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A Mathematical Model for Vehicle Routing and Scheduling Problem With Cross-Docking by Considering Risk

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KEYWORDS

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Scheduling,
Disruption risk.

ABSTRACT

After achieving some improvements in manufacturing operations, many companies are focused today on the improvement of distribution systems and there has been a strong tendency to optimize the distribution network in order to reduce logistics costs that have been a challenge. Improvement of the materials' flow is an activity considered essential to increase customer satisfaction. In this study, we apply cross-docking method to effective control of cargo flow to reduce inventory and improve customer satisfaction. Also, every supply chain is faced with risks that threat its ability to work effectively. Many of these risks are not in control and can cause great disruption and costs for the supply chain process. In this study, we are looking for a model to collect and deliver the demands by the limited capacity vehicle in terms of disruption risk that is finally presented as a compromised planning process. In fact, we propose a framework that can consider all the problems of a crisis situation. In the first step, the results were presented as a two-level planning. Then, the problem was expressed as a multi-objective optimization model and the results were explained.

1. Introduction

A cross-docking network can be defined as a subset of a supply chain that consists of one or more cross-docks, incoming and outgoing transportation routes, and stakeholders that are connected to cross-docks through these routes. In cross-docking networks, different logistic facilities are defined as potential stakeholders. These logistic facilities include common supply chain institutions such as suppliers, producers, warehouses, distribution centers, buyers and sellers and can be located around cross-dock's entrance and exit [1].

In a traditional distribution center, commodities are received at first, and then stored in places like pallet racks. When a customer requests a special product, workers pick it up from the warehouse and carry to the intended destination. From these four major activities in the dock (receiving, storage, picking and carrying orders), storage and picking up an order are usually the most expensive. Storage because of stock maintenance costs and picking up an order because of intensive and severe work are costly. Performance improvement in one or some of

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these activities or in their interactions can be a way of cost reduction. However, the cross-dock is a method that eliminates two of the most expensive operations, which are storage and pick up [2], [3].

To implement effective management of cross-docking, we need some decisions at an operating level. Operating decisions include decisions in short-time horizon such as daily and weekly. Some accomplished articles have been done on effective improvement of these operations. Researchers have classified these issues in five groups. This classification includes scheduling problem, transport problem, the dock door assignment problem, vehicle routing problem, and product assignment problem [4].

The growing need for efficient supply chain management and logistics in recent years has brought the vehicle routing problems to the center of attention again after more than half a century of their first introduction [5].

Routing and scheduling in the cross-docking can be seen in studies of lee et al. (2006), Liao et al. (2010), Mousavi and tavakoli moghadam (2013), and Agustina et al. (2014). In another work of mousavi and tavakoli (2013), a two-phase mixed integer programming model of the cross-docking location and scheduling and routing of vehicles in the cross-docking has been presented for potential applications in distribution networks. Simultaneous designing of these two problems has been stated as a primary goal, and simulated annealing and Tabu search algorithms have been proposed for their solving. Dondo and Cerda (2013) introduced an integrated formulation of vehicle's routing with a cross-docking which simultaneously determines delivery and pickup routes and scheduling of cargo vehicles in the corresponding fleet. The final model based on reciprocating exploration can find near-optimal solutions for large-scale problems in appropriate time. Studies of Xie et al. (2013) are about vehicle's scheduling in the field of home healthcare supplies in which each patient relates to one vehicle and each vehicle relates to more than one node and the time window is considered for patients and vehicles that are capacitated. In the solving phase, two mixed integer-programming models were proposed and, finally, genetic algorithms and Tabu search solving methods were presented. Ghomi et al. in 2014 considered the scheduling and vehicle routing in a network that consists of the suppliers, customers and cross-docks. A two-stage hybrid algorithm for pickup and delivery vehicle routing problems with time windows and multiple vehicles was studied by Bent et al. A mixed integer nonlinear mathematical formulation of this problem has been provided whose primary goal is to minimize the total travel cost, and then we face the minimization of function that results in a reduction of time loss in cross-docking [6]-[14].

A key issue for the success of any organization in supply chain context is to ensure ease of operation by effective management of risks and disruptions. Recently, risk and disruption management are considered as an important problem in the supply chain. Some previous studies have developed models for risk and disruption management in supply chain and production systems. Some of these studies have been done on the risk analysis of the road network, such as the one done by Mohaymany and Khodadian (2008). They proposed an integer linear programming model so that the hazardous materials system can determine the optimized allocation of all start and end pairs for various hazardous materials in a transportation network. To solve the model, they used branch and bound algorithm [15].

Different kinds of these models include models on disrupted production process, inventory-production management with disruption, and supply chain management with considering disruption. These models have been solved with different methods and some of them have been used in real world. In previous studies, a considerable number of papers considered some of the disruption and risk factors of the real world in the modeling of the supply chain and inventory-production systems. This research tries to model disruption and risk [16].

Recently, one improvement model for supply disruption has been proposed in a two-step supply chain system with only one supplier and one customer. The concept of Hishamuddin et al. (2012), which was the development of a disruption management model in real time, has further developed in recent years for management of incomplete inventory-production systems and also demand management in a coordinated byer-supplier system. [17], [18],[19].

