also, lack of domestic studies about modeling asset's joint distribution is detected that will be more explained.

Mousavi et al. (2013), in their study, used conditional GARCH-Copula method to estimate VaR of a portfolio including 17 shares and compared its results with Variance-Covariance model and historical simulation. The comparison results show that the Gaussian Copula model with normal and t-student marginal distributions have better performance than the other methods in estimation [15]. Keshavarz and Heirani (2014) hired various kinds of Copula functions and generalized heteroscedasticity of variance model and assessed dependence structure between the two chemical and pharmaceutical price indexes of Tehran Stock Exchange Market. The empirical results indicate that there is the asymmetrical dependence structure between the chemicals and pharmaceutical price indexes; findings illustrate the further accuracy and adequacy of Copula-GARCH approach compared to the other common methods in predicting portfolio's VaR as MGARCH, DCC-GARCH, EWMA, and historical simulation method [16].

3. Research Methodology

3-1. Copula theory

Copula theory is the main method of multivariate distribution modeling defined by the marginal distributions and dependences between variables and joint distribution function [1]. In other words, Copula is a function, which can link two or more marginal distribution functions to each other in order to create a joint distribution. Copula approach is a method for describing the structures of dependency.

This method compensates for the deficiencies of the other methods, in determining the structure of dependence, which only rely on the correlation of assets [17]. Consider random numbers $x_1, \dots, x_d$ with marginal distributions $F_1, \dots, F_d$. Joint distribution function of these random numbers is proceed as follows:

$$F(x_1, \dots, x_d) = pr[X_1 \le x_1, \dots \dots, X_d \le x_d]$$

$$F(x_1, \dots, x_d) = pr[F_1(X_1) \le F_1(x_1), \dots \dots, F_d(X_d) \le F_d(x_d)] \quad (1)$$

Sklar concluded that Copula will be shown as follows:

$$F(x_1, \dots, x_d) = C\big(F_1(x_1), \dots, F_d(x_d)\big), \ x_1, \dots, x_d \in R^d \quad (2)$$

If marginal distributions $F_1, \dots, F_d$ is continuous, then C will be a unit (exclusive). Copula could be directly deduced by "Equation (3)":

$$C(u_1, \dots, u_d) = F(F_1^{-1}(u_1), \dots., F_d^{-1}(u_d)) \quad (3)$$

In accordance with case Fisher (1932), if x_1 has continuous marginal distribution F_1, no matter which distribution, then $U_1 = F_1(X_1)$ has a uniform distribution [0,1].

$$U_1 = F_1(X_1) \sim Uniform(0,1) \Rightarrow X_1 = F_1^{-1}(u_1) \ \Rightarrow X_1 \sim F_1$$

We will be able to calculate joint density function of random vector by deriving the joint cumulative distribution function as follows:

$$f(x_1, \dots, x_n) = \frac{\partial F\,(x_1, \dots, x_n)}{\partial x_1 \dots \partial x_n} = \frac{\partial^n C\big(F_1\,(x_1), \dots, F_n\,(x_n)\big)}{\partial F_1\,(x_1) \dots \partial F_n\,(x_n)} \times \prod_{i=1}^{n} \frac{\partial F_i(x_i)}{\partial x_i} = c\big(F_1\,(x_1), \dots, F_n\,(x_n)\big) \times \prod_{i=1}^{n} f_i(x_i)$$

As a result:

$$f(x_1, \dots, x_n) = c(u_1, \dots, u_n) \times \prod_{i=1}^{n} f_i(x_i) \quad (4)$$

where f_i is the marginal density function, $u_i = F_i(x_i)$ is the marginal distribution function, and c is the Copula density function [18].

Hence, there is no need for the presence of the same marginal distributions to calculate copula, and this is the most important result. Also, choosing different marginal distributions does not create any limitation for Copula.

According to the importance of the methods widely used, different types of Copula functions have been presented. Accordingly, Elliptical and Archimedean Copulas family are the most important and widely used classes of copulas family members.

