

Game-Theoretic Approach for Pricing Decisions in Dual-Channel Supply Chain

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ABSTRACT

In the current study, a dual-channel supply chain is considered containing one manufacturer and two retailers. It is assumed that the manufacturer and retailers have the same decision powers. A game-theoretic approach is developed to analyze pricing decisions under the centralized and decentralized scenarios. First, the Nash model is established to obtain the equilibrium decisions in the decentralized case. Then, the centralized model is developed to maximize the total profit of the whole system. Finally, the equilibrium decisions are discussed and some managerial insights are revealed.

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1. Introduction

Nowadays, customers prefer the Internet-based sales due to convenient and secure selling over the Internet [1]. Dual-channel supply chain is one in which a manufacturer sells his/her products to consumers through the traditional retail and the online direct channel, simultaneously [2]. It is reported that about 42% of the top manufacturers such as Nike, Dell, IBM, Pioneer Electronics, and Estee Lauder employ the dual-channel structure to sell their products [3, 4]. Structure of a dual-channel supply chain is shown in Fig. 1.

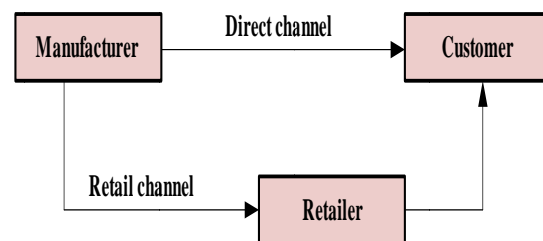


Fig. 1. The general structure of a dual-channel supply chain

Game theory is a field of applied mathematics that applied when the decision of one player affects the other players' payoff [5-8]. Below, some of the recent studies are addressed which have applied the game theoretic in the dual-channel structures:

Several researchers have examined the optimal pricing strategies in the dual-channel supply

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chains. Dai et al. [9] investigated the pricing strategies in a dual-channel supply chain by considering both the deterministic and stochastic demands. Yao and Liu [10] analyzed the price competition between the direct and the retail channels under the different game theoretic models, whereas Kurara et al. [11] used a game theoretic approach to investigate the research problem under the competition between a national brand and a store brand. Huang et al. [12, 13] proposed a pricing model in which the production costs are disrupted during the planning horizon. Furthermore, Cao et al. [14] evaluated the impact of the asymmetric cost information on the equilibrium strategies.

Coordination is a useful method to effective supply chain management [15]. Supply chain coordination has been extensively investigated in the dual-channel supply chain. Cai et al. [16] evaluated the impact of the price discounts contract on a dual-channel supply chain, while Yan [17] and Yan and Pei [18] used a game theoretic model to examine the effect of the profit sharing.

In this paper, a dual-channel supply chain is considered consisting of a manufacturer and two retailers. It is assumed that the manufacturer and two retailers have the same decision powers. In this setting, they determine their optimal pricing decisions simultaneously and independently. A game theoretic approach is applied to analyze the pricing policies in the centralized and decentralized supply chains. First, the decentralized framework is investigated by developing the Nash game model to obtain the optimal pricing policies of all the members (players) simultaneously and independently. Then, the centralized model is presented to obtain the pricing policies which maximize the profit value of the whole system. Finally, the Nash bargaining model is applied to lead to win-win situation.

The rest of the paper is organized as follows: In Section 2, the research problem is described in details. The decentralized and centralized models are investigated in Sections 3 and 4, respectively. In Section 5, the optimal decisions are compared under the considered models. Then, an instance is presented in Section 6 to well illustrate the research problem. Finally, conclusions and directions for future studies are presented in Section 7.

2. Model Framework

In this research, a dual-channel structure has been considered including a manufacturer and two retailers. The manufacturer sells a product through the retail and direct channel, simultaneously. The manufacturer and the retailers have the same decision powers. The manufacturer sets the unit price in the direct channel as well as the same unit wholesale price to the retailers. Simultaneously, the two retailers set the unit prices in the retail channel to end customers. The structure of the considered dual-channel supply chain is shown in Fig. 2.