The presented paper by Nikolic and Teodorovic (2015) is the origin of the main idea of disruption. In this paper, after encountering disruption and unexpected rising of customers' demand, we decided to develop a model for improvement after the disruption. A mathematical formulation for this problem has been considered, and finally, a Bees optimization and lexicographic optimization algorithms have been used to solve the model [20].

With an overview of the previous literature, it can be seen that a few studies have been done on simultaneous vehicle routing and scheduling under cross-docking, and despite considering some parameters in uncertainty conditions which can be a risk, no research has been done on disruption for facing risk conditions. Considering the demand disruption, this paper tries to deal with losses resulted from disruptions (risks) in these systems.

In this research, the vehicle routing and scheduling models have been examined to be simultaneously employed in conditions different from traditional distribution centers so that we can reduce the costs. For this purpose, a new and modern distribution center, named cross-docking, is considered that under this warehousing, all programs are different from traditional ones. After achieving this goal, for better development of the modeling and making it more applicable to crisis conditions that are an important and integral part of industrial systems, a part of system is considered under the disruption. The routing problem is modeled under the disruption risks. The proposed mathematical model has three objectives: minimizing the number of unserved customers, minimizing the number of customers not served in their own route, and minimizing the total cost of the model with considering a penalty as the cost of earliness and tardiness of the vehicles in delivery and pickup processes. To solve the problem and multi-objective optimizing problem, the compromise programming (that is form LP-metric methods) is used, and at the end, the model is implemented on a real-world example.

2. Problem Description

In this paper, we face with two models. In the first step, a model is developed which basically tries to present an optimized route of vehicles in delivery and pickup processes in the cross-docking network. In this modeling, we try to determine a route that passes the pickup routes with minimum cost and, under the same conditions, deliver received commodities through cross-docking to specified customers. In this model, in addition to minimization of the total costs, by assigning a penalty to tardiness and earliness for customers and suppliers, we decided to present an optimized scheduling for vehicles from which the best time for traveling is obtained.

In the second step, after minimization of cost and time in the previous model, we face with conditions in which a disruption occurs in the network and we have an unexpected rising of one or more customers' demand. Thus, by adding new variables and parameters, according to the re-routing, we consider a situation where possibly one or more customers for reasons, such as unavailability of sufficient inventory in stock or vehicles, high cost of service according to the cost-benefit balance, etc., are not served or are served by a route other than their routes. In these conditions, a model for re-routing of the first model is proposed so that the disruption has minimum impact on the designed network.

In the following, descriptions of the two proposed models are given.

2-1. Problem 1

Modeling of the problem 1 is as follow:

Indexes

i,j: indices related to each pickup node (suppliers)

n,m: indices related to each delivery node (customers)

o1,o2: indices of the cross-docks in pickup section

o3,o4: indices of the cross-docks in delivery section

v: index of each pick-up vehicle

v`: index of each delivery vehicle

Parameters

$t1_{ij}$: the travel time from nodes i to j

$t2_{nm}$: the travel time from nodes n to m

Q_1: pick up vehicle's capacity

Q_2: delivery vehicle's capacity

$S1_i$: the service time in node i in pick up process

$S2_n$: the service time in node n in delivery process

$q1_i$: the amount of supply at each pick up node

$q2_n$: the amount of demand at each delivery node

$C1_{ij}$: the travel cost from nodes i to j in pick up process

$C2_{nm}$: the travel cost from nodes n to m in delivery process

α: the penalty cost of each unit that has earliness

β: the penalty cost of each unit that has tardiness

M: a big number

$LTD1_i$: the lower limit of pick up time from suppliers

$UTD1_i$: the upper limit of pick up time from suppliers

$LTD2_n$: the lower limit of delivery time to customers

$UTD2_n$: the upper limit of delivery time to customers

Decision variables

X_{ij}^{v} : it will be 1 if vehicle v transports commodities from nodes i to j in pick up process, otherwise it is zero.

$Y_{nm}^{v`}$: it will be 1 if vehicle v' transports commodities from nodes n to m in pick up process, otherwise it is zero.

$\mathrm{w1}_{\mathrm{i}}^{\mathrm{v}}$: arrival time of pick-up vehicle v to pick-up node i

$\mathrm{w2}_{\mathrm{n}}^{\mathrm{v`}}$ : arrival time of delivery vehicle v` to delivery node n

H_i^v: maximum arrival time of pick-up vehicle v to pick-up node i

$penp1_i^v$: the deviation of pick-up vehicle v from the lower limit of pick-up deadline at node i

$penn1_i^v$: the deviation of pick-up vehicle v from the upper limit of pick-up deadline at node i

$penp2_n^{v`}$: the deviation of delivery vehicle v' from the lower limit of delivery deadline at node n

$penn2_n^{v`}$: the deviation of delivery vehicle v' from the upper limit of delivery deadline at node n.

In the following, the objective function and constraints of the problem will be introduced.

