



Designing a Bi-objective Capacitated Single-Allocation Incomplete Hub Network Considering an Elastic Demand

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ABSTRACT

This paper presents a bi-objective capacitated hub arc location problem with single assignment for designing a metro network with an elastic demand. In the literature, it is widely supposed that the network created with the hub nodes is complete. In this paper, this assumption is relaxed. In real life problems, especially for locating a metro hub, the demand is dependent on the utility that is offered by each hub; hence, the demand is assumed to be elastic in this paper. The presented model also has the ability to compute the number of trains between each pair of two hubs. The objectives of this model are to maximize the benefits of transportation and establishment of the hub facilities while minimizing the total transportation time. Furthermore, the bi-objective model is converted into a single objective one by the TH method. The significance of applicability of the developed model is demonstrated by a number of numerical experiments and some sensitivity analyses on the data taken from the Qom city monorail project.

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1. Introduction

Public transportation systems are indispensable components of an urban transportation system. Hence, the right decision about developing an efficient public transportation system should be taken into account. One of the important parts of public transportation systems is a subway that can transport passengers to each point of the city

in a short time. In a metro network, it is impossible to design a complete hub network due to the high cost of constructing subway lines. So, due to limitation in budget, the number of lines should be known in advance. Therefore, designing a metro hub network is different from other hub location problems and has more complexity. In this paper, a hub arc location is proposed as adjusted with properties of designing a metro hub network. Moreover, in real life, the demand is not static and will be changed according to the location of hub facilities. The

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elasticity of the demand depends on the utility proposed by each hub. In this paper, it is assumed that the utility of hub nodes is additive for simplicity. Utility of the nodes is based on the attraction of the facility constructed on that node and a decaying function of distance.

In hub-and-spoke models, the flow between every two origins and destinations passes through some intermediate nodes and links, named hub nodes and hub links. In this problem, when hub nodes are selected, non-hub nodes, called spoke nodes, are allocated to the hub nodes to transport their flow through the hub sub-network. In these types of models, the goal is majorly to minimize the total cost, maximize the utilization, and maximize the service level. Such structures prevent direct connections between spoke nodes causing elimination of non-efficient direct links. Therefore, economies of scale can benefit from centralizing the flow through more efficient vehicles in hub links.

In a public transportation network, fast lines are assumed to be hub links, and regular bus lines are spoke links. As direct links between all fast lines are not practically feasible, a hub network is not a complete one in this model. It is also assumed that when passengers enter the hub network through a spoke link, they tend to use the fast lines all along to the hubs that are linked to their destinations. Here, fast lines include subways.

2. Literature Review

The hub location problem was first introduced by O'Kelly [1]. Then, he proposed the mathematical formulation of hub-and-spoke networks as a quadratic integer programming model for airline passenger networks [2]. The first linear integer programming for a p -hub median problem was proposed by Campbell [3]. Hub location problems can be classified into four major types including: p -hub median problem, p -hub center problem, hub covering location problem, and capacitated and un-capacitated hub location problem. In a hub location problem (HLP), the objective is to minimize the total cost of locating hubs and transportation of flow through the hub network. If the capacities of hubs are limited, the problem is called a capacitated hub location problem; if a number of hubs are known in advance, the problem is a p -hub median problem. In the p -hub center problem (p -HCP), the objective is to find the optimal location of p hubs and the allocation of non-hub nodes to the hubs and to minimize the longest path in the network. The hub covering location problem (HCLP) contains cover constraints, which limit the

number of non-hub nodes that can be allocated to each hub [4]. The allocation of non-hub nodes to hub nodes can be of single or multiple allocations. In a single allocation, each non-hub node is allocated to exactly one hub, while such restriction is relaxed in a multiple allocation. The recent papers, relevant to the characteristics of the hub model presented in this paper, are categorized in four classes, namely incomplete hub networks, hub location problems with elastic demand, capacitated hub location problems, and hub location problem considering a service level.

