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Dynamic Inventory Planning with Unknown Costs and Stochastic Demand

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KEYWORDS

Dynamic Re-Order Policy,
Stochastic Demand,
Non-Cost Based Inventory Models,
Order Quantity,
Inventory Confidence Level

ABSTRACT

Generally, ordering policies are done by two methods: fix order quantity (FOQ) and fix order period (FOP). The mentioned methods are static. It means that the quantity of ordering or the procedure of ordering is fixed throughout time horizon, while in real environments, demand may vary in any period, i.e., Dynamic; it may be considered as uncertainty. When demand is variable in any period, the traditional static ordering policies with fixed re-order points cannot be efficient. On the other hand, sometimes, in real environments, some costs such as holding cost and ordering cost may not be well-known or precise. Therefore, using the cost-based inventory models may not be helpful. In this paper, a model is developed which can be used in the cases of stochastic and irregular demands, also unknown as costs. Indeed, in these cases, re-order point can be dynamic. Also, some attributes consisting of expected positive inventory level, expected negative inventory level, and inventory confidence level are considered as objective functions, instead of the objective function of total inventory cost. A numerical example is also presented for more explanation.

1. Introduction

Estimating the demand of customer is important for managing company processes and production planning. In irregular demands, forecasting is more difficult and the definition of optimal re-order policies can be a significant management problem. Irregular demands may appear in various production contexts; for example, in stochastic demands, start-up production, multi-echelon supply chains, and so on. In the situation of irregular demands, the models such as EOQ and EPQ cannot be very useful. If the cost of inventories is not accessible, then the cost-based inventory models may not also be applicable. In this paper, a model is proposed that can be helpful in aforementioned situations. In the classic inventory models, there is an objective function in terms of total inventory cost. So, all models intend to find economic order quantity (EOQ) based on minimum total inventory cost. In this article, three objective functions are used instead of total inventory cost. The used objective functions are expected positive inventory level

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(f'1), expected negative inventory level (f"1), and inventory confidence level (f2). The goal is minimization of both f'1 and f"1, and also maximization of f2. Of course, f'1 and f"1 are complementary to each other. So, an objective function as expected inventory level (f1 = f'1+f"1) is considered. Therefore, two objective functions are assigned.

The main problem in production planning is the answer to two questions: when and how much we order. The literature is full of re-order policy solutions; some based on mathematical models, others based on heuristic or meta-heuristic models [1- 3]. In some research studies, demand is deterministic; in others'; demand is assumed as stochastic [4]. Also, the production systems can be considered as Make to Stock (MTS) or Engineering To Order (ETO). In the system of MTS, demand is uncertain, while demand in ETO system is deterministic [5].

Syntetos et al. reported a classification of irregular and sporadic demands [6]. They assumed that the distribution of inter-demand intervals and demand sizes are geometric and normal, respectively. Two main issues can be identified in irregular demands of pattern management. The first approach is the application of forecasting techniques; the second approach is the evaluation of different inventory control models in irregular demands [7]. Nenes et al. discussed the stochastic inventory control in the case of irregular demands [8]. Some case studies about inventory control systems and modelling of irregular demand patterns can be seen in some studies [9-17]. Since the irregular demand phenomenon is made up of two constituent elements, which are demand intermittence and demand variability, a compound demand distribution is often suggested. Compound Poisson demand processes have been more often preferred due to their theoretical advantages and simplicity. Feeny et al. used one of the early contributions on the application of this compound demand distribution in the inventory control [18]. Also, other papers worked in these fields [19-31]. Gamberini et al. proposed an approach for dynamic re-order policy about irregular demand [5].

Nevertheless, the aforementioned contributions propose cost-based approaches, disregarding the fact that cost structures are not always known. For example, in new companies or producing new items, costs may not be well known. Moreover, holding, ordering, and shortage costs, which often have effect on the best re-order policy and inventory management approach, are hardly quantifiable in a wide range of operative environments. Unitary holding costs are obtained by dividing annual holding costs among stocked items. Stocked items are strictly connected with the adopted re-order and inventory management approach. Hence, rather than the fixed value of the unitary holding costs registered in the past, approaches for selecting the best re-order policy and inventory management approach should adopt functions that describe the trend of unitary holding costs in accordance with the amount of stocked items registered. Otherwise, when a fixed value of the unitary holding cost registered in the past is adopted, a simplified model of the reality is implemented. Furthermore, shortage costs are related to lost demands or customer dissatisfaction, which are hardly quantifiable in an economic parameter [5]. In this paper, three non-cost based attributes are applied such as expected positive inventory level, expected negative inventory level, and inventory confidence level. Of course, the sum of expected positive inventory level and expected negative inventory level are considered as an objective function named as expected inventory level.