Min

$$\sum_{v\epsilon V}\sum_{i\epsilon P\cup o_1\cup o_2}\sum_{j\epsilon P\cup o_1\cup o_2} C_{i,j}X_{i,j}^{v} \quad (1)$$

$$\sum_{v`\epsilon V`}\sum_{n\epsilon D\cup o_3\cup o_4}\sum_{m\epsilon D\cup o_3\cup o_4`} C_{n,m}Y_{n,m}^{v`} \quad (2)$$

$$+\sum_{v\epsilon V}\sum_{i\epsilon P\cup o_2} \alpha penp1_i^v + \beta penn1_i^v \quad (3)$$

$$+\sum_{v`\epsilon V`}\sum_{n\epsilon D\cup o_3\cup o_4} \alpha penp2_n^{v`} + \beta penn2_n^{v`} \quad (4)$$

subject to:

$$\sum_{j\epsilon P} X_{o_1,j}^{v} = 1 \;\; ; \quad \forall v \in V \quad (5)$$

$$\sum_{i\epsilon P} X_{i,o_2}^{v} = 1 \;\; ; \quad \forall v \in V \quad (6)$$

$$\sum_{i\epsilon P} X_{i,o_2}^{v} = 1 \;\; ; \quad \forall v \in V \quad (7)$$

$$\sum_{i\epsilon P} X_{i,o_2}^{v} = 1 \;\; ; \quad \forall v \in V \quad (8)$$

$$\sum_{i\epsilon P} X_{i,o_2}^{v} = 1 \;\; ; \quad \forall v \in V \quad (9)$$

$$\sum_{i\epsilon P} X_{i,o_2}^{v} = 1 \;\; ; \quad \forall v \in V \quad (10)$$

$$\sum_{v\epsilon V}\sum_{\substack{j\epsilon P\cup o_2 \\ i\neq j}} X_{i,j}^{v} = 1 \quad ; \quad \forall i \in P \quad (11)$$

$$\sum_{v`\epsilon V`}\sum_{\substack{m\epsilon D\cup o_4 \\ n\neq m}} Y_{nm}^{v`} = 1 \;\; ; \quad \forall n \in D \quad (12)$$

$$\sum_{v`\epsilon V`}\sum_{\substack{m\epsilon D\cup o_4 \\ n\neq m}} Y_{nm}^{v`} = 1 \;\; ; \quad \forall n \in D \quad (13)$$

$$\sum_{v`\epsilon V`}\sum_{\substack{m\epsilon D\cup o_4 \\ n\neq m}} Y_{nm}^{v`} = 1 \;\; ; \quad \forall n \in D \quad (14)$$

$$\sum_{v`\epsilon V`}\sum_{\substack{m\epsilon D\cup o_4 \\ n\neq m}} Y_{nm}^{v`} = 1 \;\; ; \quad \forall n \in D \quad (15)$$

$$\sum_{v`\epsilon V`}\sum_{\substack{m\epsilon D\cup o_4 \\ n\neq m}} Y_{nm}^{v`} = 1 \;\; ; \quad \forall n \in D \quad (16)$$

$$\sum_{v`\epsilon V`}\sum_{\substack{m\epsilon D\cup o_4 \\ n\neq m}} Y_{nm}^{v`} = 1 \;\; ; \quad \forall n \in D \quad (17)$$

$$\sum_{v`\epsilon V`}\sum_{\substack{m\epsilon D\cup o_4 \\ n\neq m}} Y_{nm}^{v`} = 1 \;\; ; \quad \forall n \in D \quad (18)$$

$$\sum_{v`\epsilon V`}\sum_{\substack{m\epsilon D\cup o_4 \\ n\neq m}} Y_{nm}^{v`} = 1 \;\; ; \quad \forall n \in D \quad (19)$$

$$\sum_{v`\epsilon V`}\sum_{\substack{m\epsilon D\cup o_4 \\ n\neq m}} Y_{nm}^{v`} = 1 \;\; ; \quad \forall n \in D \quad (20)$$

$$\sum_{v`\epsilon V`}\sum_{\substack{m\epsilon D\cup o_4 \\ n\neq m}} Y_{nm}^{v`} = 1 \;\; ; \quad \forall n \in D \quad (21)$$

$$penn1_i^v + W_i^v \leq UTD1_i ; \forall v \in V, \; \forall j \in P \quad (22)$$

$$penp2_n^{v`} + W_n^{v`} \geq LTD2_n ; \forall v` \in V`, \forall n \in D \quad (23)$$

$$penp2_n^{v`} + W_n^{v`} \geq LTD2_n ; \forall v` \in V`, \forall n \in D \quad (24)$$

$$X_{ij}^{v} \in \{0,1\} ; \quad \forall v \in V, i, j\epsilon P \quad (25)$$

$$Y_{nm}^{v`} \in \{0,1\} ; \quad \forall v` \in V`, n, m\epsilon D \quad (26)$$

As seen, the proposed function includes 4 formulas. Formulas 1 and 2 are considered in order to minimize the transportation costs which examine pick-up and delivery processes, respectively. Formulas 3 and 4 are trying to reduce pick-up and delivery times by consideration of penalty costs for deviation from specified lower and upper limits of deadlines considered for pick-up and delivery processes, respectively. Constraints 5 and 6 respectively indicate that any pick-up vehicle should start its route from the cross-dock and should return to the cross-dock at the end. Constraints 7 and 8 indicate that the start of any delivery vehicle is from the cross-dock and it must return to the cross-dock at the end of its route.