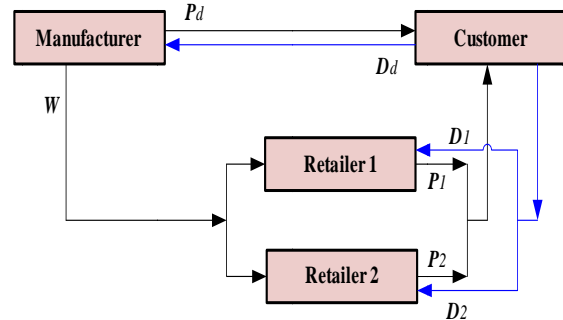


Fig. 2. Structure of the dual-channel supply chain considered

The notations are defined as:

C_m	Unit manufacturing cost
a_d	Market base of the direct channel
a_i	Market base of the retail channel for retailer- i ($i = 1, 2$)
β	Self-price sensitivity of the demands
θ	Cross-price sensitivity of the demands
W	Same unit wholesale price charged by the manufacturer to the retailers
P_d	Unit selling price charged by the manufacturer to customers in the direct channel
P_i	Unit selling price charged by retailer- i ($i = 1, 2$) to customers in the retail channel
D_d	Demand faced by the manufacturer in the direct channel
D_i	Demand faced by retailer- i in the retail channel ($i = 1, 2$)
π_m	Manufacturer's profit
π_i	Retailer- i 's profit ($i = 1, 2$)
π_{SC}	Profit of the whole system ($\pi_{SC} = \pi_m + \pi_1 + \pi_2$)

Assumption 1. In each channel, the self-price sensitivity of the demand is greater than the sum of the cross-price sensitivities. In other words, each demand is more sensitive to the changes in its own price than to the sum of the changes in two other prices, i.e., $\beta > 2\theta$ [19, 20].

Assumption 2. In the retail channel, the retailers' unit profit margin is not less than the manufacturer, i.e., $P_i - W \geq W - C_m$ ($i = 1, 2$) [21, 22, 23].

Assumption 3. We assume that $(\beta - 2\theta)C_m \leq \{a_d, a_1, a_2\}$.

We suppose that the manufacturer and retailers face the following demand functions in the direct and retail channels, respectively.

$$\begin{cases} D_d = a_d - \beta P_d + \theta(P_1 + P_2) & (1) \\ D_i = a_i - \beta P_i + \theta(P_d + P_j) \quad i = 1, 2 \quad j = 3 - i & (2) \end{cases}$$

In Relation (1), βP_d is the number of customers who quit buying through the direct channel when the price in this channel is P_d , whereas $\theta(P_1 + P_2)$ is the number of customers who switch to the direct channel from the retail channel when the prices determined by the retailers in the retail channel are P_1 and P_2 . The demands D_1 and D_2 in the retail channel are similarly defined in Relation (2).

The profit functions for the manufacturer, two retailers, and whole system are formulated as follows, respectively:

$$\begin{cases} \pi_m(W, P_d) = (P_d - C_m)D_d + (W - C_m)(D_1 + D_2) & (3) \\ \pi_i(P_i) = (P_i - W)D_i \quad i = 1, 2 & (4) \\ \pi_{SC}(P_d, P_1, P_2) = \pi_m + \pi_1 + \pi_2 & (5) \end{cases}$$

To ensure that the obtained prices are feasible, the additional constraints are considered on the model: the wholesale price and direct price should not be less than the unit manufacturing cost. Furthermore, the retail prices are equal to or more than the wholesale price. These constraints are shown as follows:

$$\begin{cases} W, P_d \geq C_m & (6) \\ W \leq P_1, P_2 & (7) \end{cases}$$

In the next sections, we obtain the equilibrium pricing policies applying the game theoretic approach. Note that the symbols N and C correspond to the Nash and centralized models, respectively.

3. Decentralized Model

In Section 2, it is assumed that the manufacturer and retailers have the same decision power. Hence, they choose their optimal pricing policies independently and simultaneously. This situation is called a Nash game. To determine the optimal

pricing strategies obtained from the Nash (N) game model, the manufacturer and two retailers' decision problems are separately solved. The Nash game model is formulated as follows:

$$\begin{cases} \max_{(W, P_d)} \pi_m(W, P_d) = (P_d - C_m)D_d + (W - C_m)(D_1 + D_2) & \text{s.t. } W, P_d \geq C_m \\ \max_{P_1} \pi_1(P_1) = (P_1 - W)D_1 & \text{s.t. } P_1 \geq W \\ \max_{P_2} \pi_2(P_2) = (P_2 - W)D_2 & \text{s.t. } P_2 \geq W \end{cases}$$

The proofs of Lemmas and Theorems presented in this section are appeared in Appendix A.