In classical models, generally all parameters and decision variables are static and constant in each period, while in real cases, the conditions of any period may be different from other periods. Hence, it is better for some parameters or variables, such as re-order point, order quantity, and etc., to be considered dynamically. In this paper, an approach is described for solving this problem. A bi-objective programming model is designed. The parameter of demand is stochastic and a discrete event simulation approach is applied to solve the above-mentioned model. Also, a numerical example is presented for explaining algorithm and its validation.

2. The Proposed Model

2-1. Parameters

Some used parameters are as follows:

ICL inventory confidence level, probability that demand (consumption) is less than re-order point in a period

x demand during a time period, which can be random

f(x) probability density function of x

n number of time periods in the planning horizon

I_i inventory level at the end of period i, for i = 0,…,n

$\bar{I}_i^+$ expected positive inventory level at the end of each time period

$\bar{I}_i^-$ expected negative inventory level at the end of each time period

$\bar{I}_i$ expected inventory level (sum of positive and negative inventories) at the end of each time period

LT lead time

SS safety stock

2-2. Decision variables

ROP_i re-order point in time period i, for i = 0,…,n

Q_i lot size (order quantity) in time period i, for i = 0,…,n

zi a tolerance that is added to ROPi to compensate for uncertainty in time period i, for i = 0,…,n

2-3. Assumptions

Some of the considered assumptions are as follows:

- Inventory at the beginning of planning horizon is zero.
- Inventory costs such as holding cost, ordering cost, shortage cost, and etc. are unknown.
- Lead time rather than time periods is small and negligible, so it can be assumed equal to zero. Any order will be received in the same time period.
- The probability density function *f(x)* is constant for each time period. In this paper, Uniform distribution is applied.
- The re-order point may change in any time period, but its procedure is constant for all time periods.

2-4. Proposed model

Two main kinds of static multi-period re-order policies are used in the standard inventory management: (ROP, Q) and (T, Q_{max}). In the (*ROP, Q*) policy, the fixed quantity *Q* is ordered whenever the inventory level drops to the re-order point *ROP* or below (see Fig.1)). Thus, this basic policy involves a continuous review of the inventory level. Instead, the control procedure in the (*T*, Q_{max}) policy is such that for every *T* units of time a variable quantity Q_i is ordered to raise the inventory position to level Q_{max} (see Fig. 2). In this policy, review of the inventory level is interval [26].

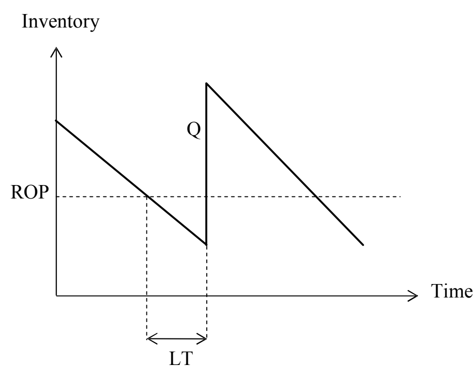

Fig. 1. (ROP, Q) policy

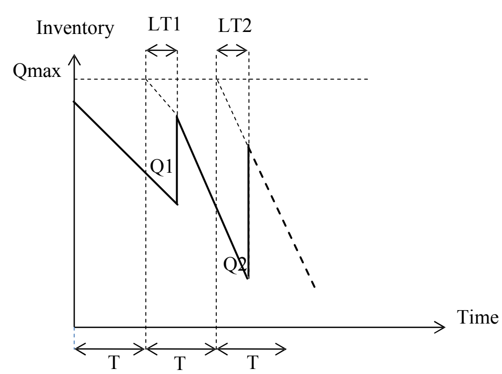

Fig. 2. (T, Q_{max}) policy

This paper refers to dynamic lot-sizing policies that belong to the (ROP_i, Q_i) approach with a periodic review of the inventory level (see Fig. 3). The quantity Q_{i+1} is ordered whenever the inventory level I_i drops to the re-order point ROP_i at the end of each time period *i* (periodic review), with Q_{i+1} and ROP_i depending on the specific time period *i* (dynamic policy). Since lead time is zero, the re-order quantity Q_{i+1} could be considered available at the beginning of time period $i+1$.