Constraints 9 and 10 indicate the flow balance at nodes. Constraint 9 indicates that each pick-up vehicle that enters a pick-up node has to exit from that node and enter the next one. Constraint 10 requires any vehicles that enter the delivery node to exit from it and then enter the next one. Constraint 11 indicates that any node in the pick-up process is only on one route, gets service from one vehicle, and goes to the next node (that can include the warehouse) and there are the same constraints for delivery nodes in constraint 12.

Constraints 13 and 14 are capacity constraints so that, in formula 13, the sum of supplies of a route should not exceed the maximum of capacity dedicated to the pick-up vehicle. Constraint 14 stands for the capacity constraint of the delivery process so that the total demands of one route should not exceed the maximum of capacity dedicated to delivery vehicles.

Formulas 15 and 16 are time constraints. According to formula 15, the arrival time of any vehicle to any node should be more than the sum of arrival time to the previous pick-up node, service time in previous pick-up node, and travel time between these two pick-up Nodes; similarly, formula 16 establishes the same constraints on delivery nodes. In formula 17, it has been expressed that the maximum of pick-up vehicles' arrival times to the cross-dock should be less than starting time of any delivery vehicle from cross-dock, and in formulation 18, it has been shown that the maximum arrival times of pick-up vehicles to the cross-dock should be more than arrival times of all vehicles to the intended dock. Formulas 19 and 20 represent that no vehicle, whether in pick-up or delivery process, does not have any route to that dock. Formula 19 imposes this constraint on pick-up process and formula 20 imposes it on the delivery process. Constraints 21 and 22 indicate that the arrival time to each pick-up node, by taking the corresponding deviations into consideration, should not exceed the upper and lower considered limit for each pick-up node in order for each node to be placed in their own time period. Constraints 23 and 24, which are the same as the two previous constraints but for delivery nodes, require the delivery nodes to lie in their own period. Constraints 25 and 26 specify the binary variables of pick-up vehicles' travel between 2 nodes in pick-up process and the binary variables of delivery vehicles' travel between 2 nodes in the delivery process, respectively.

2-2 problem 2

In this model, problem 1 is considered in a situation that a disruption happens in the route and results in disruption risks. This disruption is the result of a sudden increase in the demand of some customers and the goal of the model is to decrease its negative consequences.

The additional parameters

$Y_{nm}^{v`}$: the obtained value of the solution of model 1 when vehicle v' transports the commodities from nodes n to m in delivery process (includes 0 and 1 values),

w_n : the importance of customer n in delivery process

u_n: the importance of serving customer n in its original route in delivery process

W1 : the importance coefficient of the first objective function

W2 : the importance coefficient of the second objective function

W3 : the importance coefficient of the third objective function

The additional decision variables

It has been assumed that mathematical formulas 1 to 26 have been used to find a set of original routes. In the mathematical modeling, in situations that one or more of vehicle's routes is not feasible, $Y_{n,m}^{v`}$ will not be a decision variable. $Y_{n,m}^{v`}$ is the obtained value of the solution of formulas 1 to 26. In the following, new binary decision variables will be defined:

τ_n: it will be 1 if node n does not serve in the delivery process; otherwise, it will be zero.

$Z_n^{v`}$: it is 1 if node n does not serve in its original route k'; otherwise, it is zero.

$B_{nm}^{v`}$: it will be 1 if vehicle v' goes to node m after serving node n; otherwise, it will be zero.

$$F1 = \sum_{n\epsilon D} w_n \tau_n \quad (27)$$

$$F2 = \sum_{v`\epsilon V`} \sum_{n\epsilon D} u_n Z_n^{v`} \quad (28)$$

$$F3 = [(\sum_{v\epsilon V} \sum_{i\epsilon N} \sum_{j\epsilon N} C_{i,j} X_{i,j}^{v} + \sum_{v`\epsilon V`} \sum_{n\epsilon N`} \sum_{m\epsilon N`} C_{n,m} B_{n,m}^{v`})] \quad (29)$$

$$+[\sum_{v\epsilon V} \sum_{i\epsilon N} \alpha penp1_i^v + \beta penn1_i^v] \quad (30)$$

$$+ [\sum_{v`\epsilon V`} \sum_{n\epsilon N`} (1 - \tau_n) (\alpha penp2_n^{v`} + \beta penn2_n^{v`})] \quad (31)$$

subject to:

$$\sum_{j\epsilon P} X_{o_1,j}^{v} = 1 \;;\quad \forall v \in V \quad (32)$$

$$\sum_{i\epsilon P} X_{i,o_2}^{v} = 1 \;;\quad \forall v \in V \quad (33)$$

$$\sum_{m\epsilon D} B_{o_3,m}^{v`} = 1 \quad \forall v` \in V` \quad (34)$$

$$\sum_{n\epsilon D} B_{n,o_4}^{v`} = 1 \quad \forall v` \in V` \quad (35)$$

$$\sum_{i\epsilon N} X_{i,h}^{v} = \sum_{j\epsilon N} X_{h,j}^{v} \;;\quad \forall v \in V \;, \forall h \in P \quad (36)$$