2-1. Incomplete hub networks

One of the main assumptions of a typical hub location problem is that the hub network is complete, while this is not a realistic assumption in real life, especially in public transportation networks. However, a few studies have been conducted on incomplete hub networks. Karimi and Setak [4] categorized an incomplete hub network topology into four classes as follows: tree, ring, special and general shape. In the tree topology, there is just one way to transport between each two hubs, and there is no cycle in the network. Contreras et al. [5] proposed a hub location problem with a single assignment, in which the hub network is connected by means of a tree. Xu et al. [6] developed a tabu search heuristic to solve a tree-star problem in a telecommunications network. Martins de Sá et al. [7] presented a tree of a hub location problem and solved it by a Benders decomposition approach. Their method was an extension of the work of Papadakos [8]. In the ring topology, each hub node only links with two hubs, so there is only one cycle in the hub network. There are few studies that used this structure. Chiu et al. [9] and Wang et al. [10] presented this structure in hub network problems. In the special topology, a network structure is designed according to its special case by the decision-makers in such a way that makes it distinguishable from the tree and ring structures. Alumur et al. [11] presented a hub network with a special form in which a spanning tree passes through all the hubs. In a general network topology, the form of the hub network is defined by the model, and any structure (e.g., a tree, ring or completed structure) can be created. Yoon et al. [12] proposed a model, whose hub network has a general topology. Another hub location problem, which can make an incomplete hub network, is a hub arc location problem. In this problem, the number of hubs is known in advance. The model first locates the hub nodes, and then by considering the number

of hub links creates the hub network structure. The hub network can be connected or not. This kind of hub problem is used when the construction costs of hub links are high and the decision-maker wants to build a limited number of hub links. Campbell et al. [13, 14] presented the hub arc location problems which locate hub arcs with reduced unit costs. Sasaki et al. [15] proposed a hub arc location with competition in which two firms compete for customers. In their model, the hub network is not necessarily connected. Alumur et al. [11] presented two incomplete hub networks in which the first model is a hub arc location and the hub network is connected.

2-2. Hub location problem with a service level

Although there is a multi-criteria nature in real life cases in hub network design decisions, the hub location literature mainly focuses on single objective problems. However, for a realistic setting, service quality should be considered in addition to transportation costs. Campbell [16] considered a service level for multiple allocation p -hub median problems and hub arc location problems. The model proposed in his paper is a single-objective model subject to service level constraints. Yaman [17] proposed a three-level hub network for a cargo delivery company. The problem was modeled as a single objective model minimizing transportation costs while service level considerations were taken into account through delivery time constraints. Alumur et al. [18] proposed a hierarchical multimodal hub location problem with time definite delivery guarantee. They considered two types of hubs (i.e., ground and air hubs). Sedehzadeh and Tavakkoli-Moghaddam [19] presented a multi-objective tree p -hub median location problem with some fuzzy parameters. In their paper, the objectives are to minimize the total cost and the total time simultaneously. Mohammadi et al. [20] proposed a multi-objective p -hub covering location problem that minimizes the total cost and the maximum travel time between each two nodes simultaneously. The transportation time between each two nodes is assumed to be an uncertain parameter.

2-3. Capacitated hub location problems

Hubs are often large facilities which need several strategic decisions in addition to location decisions. One of these important decisions is the capacity that each hub can have. Correia et al. [21] revised formulation for the capacitated single-allocation hub location problem. They

showed that for some instances, the classical formulation is incomplete. They proposed a new formulation that had a better performance. Correia [22] extended a classical capacitated single-allocation hub location problem for the first time, in which the capacity of each hub is decided by solving the model. In the model, there is a set of capacities for each potential hub, and only one of them can be selected as the hub capacity. Then, Correia et al. [23] extended the previous work by adding balancing requirements to their model.