Lemma 1. π_m is not jointly concave in W and P_d , but it is increasing in line with W .

Regarding Assumption 2, we have:

$$W - C_m \leq P_i - W \longrightarrow W \leq \frac{P_i + C_m}{2} \quad i = 1, 2 \quad (8)$$

Thus, from Lemma 1 one can derive that the optimal value of W is $(\min\{P_1, P_2\} + C_m)/2$.

Lemma 2. The profit function π_i is concave in P_i ($i = 1, 2$).

Theorem 1. Given the wholesale price and the direct price set by the manufacturer, we have:

$$P_i(W, P_d) = \frac{(2\beta + \theta)(\beta W + \theta P_d) + 2\beta a_i + \theta a_j}{(2\beta + \theta)(2\beta - \theta)} \quad i = 1, 2 \quad j = 3 - i \quad (9)$$

Without the loss of generality, we assume that $a_1 \leq a_2$. Thus, we have:

$$(2\beta - \theta)a_1 \leq (2\beta - \theta)a_2 \rightarrow 2\beta a_1 + \theta a_2 \leq 2\beta a_2 + \theta a_1 \quad (10)$$

Obviously, one can derive that $P_1(W, P_d) \leq P_2(W, P_d)$ and therefore the optimal value of W is $(P_1 + C_m)/2$.

New profit functions π_m^N , π_1^N , and π_2^N are obtained by substituting $W = (P_1 + C_m)/2$ into profit functions π_m , π_1 , and π_2 , respectively:

$$\pi_m^N(W, P_d) = (P_d - C_m)D_d + \left(\frac{P_1 - C_m}{2}\right)(D_1 + D_2) \quad (11)$$

$$\pi_1^N(P_1) = \left(\frac{P_1 - C_m}{2}\right)D_1 \quad (12)$$

$$\pi_2^N(P_2) = \left(\frac{2P_2 - P_1 - C_m}{2}\right)D_2 \quad (13)$$

Lemma 3. Profit functions π_m^N , π_1^N , and π_2^N are concave in P_d , P_1 , and P_2 , respectively.

Theorem 2. The equilibrium pricing strategies made by the manufacturer and the retailers in the Nash model can be obtained as follows:

$$\begin{cases} W^N = \frac{E_1 C_m + \theta a_d + (2\beta - \theta)a_1 + \theta a_2}{2E_2} \\ P_d^N = \frac{(2\beta + \theta)E_3 C_m + (2\beta + \theta)(3\beta - 2\theta)a_d}{2(2\beta + \theta)E_2} \\ \quad + \frac{\theta(9\beta + 2\theta)a_1 + \theta(3\beta + 4\theta)a_2}{2(2\beta + \theta)E_2} \\ P_1^N = \frac{E_4 C_m + \theta a_d + (2\beta - \theta)a_1 + \theta a_2}{E_2} \\ P_2^N = \frac{(2\beta + \theta)E_4 C_m + \theta(2\beta + \theta)a_d + E_5 a_1 + E_6 a_2}{(2\beta + \theta)E_2} \end{cases} \quad (14)$$

where, E_1, E_2, \dots, E_6 are defined in Appendix A.

4. Centralized Model

In the centralized (C) model, a cooperative game is considered in which all the members adopt to cooperate and maximize the profit value of the whole system. The centralized model is formulated as follows:

$$\begin{cases} \max_{(P_d, P_1, P_2)} \pi_{SC} = (P_d - C_m)D_d + (P_1 - C_m)D_1 + (P_2 - C_m)D_2 \\ \text{s.t. } P_d, P_1, P_2 \geq C_m \end{cases}$$

The proofs of Lemmas and Theorems in this section appear in Appendix B.

Lemma 4. Profit function π_{SC} is jointly concave in $P_d, P_1,$ and P_2 .