$$\sum_{n\epsilon N`} B_{n,h}^{v`} = \sum_{m\epsilon N`} B_{h,m}^{v`} \;; \forall v` \in V` \;, \forall h \in D \quad (37)$$

$$\sum_{v\epsilon V} \sum_{\substack{j\epsilon P\cup o_2 \\ i\neq j}} X_{i,j}^{v} = 1 \;;\quad \forall i \in P \quad (38)$$

$$\sum_{v`\epsilon V`} \sum_{\substack{m\epsilon D\cup o_4 \\ n\neq m}} B_{nm}^{v`} + \tau_n = 1 \quad \forall n \in D \quad (39)$$

$$\sum_{v`\epsilon V`} \sum_{\substack{m\epsilon D\cup o_4 \\ n\neq m}} Y_{nm}^{v`} = 1 \;;\quad \forall n \in D \quad (40)$$

$$\sum_{j\epsilon P\cup o_1} \sum_{\substack{i\epsilon P \\ i\neq j}} q_i X_{ji}^{v} \leq Q \;;\quad \forall v \in V \quad (41)$$

$$\sum_{m\epsilon D\cup o_3} \sum_{\substack{n\epsilon D \\ n\neq m}} q_n B_{mn}^{v`} \leq Q` \quad \forall v` \in V` \quad (42)$$

$$W_j^v \geq W_i^v + S_i + t_{ij} - M(1 - X_{ij}^v) \;;\; \forall v \in V, \forall j \in P, \forall i \in P \cup \{o_1\} \quad (43)$$

$$W_m^{v`} \geq W_n^{v`} + S_n + t_{nm} - M(1 - B_{nm}^{v`}) \;;\; \forall v` \in V`, \forall m \in D, \forall n \in D \cup \{o_3\} \quad (44)$$

$$H_i^v \leq W_n^{v`} \;; \forall v \in V, \forall i \in o_2, \forall v` \in V`, \forall n \in o_3 \quad (45)$$

$$H_i^v \geq W_i^v \;;\; \forall v \in V, \forall i \in o_2 \quad (46)$$

$$\sum_{i\epsilon} (X_{nm}^{v`} - B_{nm}^{v`}) \leq Z_{mk} + \tau_{m\,;}\; \forall v` \in V`, m \in D \quad (47)$$

$$X_{ii}^v = 0 \;;\quad \forall i \in P \cup o_1 \cup o_2, \forall v \in V \quad (48)$$

$$B_{nn}^{v`} = 0 \;;\quad \forall n \in D \cup o_3 \cup o_4, \forall v` \in V` \quad (49)$$

$$penp1_i^v + W_i^v \geq LTD1_i;\; \forall v \in V,\; \forall j \in P \quad (50)$$

$$penn1_i^v + W_i^v \leq UTD1_i;\; \forall v \in V, \forall j \in P \quad (51)$$

$$penp2_n^{v`} + W_n^{v`} \geq LTD2_n;\; \forall v` \in V`, \forall n \in D \quad (52)$$

$$penn2_n^{v`} + W_n^{v`} \leq UTD2_n;\; \forall v` \in V`, \forall n \in D \quad (53)$$

$$\tau_n \in \{0,1\} \;;\quad \forall n \in D \quad (54)$$

$$B_{nm}^{v`} \in \{0,1\} \;\forall v` \in V`, n, m\epsilon D \quad (55)$$

$$Z_{mk} \in \{0,1\} \;\forall v` \in V`, m\epsilon D \quad (56)$$

Considering the importance of each customer, in equation 27, we will focus on minimizing the total customers that are not served, and in formula 28, with the consideration of serving each customer in its own route, the objective is to minimize the number of customers not served by their original route.

Formula 29 has been formulated with the aim of minimizing the total costs of transportation of the routes. Formulas 30 and 31 are concerned with the penalties considered as the cost of vehicles' earliness and tardiness in the pick-up and delivery processes that, in formulation 31, regarding the possibility of failure in serving some nodes in the delivery process, these costs are considered only for the serviced nodes. As it can be seen in formulas 27 to 31, the objective function is multi-objective, so in order to achieve the objectives in an integrated function, this research focuses on the use of a compromise programming to formulate it as a single-objective problem. First, considering the non-linearity of the model, linearization is done and then compromise programming to solve multi-objective optimization problem is described in detail and the solving procedure is presented.

With an overview of the constraints, we see that the majority of the constraints are similar to the first proposed model, and they have exactly the same functionality with a difference that in all constraints that $Y_{nm}^{v`}$ had been used, $B_{nm}^{v`}$ variable is replaced with this model, and constraints 39 and 47 have been added according to the problem's requirement. In formulas 54, 55, and 56, the new variables of this proposed model have been defined.

Constraint 39 identifies that if customer $n \in D$ is not served, the value of τ_n is 1 and also, in constraint 47, we illustrate that if customer $m \in D$ is not served in its original route, the value of Z_{mk} is 1.