In this study, the Gaussian and t-Student Copula are used to calculate the multivariate density distribution function related to elliptical Copula family. Brief introduction of these two functions of this group will be presented as follows.

3-1-1. Gaussian copula

Assume that R is symmetric definite positive matrix and the main elements of the diameter are $(\mathrm{diag}(R) = (1,1, \dots.,1)^T)$ showing the correlation between variables. Φ_R is n-dimension standard normal joint distribution function with the correlation matrix R, and then Gaussian Copula is the multivariate normal distribution Copula which is defined as "Equation (5)":

$$C_R^{Ga}\,(\mathrm{u}) = \Phi_R^n\,\big(\Phi^{-1}\,(u_1), \dots, \Phi^{-1}\,(u_n)\big) \quad (5)$$

Φ^{-1} is the reverse of univariate standard normal distribution function. By using "Equation (5)," Gaussian Copula is concluded as follows:

$$\frac{1}{(2\pi)^{\frac{n}{2}}\,|R|^{\frac{1}{2}}} \exp = \mathrm{c}_R^{Ga}\,\big(\Phi\,(x_1), \dots, \Phi\,(x_n)\big) \times \prod_{i=1}^{n} \left(\frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2} x_i^2\right)\right) \quad (6)$$

If $u_i = \Phi\,(x_i)$, then $x_i = \Phi^{-1}\,(u_i)$. With these assumptions, the density function is calculated as follows:

$$\mathrm{c}_R^{Ga}\,(u_1, \dots, u_n) = \frac{1}{|R|^{\frac{1}{2}}} \exp\left(-\frac{1}{2}\,\mathcal{S}^T\,(R^{-1} - I)\mathcal{S}\right) \quad (7)$$

where $\mathcal{S} = (\Phi^{-1}\,(u_1)\,, \dots, \Phi^{-1}\,(u_n)\,)^T$ (Grzyska, 2015).

3-1-2. t-student copula

t-Copula is based on multivariate t distribution. Similar to Gaussian Copula, it is extracted from multivariate normal distribution. This Copula is defined as follows:

$$C_{v,R}^t\,(\mathrm{u}) = t_{v,R}\,\big(t_v^{-1}\,(u_1), \dots, t_v^{-1}\,(u_n)\big) \quad (8)$$

where $t_{v,R}$ is n-dimensional t-Student joint distribution function with correlation matrix R, and t_v^{-1} is the inverse of t-standard univariate distribution function [13].

t-Student density function can be calculated as in the previous section.

This study will estimate Copula's correlation parameter with the application of DCC-GARCH model. As mentioned in the estimation methods of the literature review, Copula's parameters are not estimated dynamically. In this study, the correlation matrix, which is calculated by DCC-GARCH method, is used to estimate the Copula's parameters which is expected to have better performance than the other methods.

3-2. DCC model

Dynamic conditional correlation model introduced by Engel (2002) is used for modeling the dynamic correlation structure (R_t) in Copula. Structure of DCC model is as follows:

$$H_t = D_t R_t D_t \quad (9)$$

where D_t is a diagonal matrix in which the elements of the main diagonal are conditional standard deviation, the square root of the conditional variance of GARCH (1,1) model. Also, R_t is conditional correlation matrix which is time-varying. There are several ways to calculate R_t parameter where exponential smoothing method of Engel (2002) is one of them.

$$\begin{aligned}\mathbf{Q_t} &= \mathbf{S}\odot(\mathbf{\acute{u}} - \mathbf{A} - \mathbf{B}) + \mathbf{A}\odot(\mathbf{u_{t-1}u'_{t-1}}) \\ &\quad + \mathbf{B}\odot\mathbf{Q_{t-1}} \\ \mathbf{u_t} &= \mathbf{D_t^{-1}}\boldsymbol{\varepsilon}_\mathbf{t}\end{aligned} \quad (10)$$

where S is unconditional correlation matrix, and Q_t is $N \times N$ symmetric definite positive matrix.

$$R_t = \mathrm{diag}(Q_t)^{-1/2} Q_t \mathrm{diag}(Q_t)^{-1/2} \quad (11)$$

3-3. Marginal distributions modeling

After the Conditional Volatility autoregressive (ARCH) presentation by Engle (1982) and Generalized Autoregressive Conditional Volatility (GARCH) by bollerslev (1986), analysis of financial and economic time series is made possible. The chosen marginal model in this study is classical GARCH and GJR-GARCH models, where error terms follow normal and t-student distribution that will be briefly discussed as follows.