Note that the order of Q_{i+1} units is placed at the end of time period *i* because $I_i \leq ROP_i$.

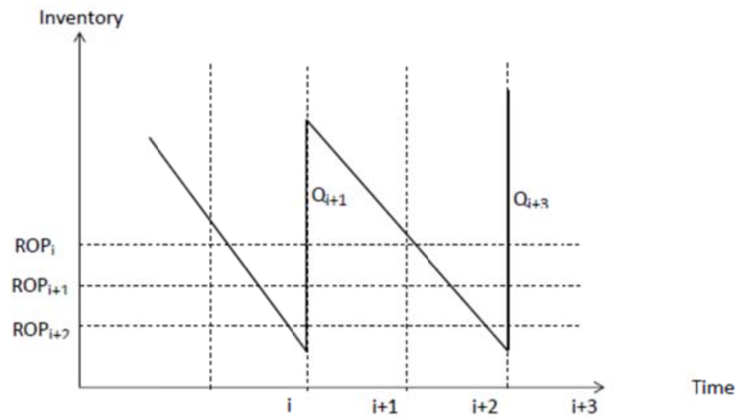

Fig. 3. (ROP_i, Q_i) policy

This paper deals with a stochastic dynamic lot-sizing problem that is non-cost based and a maximal inventory confidence level criterion, that is used as the probability that at the end of every period, the net inventory will not be negative. The demand *x* is considered as a random variable with a known probability density function *f(x)* that may vary from a period to another. Of course, in this study, f(x) is considered as Uniform probability density function and is fixed in each period.

The proposed model can be expressed as the minimization of two expected inventory levels (positive and negative) and maximization inventory confidence level with several constrains as follows:

$$Min \quad f'_1 = E(I^+) = \frac{1}{n}\sum_{i=0}^{n}\int_{0}^{I_i} I_i . f(x_i)\,dx = \frac{1}{n}\sum_{i=0}^{n}\int_{0}^{I_i} (I_{i-1} + \delta_{i-1}.Q_i - x_i).\, f(x_i)\,dx \qquad (1)$$

$$Min \quad f''_1 = E(I^-) = \frac{1}{n}\sum_{i=0}^{n}\int_{Ii}^{+\infty} |I_i| . f(x_i)\,dx = \frac{1}{n}\sum_{i=0}^{n}\int_{Ii}^{+\infty} |(I_{i-1} + \delta_{i-1}.Q_i - x_i)| . f(x_i)\,dx \qquad (2)$$

$$Max \quad f_2 = E(ICL) = \frac{1}{n}\sum_{i=0}^{n}\int_{-\infty}^{ROP_i} f(x_i)\,dx \qquad (3)$$

$$s.t: \quad I_i = I_{i-1} + \delta_{i-1}.Q_i - x_i \qquad \forall i = 0,\cdots,n \qquad (4)$$

$$ROP_i = x_i + ss + z_i \qquad \forall i = 0,\cdots,n \qquad (5)$$

$$\delta_i = \begin{cases} 1 & I_i \le ROP_i \\ 0 & Otherwise \end{cases} \qquad \forall i = 0,\cdots,n \qquad (6)$$

$$Q_i, z_i \ge 0 \qquad (7)$$

Since f'$_1$ and f''$_2$ show inventory levels against each other, and f'$_1$ advises the minimization of ROP and f''$_2$ advises the maximization of ROP, so they may not be helpful singly and is better to be combined as an objective function. Also, because f_2 is always advised for the maximization of ROP, hence its solution is known approximately, and can optionally be removed from the list of objective functions. However, the above model may be rewritten as follows:

$$\begin{aligned} &Min \quad f_1 = E(I^+) + E(I^-) = \\ &\frac{1}{n}\left(\sum_{i=0}^{n}\int_{0}^{I_i} I_i . f(x_i)\,dx + \sum_{i=0}^{n}\int_{Ii}^{+\infty} |I_i| . f(x_i)\,dx\right) \\ &Max \quad f_2 = E(ICL) = \frac{1}{n}\sum_{i=0}^{n}\int_{-\infty}^{ROP_i} f(x_i)\,dx \\ &s.t: \quad I_i = I_{i-1} + \delta_{i-1}.Q_i - x_i \qquad \forall i = 0,\cdots,n \\ &\qquad ROP_i = x_i + ss + z_i \qquad \forall i = 0,\cdots,n \\ &\qquad \delta_i = \begin{cases} 1 & I_i \le ROP_i \\ 0 & Otherwise \end{cases} \qquad \forall i = 0,\cdots,n \\ &\qquad Q_i, z_i \ge 0 \end{aligned} \qquad (8)$$