Constraints 54, 55, and 56 show the binary variables of serving a customer, travel of delivery vehicles between two nodes in the delivery process, and serving a customer in its own route, respectively.

Linearization of the proposed model

Unfortunately, the proposed model is nonlinear and nonlinear models are usually so harder than the linear models to solve in the optimized state. Defining the new collection of variables, we reformulated the model under a mixed integer linear programming model. In this model, we faced with a nonlinear statement which is the multiplication of a continuous variable and a binary variable. In this section, a linearization approach is represented by the proposed model, and the nonlinear statements in the objective function, obtained by multiplication of the two existing statements in formula 29, are linearized by the use of new continuous variables $\mathrm{TPP}_n^{v`}$ and $\mathrm{TPP}_n^{v`}$. Consider a statement like $z = x \times y$ in which x is a binary and y is a continuous variable. This statement can be converted into linear auxiliary

$$z = x \times y \leftrightarrow \begin{cases} z \le y \\ z \le Ux \\ z \ge y - U(1-x) \end{cases} \quad (57)$$

constraints as follows [21]:

In formula 29, we have two nonlinear statements:

$$\tau_n \ \mathrm{penp}2_n^{v`} \quad (58)$$

$$\tau_n \ \mathrm{penn}2_n^{v`} \quad (59)$$

Thus, according to formula 57, in $\tau_n \ \mathrm{penp}2_n^{v`}$, given that $\tau_n$ is a binary and $\mathrm{penp}2_n^{v`}$ is a continuous variable; also in $\tau_n \ \mathrm{penn}2_n^{v`}$, $\tau_n$ is a binary and $\mathrm{penn}2_n^{v`}$ is a continuous variable, they can be converted into a linear set of auxiliary constraints. To write these constraints, we should consider the followings:

- τ_n and $\mathrm{penp}2_n^{v`}$ variables are replaced with new continuous variable $\mathrm{TPP}_n^{v`}$.
- τ_n and $\mathrm{penn}2_n^{v`}$ variables are replaced with new continuous variable $\mathrm{TPN}_n^{v`}$.

Now, we define the auxiliary constraints used for their linearization. The needed constraints on formula 58 are as follows:

$$\mathrm{TPP}_n^{v`} \le \mathrm{penp}2_n^{v`} \quad (60)$$

$$\mathrm{TPP}_n^{v`} \le M\tau_n \quad (61)$$

$$\mathrm{TPP}_n^{v`} \ge \mathrm{penp}2_n^{v`} - M(1-\tau_n) \quad (62)$$

In addition, the needed constraints on formula 59 are as follows:

$$TPN_n^{v`} \le penn2_n^{v`} \quad (63)$$

$$TPN_n^{v`} \le M\tau_n \quad (64)$$

$$TPN_n^{v`} \ge penn2_n^{v`} - M(1-\tau_n) \quad (65)$$

And finally, these constraints will be added to the previous constraints.

The linear model

Finally, the modified and linearized objective function is as follows:

$$F1 = \sum_{n \in D} w_n \tau_n$$

$$F2 = \sum_{n \in D} w_n \tau_n \quad (66)$$

$$F3 = \sum_{v` \in V`} \sum_{n \in D} u_n Z_n^{v`} \left( \sum_{v \in V} \sum_{i \in N} \sum_{j \in N} C_{i,j} X_{i,j}^v + \sum_{v` \in V`} \sum_{n \in N`} \sum_{m \in N`} C_{n,m} B_{n,m}^{v`} \right) + \sum_{v \in V} \sum_{i \in N} \alpha \mathrm{penp}1_i^v + \beta \mathrm{penn}1_i^v$$

$$+\sum_{v`\epsilon V`}\sum_{n\epsilon N`}\left(\alpha penp2_n^{v`}+\beta penn2_n^{v`}\right)-\alpha TPP_n^{v`}-\beta TPN_n^{v`}]$$

As it is evident in formula 66, in comparison with the formulas 27-31, formulations 27, 28, 30, and 31 remain unchanged; only formula 29 has been changed, and consequently, the final constraints are obtained by the addition of formulas 60-65 to formulas 32-56.

3. Computational Results

Generally, multi-objective optimization problems can be solved by different concepts and, to do this, the given model is converted into a single-objective model. In this problem, objective function goals are unavailable and, in some cases, acquiring the objective function goals is impossible. Thus, in this paper, one of the methods without the information of decision-maker will be used for multi-objective optimization problem (the set of formulas 32-56, 60-65, 66). According to this method, customers that have not been served and customers that have not been served in their original route will get value without the inclusion of the decision-maker. Hence, to solve the stated problem and multi-objective optimization, compromise programming method (a kind of LP-metric method) is used. In this method, to derive the final model and with the purpose of normalization, the calculations of ideal and anti-ideal values of each function are necessary. To obtain these values, given that the minimization of the objectives is considered in this problem, the optimal value of each objective function (ideal values) will be calculated in the case of minimization, and then, with maximization of each function, the optimal values in the case of maximization (anti-ideal values) are also obtained. The final model of compromise programming, in the case of importance weights, follows a single function that fits the terms of this problem as follows:

$$Max\ W1\left(\frac{f_{21}-f_{21}^{+}}{f_{21}^{-}-f_{21}^{+}}\right)+W2\left(\frac{f_{22}-f_{22}^{+}}{f_{22}^{-}-f_{22}^{+}}\right)+W3\left(\frac{f_{23}-f_{23}^{+}}{f_{23}^{-}-f_{23}^{+}}\right) \qquad (67)$$