3-3-1. GJR-GARCH model

To consider the asymmetry of variance, different models were proposed; one of them is GJR model introduced by Glosten et al in 1993.

$$\begin{aligned} r_t &= \mu_t + a_t \\ a_t &= \sigma_t\, \varepsilon_t \\ \sigma_t^2 &= \alpha_0 + \alpha_1 a_{t-1}^2 + \beta\sigma_{t-1}^2 + \gamma s_{t-1} a_{t-1}^2 \\ &\varepsilon_t \sim N(0,1) \quad or \quad \varepsilon_t \sim t_d \\ &\alpha_1 \geq 0,\ \alpha_0 > 0 \quad,\quad \beta < 1 \quad, \\ &\alpha_1 + \beta + \frac{1}{2}\gamma < 1 \end{aligned} \quad (12)$$

where μ_t is the conditional mean, and σ_t^2 is conditional variance, such that:

$$\sigma_t^2 = Var(r_t|\Omega_{t-1}) = Var(a_t|\Omega_{t-1}) \quad (13)$$

where Ω_{t-1} is information set in period $t-1$.
Also, s_t is the binary variable and defined as follows:

$$s_{t-1} = \begin{cases} 1, & a_{t-1} < 0 \\ 0, & a_{t-1} > 0 \end{cases} \quad (14)$$

Despite classical GARCH, GJR model includes asymmetric effects. In this model, the good news ($a_t > 0$) and bad news ($a_t < 0$) have different impacts on the conditional variance. Good news and bad news have the effect of α_1 and $\alpha_1 + \gamma$, respectively. If $\gamma > 0$, then it is known as leverage effect, and if $\gamma \neq 0$, it is concluded that news effect is asymmetric. Finally, marginal distribution per share is calculated as follows:

$$P\,(r_{t+1} \leq r|\Omega_t) = P(\varepsilon_{t+1} \leq \frac{(r - \mu_t)}{\sqrt{\alpha_0 + \alpha_1 a_t^2 + \beta\sigma_t^2 + \gamma s_t a_t^2}}\,|\Omega_t \quad (15)$$

$$= \begin{cases} N\left(\frac{(r-\mu_t)}{\sqrt{\alpha_0+\alpha_1 a_t^2+\beta\sigma_t^2+\gamma s_t a_t^2}}\,|\Omega_t\right), \text{if } \varepsilon_t \sim N(0,1) \\ t_d\left(\frac{(r-\mu_t)}{\sqrt{\alpha_0+\alpha_1 a_t^2+\beta\sigma_t^2+\gamma s_t a_t^2}}\,|\Omega_t\right), \text{if } \varepsilon_t \sim t_d \end{cases} \quad (16)$$

All of the margin distribution parameters are calculated by maximum likelihood method.

3-4. Estimation methods

The most common methods used to estimate parameters of Copula and marginal distribution are Maximum Likelihood Estimation (MLE) and Inference Function for Margins (IFM), where (IFM) will briefly be described in the following.

3-4-1. Inference function estimation for margins (IFM)

IFM is one of the methods used to estimate the Copula-GARCH model's parameters. IFM parameters are estimated in two steps, and this method is easier than maximum likelihood method computationally.
In the first step, we estimate the marginal parameters of θ_1 by univariate marginal distributions:

$$\hat{\theta}_1 = arg\,max\,\theta_1 \sum_{t=1}^{T}\sum_{j=1}^{n} ln\, f_j(x_{jt};\theta_1) \quad (17)$$

In the second step, using θ_1, it is attempted to estimate the Copula parameters θ_2:

$$\hat{\theta}_2 = \arg\max \theta_2 \sum_{t=1}^{T} \ln c(F_1(x_{1t}), F_2(x_{2t}), \ldots, F_n(x_{nt}); \theta_2, \hat{\theta}_1)$$
(18)

IFM estimator is defined as "Equation (19)":

$$\hat{\theta}_{IFM} = (\hat{\theta}_1, \hat{\theta}_2)' \quad (19)$$

In each step of this method, Maximum likelihood is used to calculate intended parameters.