2-4. Hub location problems with an elastic demand

In most hub location problems, the demand is assumed to be static while this is not a realistic assumption. In real life cases, the demand is dependent on the utility that is proposed by the hub facilities, especially for unessential goods. It means that the amount of the demand assigned to each hub depends on the location of hub nodes (which is not predefined in advance) and the attractiveness of them. For the first time, Redondo et al. [24] explored the effects of variable demand in location problems. They showed that fixed or variable assumptions in location problems affect the decision very much. As the assumption of the variable demand increases the complexity of the problem, the selection of the demand type (i.e., fixed or variable) must be made carefully in modeling location problems. Khosravi et al. [25] considered an elastic demand in the classical hub location problem. To the best of our knowledge, this paper is the only one in the literature, assuming that the demand depends on the location of hub facilities.

2-5. Hub location in public transportation

For the first time, Nickel et al. [26] proposed new mathematical models for an application of HLPs to an urban public transportation network for the first time. They relaxed some classical assumptions of HLPs to customize their models for public transport systems. Their model's network was also a general incomplete network. Afterwards, Gelareh and Nickel [27] proposed a new model for an uncapacitated multiple-allocation hub location problem for urban transportation and liner shipping network design. Their model was an extension of the model [26] and resulted in a better performance. Setak et al. [28] proposed an incomplete hub location network for an urban transportation problem. In

their model, non-hub nodes can be connected directly to each other, and the network topology is incomplete. This model was an extension of the Gelareh and Nickel's model [27]. Karimi and Setak [4] introduced a new formulation for public transportation network with a hub structure. The advantage of their model in comparison to the previous models in this area is that the number of variables and constraints is reduced so that the computational time is reduced considerably. Karimi and Setak [29] presented flow shipment scheduling in a hub location-routing problem. Their integer programming model scheduled the departure time from the nodes. Djenić et al. [30] proposed a bus terminal location problem by incorporating characteristics of p -median and maximal covering problems and used a parallel variable neighborhood search algorithm to solve it.

As it is obvious in the literature, in spite of the fact that public transportation systems make use of a hub-and-spoke structure, not much attention has been paid to this area in the previous studies of hub networks. The hub models studied in public transportation have focused mainly on the design of an incomplete hub network and have neglected other aspects of a public transportation network. The models presented in this field have been single-objective and negligent of the service levels and capacity constraints. Also, in all of these models, the demand is assumed to be static, which is far from reality. In this study, the main focus is on designing a new hub network for public transportation, which fills the gaps in the previous models in this area and complies more with reality. Main contributions of this study can be briefed as:

- Considering three potential capacity levels for each hub node from which only one is chosen as the final capacity of the hub.
- Considering a limited capacity for hub links.
- Assuming the demand to be variable and dependent on the location of hubs.
- Incorporating the TH method [31] for converting the multi-objective functions into a single one.
- Considering a multi-objective model that maximizes the benefits of transportation and establishes the hub facilities while minimizes the total transportation time, in which developing a metro line with shorter time results in an increase in the total cost.

- Calculating the number of trains used in hub links.

The rest of this paper is structured as follows. Section 3 presents the mathematical model for the bi-objective incomplete hub median location problem and the TH method to convert the original multi-objective model into a single-objective one. The computational results are presented in Section 4 followed by the conclusion and future studies in Section 5.

3. Assumptions and Mathematical Formulation

3-1. Problem description

In this paper, it is assumed that there is a set of n nodes in which some can be chosen as hubs. The model chooses the hub nodes, and then constructs the hub network according to the number of hub links. The demand of each node is dependent on the utility of each hub and a decaying function of distance. The first objective of the model is to maximize the benefits of the total transportation, construct hub stations and the total number of transportation vehicles. The second objective is to minimize the total travel time. The main assumptions used to formulate the problem are as follows:

- The number of hubs that should be located is predefined.
- The number of hub links is predefined.
- Each non-hub node can be allocated to only one hub (i.e., single-allocation assumption).
- Hub network is an incomplete, but connected graph.
- Hubs and arcs capacities are limited.
- Each hub includes three capacity levels from which only one is selected as an appropriate level considering the total cost.
- The capacity of trains is limited.
- All flows travel from (to) a node passing through hub(s).
- The demand of each node is a function of the utility perceived by customers at that node.