3-1. Case study

To evaluate the performance of this model, the information of some activities of Nowshahr's shipping and ports organization is used. In this study, we have three suppliers that are Bahonar, Rajaee, and Emam khomeini ports. The considered warehouse as the cross-dock is placed in Salmanshahr and the predetermined customers, which are the sale representatives of products that come from south, consist of six customers placed in Nowshahr, Chalus, Kelarabad, Tonekabon, Ramsar, and Sari, respectively.

In this case, each port and representative is considered as a node. The supplier nodes are nodes 1, 2, and 3 according to the order mentioned above and customer nodes are defined as nodes 4, 5, 6, 7, 8, and 9 based on the above-mentioned order. With regard to the problem constraints, each node is only served by one vehicle and also a time window has been considered for it that it has to be served in that time limit. The problem data are as follows: the number of supply in suppliers 1, 2 and, 3 is 30, 30, and 60 boxes (each box contains 6 products), respectively, and the amounts of demand for customers 4, 5, 6, 7, 8 and 9 are 10, 22, 9, 26, 26 and 27 boxes, respectively. The service time at each supplier with the mentioned order is 120, 120, and 240 minutes, and for each customer with the mentioned order is 5, 10, 4, 12, 12, and 13 minutes, respectively. The values of other parameters are presented in Tables 1 and 2.

Tab. 1. values of travel time from nodes i to j

	CD[2]	1	2	3
CD	-	2040	2400	1800
1	2040	-	300	90
2	2400	300	-	132
3	1800	90	132	-

Tab. 2. values of travel time from nodes n to m

	CD	4	5	6	7	8	9
CD	-	22	15	5	25	50	195
4	22	-	10	22	60	90	170
5	15	10	-	15	55	85	185
6	5	22	15	-	45	70	190
7	25	60	55	45	-	25	234
8	50	90	85	70	25	-	260
9	195	170	185	190	234	260	-

3-2. Result of multi-objective optimization problem

In this paper, we have two scenarios. According to the first scenario, the utilized data in this organization are entered in the first proposed model and the model is run in the Gams Software 1/1/24, in a corei5 personal computer in 10.866 sec. The first scenario is considered in the case of meeting all demands and the results of software output are shown in Tables 3 and 4.

[2] Cross-Dock (CD)

Tab. 3. the obtained values of parameter X

x	1	2	3	11
1,3				1
1,10			1	
2,1		1		
2,2				1
2,10	1			

Tab. 4. the obtained values of parameter Y

Y	4	5	6	7	8	9	13
1,7					1		
1,8							1
1,12				1			
2,4						1	
2,5	1						
2,6							1
2,9			1				
2,12		1					

The obtained values of parameter X are presented in Table 3 that shows the routes of the pick-up process.

The obtained values of parameter Y are have been presented Table 4 shows the routes of the delivery process. The output flow of the model that is considered without disruption is presented in Figure 1.

In the second scenario, which is considered to reply to the disruption occurred due to a 10-time increase in demand of some nodes, the optimal solution has been presented. The present scenario shows that in the case of a 10-time increase in demand of the delivery route, the states of the routes are shown in Table 5. The output flow of the model under disruption is presented in Figure 2.

According to the obtained solutions to the mentioned model, the amount of τ for nodes 4, 7, 8, and 9 is equal to 1, and this means that these nodes are deprived of demand delivery. Regarding the problem scenario, since demands of nodes 7, 8 and 9 are all increased and because of the capacity limitation of the route, it is not possible for the vehicle to cover the demands of these nodes.

The preliminary results of outputs show that with an increase of the node demands, a large number of the nodes lose their efficiency.

The reason for this loss of demand is the shortage of vehicle's capacity.

On the other hand, since it is possible in the second scenario that some nodes will not be visited with accepting a penalty cost, it is possible that some other nodes will not be visited due to the high cost of movement. This fact does somehow the cost-benefit analysis of the nodes or, in other words, the balance between the cost of penalty and the cost of movement between nodes. This means that if the cost of movement is more than the penalty of not covering the demand node, it is deleted. In these outputs, node 4 is deleted. According to the applied method, all customers not served at all and also all customers not served by their own routes get values without the inclusion of the decision-maker. Hence, to solve the mentioned problem and multi-objective optimization, compromise programming method is used; as it was stated, to derive the final model and with the purpose of normalization, the calculation of ideal and anti-ideal values of each function is needed which are reported in Table 6.