This study used a novel method to estimate parameters, which is one of the contributions. The way we have presented in this study is similar to the IFM method done in two steps. The method can be used to estimate correlation parameter of Gaussian Copula and t-student Copula. Based on the procedure, similar to IFM at the first step, the marginal parameters θ_1 are estimated by univariate marginal distributions. Next, by the application of maximum likelihood method, according to "Equation (17)", Copula's parameter will be estimated using the DCC-GARCH model differently.

3-5. Validating var predictive models

In predictive models, there is a possibility of error due to different reasons, where sampling error, lack of information, and modeling error can be noticed. So, calculating the market risk using quantitative risk measure models, particularly VaR models, will be reliable and useful when, firstly, these models predict the amount of risk accurately; secondly, these predictions can be effective. One of the model validation methods is its back testing, including the application of quantitative methods, in order to ensure its compliance with the model's predictions.

3-5-1. Kupiec test

One way to assess the predictive ability of the VaR models is counting the number of times that the amount of incurred losses was greater than actual predicted ones by VaR. If the actual losses are greater than the estimated ones by the model, it will be considered as a failure (violation), but if actual losses are smaller than the estimated losses, then it will be recorded as a success (overruns). To test this hypothesis, the violation or failure rate is obtained through the violation number of the total number of forecasts.

Kupiec, to investigate recent hypotheses, proposed the probability of failures ratio test. This ratio has chi-square distribution probability with one degree of freedom and its statistics defined in the form of "Equation (20)".

$$LR_{PF} = 2Ln\left[\frac{\hat{\alpha}^{T_1}(1-\hat{\alpha})^{T-T_1}}{\alpha^{T_1}(1-\alpha)^{T-T_1}}\right] \quad (20)$$

LR_{PF}: The probability of failure

T: Total predictions

T_1: Number of failures

α: Failure rate $\hat{\alpha}$: coverage

If the probability of failure is greater than the chi-square distribution with one degree of freedom and α significant level, the null hypothesis will be rejected.

4. Data and Experimental Results

We consider price indexes to be very important because these indexes are more reliable to assess movements of prices. In this study, we use the daily logarithmic returns of 5 indexes of Tehran's Stock Exchange Market including "Pharmaceutical", "Machinery", "Tile", "Metals", "Oil product", and "Chemical product" from 2011/03/26 up to 2017/03/07 which possess 1437 observations.

Table 1 shows statistical characteristics of the used data in summary. It is observed clearly that kurtosis of return series is more than normal distribution, and the series are skewed. In addition, Jarqe-bera statistics suggests that the null hypothesis is rejected and none of the return's series is normal. Jarqe-bera statistics is used for normality test.

Tab. 1. Descriptive statistics of daily log-returns of asset

	Chemical	Oil product	Tile	Machinery	Pharmaceutical
Mean	0.1226	0.1226	0.104	0.12226	0.1367
Median	0.0235	-0.0008	-0.0555	0.0235	-0.0049
Maximum	10.2716	29.254	14.239	10.2716	6.4701
Minimum	-4.3427	-46.4321	-6.4232	-4.3427	-1.8525
Std.Dev	1.1534	2.0645	1.1546	1.1534	0.6968
Skewness	1.1828	-5.1647	2.3908	1.1828	2.2583
Kurtosis	11.1004	216.6665	24.0107	11.1004	13.8219
Jarque-Bera	4263.903	2739885	278000.97	4263.903	8239.42
Augmented Dickey-Fuller (ADF)	-17.689	-33.617	-14.035	-16.616	-14.427
ARCH-LM	-14.879	-9.731	-12.891	-15.762	-17.863

In order to evaluate the stationary of return series, the Dickey-Fuller test is hired. This test is examined, and statistics show that the null hypothesis is rejected (there is a unit root), so the series returns are stationary. To investigate the effects of heteroscedasticity of variance in return series, ARCH-LM test is used. The results show that the existence of ARCH effects assumption cannot be rejected at 90% significant level. So, GARCH family models can be used for marginal distribution. Figure 1 presents the returns series of the studied indexes.