3-2. Mathematical model

In this model, the demand between each two nodes is the function of the utility. So, it is shown as $W_{ij}(U_i+U_j)$, and it means that the demand is dependent on the utility perceived by a customer at nodes i and j . The utility is assumed additive for the simplicity of the calculation. In this formulation, U_i means the utility that is perceived by node i and is calculated by the following function:

$$U_i = \sum_k u_{ik} z_{kk} \quad \forall i \quad (1)$$

where u_{ik} is the utility of hub k that is perceived by node i .

As it is specified in Eq. (8), the utility perceived by each node is a function of the location of the hub. so the demand is not predefined. As it is proposed by Khosravi et al. [25], the demand function is defined as follows:

$$W_{ij}(U_i + U_j) = (W_{ij}^{\max} - W_{ij}^{\min})c_{ij} \cdot (U_i + U_j) + W_{ij}^{\min} \quad (2)$$

$$c_{ij} = \frac{1}{U_i^{\max} + U_j^{\max}} \quad (3)$$

$$U_i^{\max} = \sum_k u_{ik} \quad (4)$$

where U_i^{\max} is the maximum utility perceived by a customer at node i ; W_{ij}^{\max} and W_{ij}^{\min} are the maximum and minimum demands between nodes i and j , respectively.

This formulation means that among nodes, the ones with high attractiveness are established. u_{ik} is a function of attractiveness of the facility and a decaying function of distance. As it is defined by Berman and Krass [32], “The “attractiveness” of the facility is a function of the facility characteristics (e.g., size), characteristics of the facility’s location (e.g., availability of parking and proximity to major intersections), as well as other variables (e.g., marketing expenditures and prices) that may affect the “perceived attractiveness” of the facility”.

$$u_{ik} = \frac{A_k}{d_{ik}} \quad (5)$$

where A_k is the attractiveness of hub k and d_{ik} is the distance between node i and hub k .

Decision variables and parameters of the proposed model are defined as follows:

Indices and sets:

- N Set of all nodes
- $i, j \in N$ Origin and destination indices
- $k, l \in N$ Hub nodes indices
- q Set of capacity levels

Parameters:

- P_{ik} Price per unit of flow from origin i to hub k
- $W_{ij}(U_i + U_j)$ Function of flow from i to j
- C_{ik} Price per unit of flow from origin i to hub

- k
- I_{kl} Fixed cost for establishing hub link between nodes k and l
- F_k^q Fixed cost for establishing a hub at node k with capacity level q
- C^m Cost of a train
- t_{ik} Transportation time from node i to hub k
- Wt_k Waiting time in hub k
- V^m Capacity of a train
- Γ_k^q Capacity of hub k with capacity level q
- r_{kl} Capacity of link $k-l$
- P Number of hub nodes which should be established
- A Number of hub links which should be established
- α Discount factor of transportation between hub nodes

Decision variables

- x_{ijkl} Fraction of the flow originated at i destined to j that is routed via hubs k and l
- y_{kl} 1 if a hub link $k-l$ is established between hub nodes k and l ; 0, otherwise
- h_k^q 1 if hub k with capacity level q is established; 0, otherwise
- z_{ik} If non-hub node i is assigned to hub node k
- n_{kl} Number of trains between hub link $k-l$

According to the above notations, a new mathematical model for the single-allocation hub arc location problem is presented as follows. The first objective of this model is to maximize the total profit of transportation, operating hubs, hub links, and transportation vehicles, while the second one minimizes the total transportation time. regarding the above assumptions, the multi-objective problem can be developed as follows:

$$\begin{aligned} \text{Max } z_1 = & \sum_i \sum_{k \neq i} \sum_j P_{ik} W_{ij}(U_i + U_j) z_{ik} + \sum_i \sum_{j > i} \sum_k \sum_{l \neq k} P_{kl} W_{ij}(U_i + U_j) \\ & + \sum_i \sum_{k \neq i} \sum_j P_{kj} W_{ij}(U_i + U_j) z_{jk} - (\sum_i \sum_{k \neq i} \sum_j C_{ik} W_{ij}(U_i + U_j) \\ & + \sum_i \sum_{j > i} \sum_k \sum_{l \neq k} \alpha C_{kl} W_{ij}(U_i + U_j) x_{ijkl} + \sum_i \sum_{k \neq i} \sum_j C_{kj} W_{ij}(U_i + U_j) \\ & + \sum_k \sum_q F_k^q h_k^q + \sum_k \sum_{l > k} I_{kl} y_{kl} + \sum_k \sum_{l > k} n_{kl} C^m) \end{aligned} \quad (6)$$

$$\begin{aligned} \text{Min } z_2 = & \sum_i \sum_{k \neq i} t_{ik} z_{ik} + \sum_i \sum_j \sum_k \sum_{l \neq k} \alpha (Wt_k + t_{kl} + Wt_l) x_{ijkl} \\ & + \sum_i \sum_{k \neq i} t_{kj} z_{jk} \end{aligned} \quad (7)$$

s.t.

$$z_{ik} \leq z_{kk} \quad \forall i, k \neq i \quad (8)$$

$$\sum_k z_{ik} = 1 \quad \forall i \quad (9)$$

$$\sum_{k \neq l} x_{ijkl} + z_{il} = \sum_{k \neq l} x_{ijlk} + z_{jl} \quad \forall i < j, j, l \quad (10)$$

$$\sum_k z_{kk} = p \quad (11)$$

$$y_{kl} \leq \sum_q h_k^q \quad \forall k, l > k \quad (12)$$

$$y_{kl} \leq \sum_q h_l^q \quad \forall k, l > k \quad (13)$$

$$\sum_q h_k^q \leq z_{kk} \quad \forall k \quad (14)$$

$$\sum_k \sum_{l > k} y_{kl} = A \quad (15)$$

$$x_{ijkl} + x_{ijlk} \leq y_{kl} \quad \forall i, j > i, k, l > k \quad (16)$$

$$\sum_i \sum_j W_{ij}^{\max} z_{ik} \leq \sum_q \Gamma_k^q h_k^q \quad \forall k \quad (17)$$

$$\sum_i \sum_j W_{ij}^{\max} x_{ijkl} \leq r_{kl} y_{kl} \quad \forall k, l > k \quad (18)$$

$$V^m n_{kl} \geq \sum_i \sum_j W_{ij}^{\max} x_{ijkl} \quad \forall k, l \neq k \quad (19)$$

$$n_{kl} = n_{lk} \quad \forall k, l \quad (20)$$

$$x_{ijkl}, y_{kl}, h_k^q, z_{ik} \in \{0, 1\} \quad (21)$$

$$n_{kl} \geq 0$$

Constraints (8) allow allocation of node i to hub k only if hub k is installed. Constraints (9) ensure that each node can be assigned to just one hub. Constraints (10) are the flow conservation constraints. Constraint (11) ensures that p hub should be installed. Constraints (12) and (13) specify that both end-points of a hub edge must be hub nodes. A hub node with capacity level q can be installed only if node k is selected as a hub. This issue is indicated by constraints (14). Constraint (15) ensures that a hub link should be established. x_{ijkl} can be positive only if the hub link between k and l exists. This issue is ensured by Constraint (16). Constraints (17) and (18) restrict the entering flow to a hub node and hub link, respectively. Constraints (19) and (20) determine the number of trains on each hub link. Constraints (21) represent the domains of decision variables.