Theorem 3. The equilibrium pricing policies in the centralized model can be obtained as follows:

$$\begin{cases} P_d^C = \frac{(\beta + \theta)(\beta - 2\theta)C_m + (\beta - \theta)a_d + \theta(a_1 + a_2)}{2(\beta + \theta)(\beta - 2\theta)} \\ P_i^C = \frac{(\beta + \theta)(\beta - 2\theta)C_m + (\beta - \theta)a_i + \theta(a_d + a_j)}{2(\beta + \theta)(\beta - 2\theta)} \end{cases} \quad i = 1, 2 \quad j = 3 - i \quad (15)$$

Since the wholesale price is ignored under the centralized model, the players' profits cannot be obtained in this model.

5. Comparison of the Equilibrium Decisions

In section, the equilibrium prices, demands, and profits are compared under the decentralized and centralized models.

Proposition 1. One can derive that:

- (i) $P_d^N \leq P_d^C$
- (ii) $\begin{cases} P_i^N \leq P_i^C & \text{if } 2\theta < \beta \leq (2 + \sqrt{5})\theta \\ P_i^C < P_i^N & \text{if } \beta > (2 + \sqrt{5})\theta \end{cases} \quad i = 1, 2$
- (iii) $D_d^C \leq D_d^N$

- (iv) $\begin{cases} D_i^N \leq D_i^C & \text{if } 2\theta < \beta \leq \frac{(3+\sqrt{21})}{2}\theta \\ D_i^C < D_i^N & \text{if } \beta > \frac{(3+\sqrt{21})}{2}\theta \end{cases} \quad i = 1, 2$
- (v) $\pi_{SC}^N \leq \pi_{SC}^C$

Proof. After algebraic manipulations, Proposition 1 is proved. Thus, it is omitted for brevity.

6. Numerical Example

In this section, an instance is presented to well understand the effects of the models. The default values of the parameters are shown in Table 1. Moreover, the results obtained from the different models are summarized in Table 2.

Tab. 1. The value of the parameters in the instance

Parameter	C_m	a_d	a_1	a_2	β	θ
Value	100	500	300	200	1.5	0.4

Tab. 2. The results for the illustrative instance

Models	Prices				Demands		
	W	P_d	P_1	P_2	D_d	D_1	D_2
Nash	181.07	304.26	262.13	232.72	242	122	78
Centralized	-	331.95	279.32	253.01	215	115	65
	Profits						
	π_m	π_1	π_2	π_{sc}			
Nash	65476.88	9857.32	4002.10	79336.31			
Centralized	-	-	-	80437.97			

We perform a sensitivity analysis to investigate the effects of the changes of the self-price and cross-price sensitivities of the demands on the profits. For this reason, we change the parameters β and θ in intervals (1.1,1.9) and (0.3,0.7), respectively. The changes of the profits with β and θ are shown in Fig. 3 and Fig. 4, respectively.

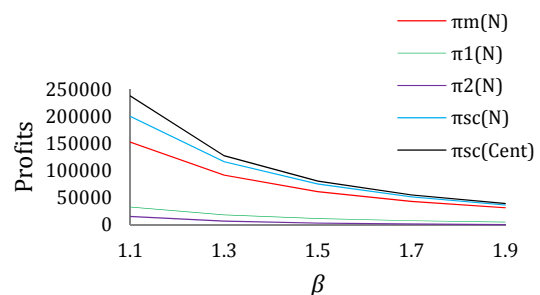


Fig. 3. Changes of the profits with β

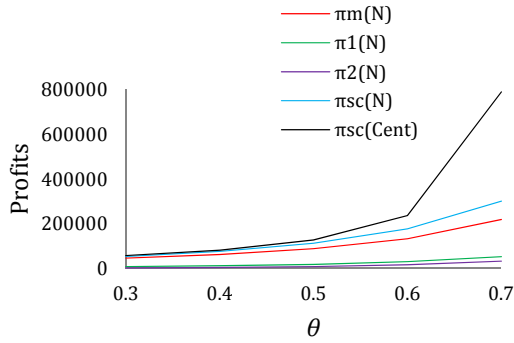


Fig. 4. Changes of the profits with θ

Obviously, all the profits decrease by increasing β , whereas they increase when θ increases.

7. Conclusions

In this study, a dual-channel supply chain was considered which consists of one manufacturer and two retailers. In the direct and retail channels, the manufacturer and retailers face the price-sensitive demands, respectively. It was assumed that all the members have the same decision powers. In this setting, they make their optimal decisions independently and simultaneously.