Integral in Eq. (1) represents expected positive inventory level at the end of time period i, and sigma calculates the sum of expected positive inventory levels throughout planning horizon. Eq. (2) is similar to Eq. (1) with the exception that an absolute function is used for negative inventory levels. Integral in Eq. (3) obtains inventory confidence level for each period i and sigma calculates the sum of these confidence levels. Constraint (4) shows how to calculate inventory levels in each period in terms of the inventory of past period, demand, and order quantity in the same period. Constraint (5) represents the re-order point in the (*ROPi, Qi*) policy (according to Fig. 3); the dynamic quantity zi as a decision variable is added to ROPi as a precautionary size against the high variability and uncertainty of the demand. Constraint (6) defines a binary variable which can be useful in the model and depends on the replenishment order placed in time period i. Constraint (7) shows that Qi and zi are decision variables and cannot be negative [32-34].

Thus, the stochastic dynamic lot-sizing problem formulated above can be used to evaluate the two dynamic decision variables Qi and zi for each time period i, supposing that the probability density function *f(x)* is known. In fact, all the achieved results depend on the applied probability density function *f(x)*, which affects the calculation of the sizes necessary for the application of the re-order policies.

2-5. Structure of model solving

There are two main problems in model's solution. First, since demand is stochastic and model is a bi-objective programming, solving the aforementioned model is hard directly. Next problem is that due to discrete planning horizon, the objective functions have also been defined discretely by sigma. Solving these models is directly difficult too. Therefore, the discrete event simulation approach is applied in this paper. For this goal, many data are generated for x, f(x), and model is analyzed for the different values of decision variables.

The flowchart of figure (4) is recommended for completing process.

3. Numerical Example

Assume that a manufacturing company would like to perform an inventory planning during 20 time period (weekly) as planning horizon. Safety stock is equal to 50. Demand's random variable follows Uniform probability density function between 100 and 500 (x~ U[100, 500]). The value of inventory in the beginning of planning horizon is zero. The goal is minimization of the expected inventory level (f_1) and maximization of the expected inventory confidence level (f_2).

Since demand is stochastic, the simulation model was run 30 times for 20 time period, and in each period, for 400 different values of Q_i and 50 different values of z_i. A sample of obtained efficient solution is seen in Table (1).

Tab. 1. A sample of efficient solution

i	Q	ROP	ICL		
1	341	434	0.835	Min f_1 =	100.55
2	341	365	0.6625		
3	341	383	0.7075	Max f_2 =	0.769875
4	341	267	0.4175		
5	341	272	0.43		
6	341	462	0.905		
7	341	157	0.1425		
8	341	456	0.89		
9	341	478	0.945		
10	341	435	0.8375		
11	341	169	0.1725		
12	341	317	0.5425		
13	341	497	0.9925		
14	341	468	0.92		
15	341	301	0.5025		
16	341	537	1		
17	341	500	1		
18	341	174	0.185		
19	341	532	1		
20	341	297	0.4925		