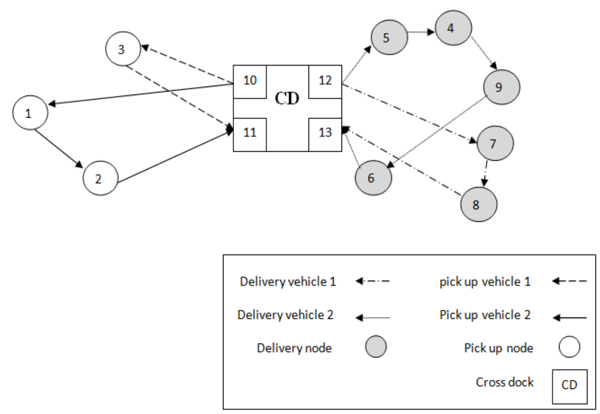

Fig. 1. The representation of the output flow under the normal conditions

Tab.5. The obtained values of parameter Y with disruption

Y	5	6	13
1,6			1
1,12		1	
2,5			1
2,12	1		

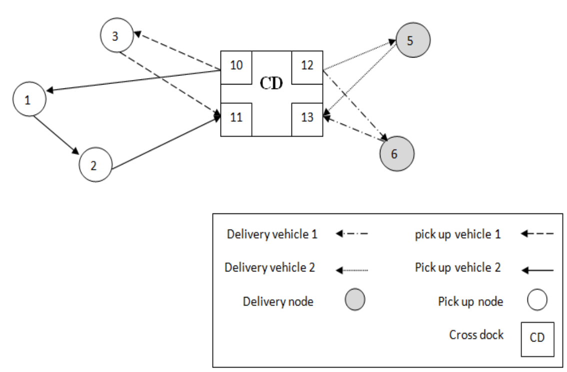

Fig. 2. The representation of the output flow obtained from the first model under the disruption

Tab. 6. ideal and anti-ideal values for the objective function of problem 2

	Ideal values	anti-ideal values
$z1$	0	2.798
$z2$	0	3.733
$z3$	1.587E+7	2.025E+7

The ideal and anti-ideal values were calculated as mentioned before. Since the objective function is a minimization function, anti-ideal values for all of them are greater than their ideal values.
Then, by forming the final function obtained from substation of the values in formula 67 and by changing the weights of each normalized functions, the Pareto optimal solutions are obtained.
As mentioned, by changing of w1, w2, and w3, we can have a range of Pareto optimal solutions. In this case study, six cases have been investigated, and the values of ideal and anti-ideal objective functions of Pareto optimal solutions are represented in Table 7. When a critical incident occurs, demand of some or all nodes increases because of the crisis and this issue results in problems such as setting priorities for the damaged and healthy nodes; in this regard, problems like uncovered demand, vehicles' capacity, the penalty of each uncovered unit, the transportation costs and time windows arise. In fact, in this research, a framework was developed for decision making in these situations to consider somehow all of the abovementioned problems.

4. Conclusion and Future Research

The aim of these problems is to concurrently design a vehicle routing scheduling model with cross-docking by considering risk.
In this research, the vehicle routing and scheduling models have been simultaneously considered under a new distribution center, named cross-dock, which is different from traditional warehousing, so that we can reduce the costs. Besides, the real world conditions, including the constraint of the number/capacity of the fleet with consideration of time window, are considered in the model. To make the cross-docking model more developed and more applicable under crisis conditions that are from the most challenging problems in all fields, especially industrial systems, a part of re-routing system has been exposed to a disruption. To better explanation,

Tab. 7. values of ideal and anti-ideal objective function of Pareto optimal solutions

	W1	W2	W3	z1	z2	z3
1	0.33	0.33	0.33	2.1	1.03	1.58E+07
2	0	0.5	0.5	4.31	1.02	1.58E+07
3	0.5	0	0.5	0	0	1.61E_07
4	0.5	0.5	0	2.1	4.1	1.59E+07
5	0.5	0.2	0.3	2.1	1.03	1.58E+07
6	0.8	0.1	0.1	2.1	1.02	1.58E+07

In this paper, a model was developed for collecting and delivering demands by vehicles with limited capacity in the case risk conditions arise due to a disruption. The proposed model was solved by the use of a compromise programming process. In fact, for collecting and delivering the commodities for a center, there is always a basic route in which the vehicles start from the cross-dock and pass through some specified nodes, and finally return to the depot. In fact, in this research, a framework was developed for decision making in the case of a crisis that considers all important issues. In this research, after developing a mathematical model and a conceptual framework for modeling the mentioned conditions, the model was investigated in a case study. The case study was comprised of three suppliers (that played the role of pick-up nodes) and six nodes that were demand centers. The problem results were first represented in the case of two-level programming, and then the model was developed as a multi-objective optimization model and its results were explained.
Since the simultaneous vehicle routing and scheduling problems are NP-hard ones [10], efficient heuristic and meta-heuristic methods are suggested. Since one of the important gaps of developing the existing work is not taking into account uncertainty in programming, this issue is suggested for one of the future studies. The accuracy and precision of many data are suspicious in reality and this issue is more critical for programming in the case of crisis, so investigation in this area is recommended. Among other issues in crisis management is the redundancy in delivery that is one of the important and necessary issues. Since, in many cases, it is not possible to deliver demands at once and fully due to the capacity limitation and high volume of demand, partial delivery can be used in problem assumptions.

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