A game-theoretic approach was established to can investigate the pricing policies in the decentralized and centralized dual-channel supply chain. First, a Nash game model was applied to determine the equilibrium pricing policies in the decentralized model. Then, the centralized model was presented to maximize the total profit value of the whole system. Finally, the Nash bargaining model was used to lead to win-win situation.

There are many directions for future studies. In this research, a linear price sensitive demand function was considered. In future researches, a non-linear demand function can be investigated. Moreover, all the models were established under a deterministic environment such as a known market demand. In future studies, one can analyze the effects of the demand disruption on the optimal decisions obtained from the models under the stochastic environments.

Appendix A. Proofs in the decentralized model

$$\begin{aligned}
 E_1 &= 4\beta^2 - \beta\theta - 4\theta^2 \\
 E_2 &= 3\beta^2 - 2\beta\theta - 3\theta^2 \\
 E_3 &= 3\beta^2 - 2\beta\theta + 2\theta^2 \\
 E_4 &= \beta^2 + \beta\theta - \theta^2 \\
 E_5 &= \beta^2 + 2\beta\theta + 2\theta^2 \\
 E_6 &= 3\beta^2 - 2\theta^2
 \end{aligned}$$

Proof of Lemma 1. The first order partial deviations of profit function π_m to W and P_d are:

$$\frac{\partial \pi_m}{\partial W} = D_1 + D_2 = a_1 + a_2 - (\beta - \theta)(P_1 + P_2) + 2\theta P_d \quad (a1)$$

$$\frac{\partial \pi_m}{\partial P_d} = a_d - 2\beta P_d + \theta(P_1 + P_2) + \beta C_m + 2\theta(W - C_m) \quad (a2)$$

The Hessian matrix is shown as follows:

$$H_m = \begin{bmatrix} \frac{\partial^2 \pi_m}{\partial W^2} & \frac{\partial^2 \pi_m}{\partial P_d \partial W} \\ \frac{\partial^2 \pi_m}{\partial W \partial P_d} & \frac{\partial^2 \pi_m}{\partial P_d^2} \end{bmatrix} = \begin{bmatrix} 0 & 2\theta \\ 2\theta & -2\beta \end{bmatrix} \quad (a3)$$

Obviously, the Hessian matrix H_m is not negative definite and therefore the profit function π_m is not jointly concave with respect to W and P_d . But, since the demands D_1 and D_2 are nonnegative, we have $\partial \pi_m / \partial W = D_1 + D_2 \geq 0$ and $\partial^2 \pi_m / \partial W^2 = 0$ and thus π_m is increasing in line with W . \square

Proof of Lemma 2. The first and second order partial deviations of the profit function π_i with respect to P_i ($i = 1, 2$) can be shown as follows:

$$\frac{d\pi_i}{dP_i} = a_i - 2\beta P_i + \theta(P_d + P_j) + \beta W \quad j = 3 - i \quad (a4)$$

$$\frac{d^2 \pi_i}{dP_i^2} = -2\beta \quad (a5)$$

The second-order partial deviation is negative. Thus, π_i is concave in P_i . \square

Proof of Theorem 1. Suppose the wholesale price W and the direct price P_d are set by the manufacturer. By solving $d\pi_i/dP_i = 0$ ($i = 1, 2$), Relation (9) can be obtained. This completes the proof of Theorem 1. \square

Proof of Lemma 3. From Relations (11)-(13), the first order partial deviations of the profit functions π_m^N , π_1^N , and π_2^N with respect to P_d , P_1 , and P_2 respectively are:

$$\frac{d\pi_m^N}{dP_d} = a_d - 2\beta P_d + \theta(P_1 + P_2) + \beta C_m + \theta(P_1 - C_m) \quad (a6)$$

$$\frac{d\pi_1^N}{dP_1} = \frac{1}{2}(a_1 - 2\beta P_1 + \theta(P_d + P_2)) + \beta C_m \quad (a7)$$

$$\frac{d\pi_2^N}{dP_2} = a_2 - 2\beta P_2 + \theta(P_d + P_1) + \beta \left(\frac{P_1 + C_m}{2} \right) \quad (a8)$$

Taking the second order partial deviations we have:

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