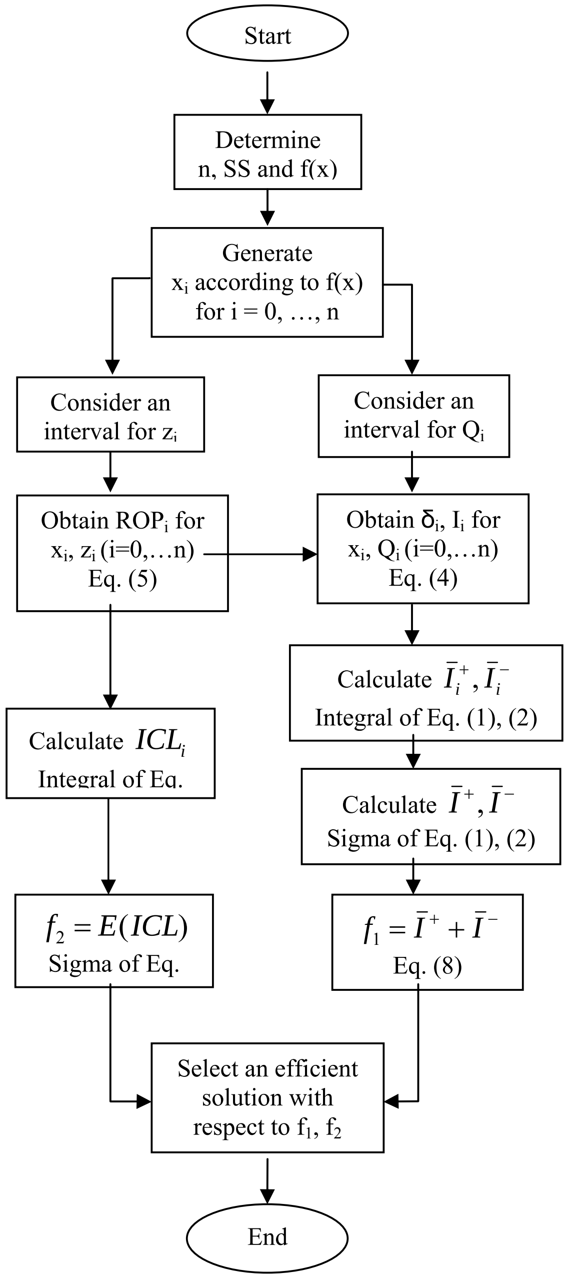

Fig. 4. Structure of Model's Solution

Similarly, the above model was run 30 times and 30 efficient solutions were obtained (see Table 2):

Tab. 2. Efficient solutions for 30 times running

Run	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Min f_1	100.55	94.8	102.4	84.15	103.2	75.3	95.25	95.2	108.55	88.6	69.05	94.2	106.25	113.4	134.85
Max f_2	0.770	0.778	0.627	0.700	0.710	0.701	0.644	0.623	0.690	0.753	0.720	0.705	0.690	0.732	0.701
Run	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Min f_1	115.7	71.55	103.1	88	135.3	122.25	120.3	144.9	92.2	79.3	115.5	86.4	98.7	94.75	91.3
Max f_2	0.766	0.718	0.715	0.765	0.726	0.644	0.704	0.744	0.684	0.753	0.683	0.693	0.754	0.735	0.751

In this stage, inventory manager as decision-maker (DM) can select one of the presented efficient solutions. But, in this paper, TOPSIS method, which is one of suitable methods in multi-attribute decision-making (MADM), is used to select the best solution [35].

By using TOPSIS method, the best efficient solution is obtained as Table (3):

Tab. 3. Best efficient solution obtained by TOPSIS method

i	Q	ROP	ICL		
1	303	350	0.625	Min f_1 =	69.05
2	303	317	0.5425		
3	303	430	0.825	Max f_2 =	0.720375
4	303	223	0.3075		
5	303	321	0.5525		
6	303	369	0.6725		
7	303	478	0.945		
8	303	368	0.67		
9	303	322	0.555		
10	303	355	0.6375		
11	303	242	0.355		
12	303	399	0.7475		
13	303	347	0.6175		
14	303	424	0.81		
15	303	507	1		
16	303	300	0.5		
17	303	272	0.43		
18	303	268	0.42		
19	303	211	0.2775		
20	303	363	0.6575		

The results of Table (3) show that economic order quantity in each period is 303 units constantly, while the value of re-order point is different in each period dynamically. According to ROP, the confidence level of inventory may also vary from 27.8% to 100%, but, its expected value during all over of planning horizon is 72%, while expected inventory level is 69 units. It means that if Q and ROP are selected according to Table (3) in each time period, the company could obtain the best values for inventory and confidence levels.

4. Discussions and Conclusions

Assume that the above example is considered as static; for instance, in Table (3), Q = 303; x has Uniform probability density function between 100 and 500; ROP is taken as the average of ROP's in all over time periods, i.e., ROP = 343.3, then expected inventory level is equal to 103.12 and expected inventory confidence level is equal to 0.60825. It is clear that the obtained solution is not better than the solution of Table (3) and is dominated. In other words, considering the problem dynamically can reduce the expected inventory level and raise the expected inventory confidence level.

In this paper, a model is proposed which can be used when the cost estimation is impossible or hard. Also, the proposed model can be helpful when demand is irregular and variable in each time period. A bi-objective model was presented and the structure of model solving was explained. Also a numerical example was described and its validation by a numerical instant was tested.

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