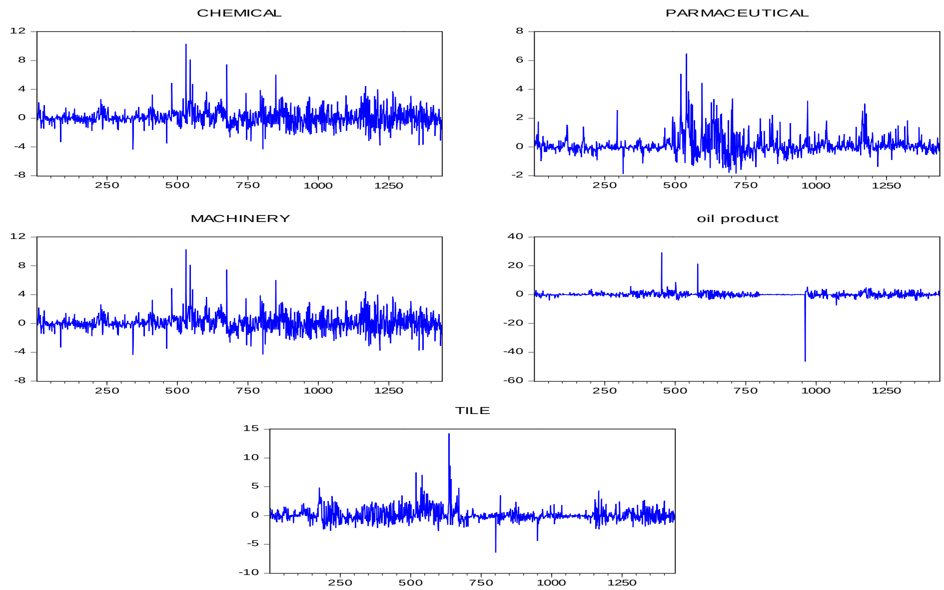

Fig. 1. Daily returns of price indexes

4-1. Marginal distributions estimation using different models

With regard to heteroscedasticity in the returns series, the GARCH(1,1) and GJR-GARCH(1,1) models are used for the marginal distribution of series in this study. The model's parameters estimated by MLE method in table 2 are presented. The results also show that the amount of γ is negative in all cases except Oil product. This means that almost negative shocks do not turbulent the market as much as positive shocks do; however, the news effect is asymmetric.

Tab. 2. Parameter estimation of GARCH and GJR marginal distributions

parameters		Chemical	Oil product	Tile	Machinery	Pharmaceutical
GARCH Model	α_1	0.0222 (0.0098)*	0.0245 (0.008)	0.3661 (0.2047)	0.0222 (0.0098)	0.1121 (0.0801)
	β	0.9719 (0.014)	-0.1002 (0.1481)	0.5918 (0.1282)	0.9719 (0.0145)	0.8763 (0.0874)
	AIC	-2127.043	-3057.79	-1972	-2127.04	-946.753
GJR Model	α_1	0.0629 (0.0351)	0.0214 (0.008)	0.4112 (0.2472)	0.0629 (0.0351)	0.1075 (0.0434)
	β	0.9594 (0.03)	-0.114 (0.044)	0.6375 (-0.2800)	0.9594 (0.0300)	0.9237 (0.0367)
	γ	-0.0795 (0.043)	0.0104 (0.009)	-0.2800 (0.1682)	-0.0795 (0.0430)	-0.1169 (0.0328)
	AIC	-2108	-3057.459	-1962.916	-2108.4	-927.345

4-2. Conditional correlation estimation using the DCC model

This study estimates Gaussian and t-student Copula's correlation parameter by using the DCC model that will achieve the correlation structure as time-varying. Table 3 reports the dynamic correlation parameter estimated by DCC model.

Tab. 3. Dynamic correlation parameters estimation using DCC model

	Chemical	Pharmaceutical	Machinery	Oil product	Tile
Chemical	1.000000	0.181298	0.126543	0.164735	0.194591
Pharmaceutical	0.181298	1.000000	0.181298	0.066254	0.194488
Machinery	0.126543	0.181298	1.000000	0.164735	0.194591
Oil product	0.164735	0.066254	0.164735	1.000000	0.090198
Tile	0.194591	0.194488	0.194591	0.090198	1.000000

4-3. Var calculation

After determining and estimating the marginal distribution models with application of GARCH (1,1) and GJR-GARCH (1,1) models and calculation of dynamic correlation using the DCC, Gaussian and t-student Copulas are used to create the multivariate joint distribution of return series. After obtaining density function, portfolio's Value at Risk, including five indexes with equal weight, is calculated. The results of estimating Value at Risk by using the proposed models at 95 and 99 percent of significant level are shown in table 4.

Tab. 4. Investment portfolio's value at risk with assuming equal weights for assets

Confidence level	Gaussian Copula		t-Copula	
	GARCH	GJR	GARCH	GJR
95%	2.248	2.021	1.265	0.887
99%	2.751	2.351	1.564	1.165

According to table 5, at 95% significant level, the Value at Risk predicted by t-Copula-GARCH model is equal to 1.267. That means that the maximum loss occurring in one day for investment portfolio at 95% is 1.76 percent of the total portfolio values. These results specify that the predicted VaR amount by t-Copula is less than Gaussian Copula model in all cases. The reason is that t-Copula focus on the series tale correlation despite Gaussian Copula due to the following symmetric distribution.

4-4. Model validation and kupiec test's results

To do Kupiec test, the sample data are divided into two groups of 1200 inside and 236 outside observations. To specify the efficiency of suggested models, the amount of VaR related to inside sample period is compared to the return of portfolios concerning the outside sample period. The results of the studied models for both 95 and 99 percent are presented in table 5.

Tab. 5. Kupiec test results of all models

confidence level	Kupiec test	Gaussian Copula		t-Copula	
		GARCH	GJR	GARCH	GJR
95%	Kupiec value	2.843	2.153	2.356	1.965
	Critical value	3.841	3.841	3.841	3.841
99%	Kupiec value	5.453	4.986	4.765	4.156
	Critical value	6.634	6.634	6.634	6.634

As the results illustrate, at the significant level of 95% and 99%, Kupiec statistic is less than the critical value of chi-square distribution with one degree of freedom in all models; therefore, the null hypothesis of rate of failure equality and the significance level is not rejected, which indicates that the suggested model has been estimated properly.

5. Summary and Conclusion

This study hired DCC-Copula-GARCH model which considers dynamic correlation structure of assets in order to calculate Value at Risk of one day for equal weighted portfolio. The paper focused on Tehran Stock Exchange Market, and the major "Pharmaceutical", "Machinery", "Tile", "Metals", "Oil product", and "Chemical product" indexes were selected for portfolio composition. To validate the models efficiency, an intraday database comprised of 5 indexes in The time span 2011 to 2017 was employed, and the data set was divided into two part consisting of in and out sample sets. The first data set was hired to estimate Value at Risk, and the second one was used to examine efficiency of the proposed models. The empirical results show that, for an equally weighted portfolio of five indexes, the VaR obtained from the Student *t*-copula is smaller than those obtained from the Gaussian copula. According to the results, Student t-copula with GJR marginal distribution has the less amount of VaR. Also, the result of kupiec test presents that the proposed models are qualified enough to estimate VaR amount of portfolio. It is suggested for the next studies to use other measures, such as CvaR, and compare the results to those of the VaR model. Also, more copulas, such as multivariate Clayton, multivariate Gumbel, and vine copula, can be hired to calculate joint distribution of portfolio.

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