3-3. Linearization of the model

According to the definition of the demand function, each term of the objective function, which includes demand function, is nonlinear. To

avoid this formulation, the model should be linearized. In this term, the linearization scheme of the model is explained. To understand this scheme, the first non-linear part of the model is expanded as follows:

$$\begin{aligned} & \sum_i \sum_{k \neq i} \sum_j P_{ik} W_{ij} (U_i + U_j) z_{ik} \\ &= \sum_i \sum_{k \neq i} \sum_j P_{ik} ((W_{ij}^{\max} - W_{ij}^{\min}) c_{ij} (U_i + U_j) + W_{ij}^{\min}) z_{ik} \\ &= \sum_i \sum_{k \neq i} \sum_j P_{ik} ((W_{ij}^{\max} - W_{ij}^{\min}) c_{ij} (u_{ik} z_{kk} + u_{jk} z_{kk}) + W_{ij}^{\min}) z_{ik} \\ &= \sum_i \sum_{k \neq i} \sum_j P_{ik} ((W_{ij}^{\max} - W_{ij}^{\min}) c_{ij} (u_{ik} + u_{jk}) z_{kk} z_{ik} + W_{ij}^{\min} z_{ik}) \end{aligned}$$

Linearization scheme:

$$g_{ik} = z_{kk} z_{ik} \quad \forall i, k \quad (22)$$

$$z_{kk} + z_{ik} - g_{ik} \leq 1 \quad \forall i, k \quad (23)$$

$$g_{ik} \leq z_{kk} \quad \forall i, k \quad (24)$$

$$g_{ik} \leq z_{ik} \quad \forall i, k \quad (25)$$

The other non-linear parts of the model can be linearized similarly.

3-4. TH method

There are differences between single- and multi-objective optimization problems. In a single-objective one, the goal is to find one and only one optimal solution, while in multi-objective optimization problems, a set of solutions depending on non-dominance criterion is found that is named the Pareto sense. Various applicable methods have been developed to solve the multi-objective programming models in the literature. Despite the classical multi-objective programming methods that may result in weakly efficient solutions, the TH method guarantees to find efficient solutions. This method is summarized as follows:

Step 1: Determine the positive ideal solution (PIS) and negative ideal solution (NIS) for both objective functions.

$$z_1^{PIS} = \min z_1 \quad z_1^{NIS} = z_1(V_2^*)$$

$$z_2^{PIS} = \min z_2 \quad z_2^{NIS} = z_2(V_1^*)$$

where V_1^* and V_2^* are the optimal values of variables in the first and second objective functions, respectively.

Step 2: Determine a linear membership function for each objective function as follows:

$$\mu_1(v) = \begin{cases} 1 & \text{if } z_1 \prec z_1^{PIS} \\ \frac{z_1^{NIS} - z_1}{z_1^{NIS} - z_1^{PIS}} & \text{if } z_1^{PIS} \leq z_1 \leq z_1^{NIS} \\ 0 & \text{if } z_1 \succ z_1^{NIS} \end{cases}$$

$$\mu_2(v) = \begin{cases} 1 & \text{if } z_2 \prec z_2^{PIS} \\ \frac{z_2^{NIS} - z_2}{z_2^{NIS} - z_2^{PIS}} & \text{if } z_2^{PIS} \leq z_2 \leq z_2^{NIS} \\ 0 & \text{if } z_2 \succ z_2^{NIS} \end{cases}$$

where $\mu_h(v)$ denotes the satisfaction degree of the h -th objective function for the given solution vector.

Step 3: Convert the auxiliary MOMILP model into an equivalent single-objective MILP using the following auxiliary formulation:

$$\begin{aligned} \max \lambda(v) &= \gamma \lambda_0 + (1 - \gamma) \sum_h \theta_h \mu_h(v) \\ \text{s.t.} & \\ \lambda_0 &\leq \mu_h(v) \quad \forall h \\ v &\in F(v), \lambda_0 \text{ and } \gamma \in \{0, 1\} \end{aligned}$$

where θ_h and γ show the importance of the h -th objective function and coefficient of recompense ($\sum_h \theta_h = 1; \theta_h \succ 0$), where $F(v)$ shows the feasible region of the problem.

4. Computational Experiments

In this section, the proposed model is solved using the real data taken from transportation studies of Qom city by 24.0.1 version of GAMS software and CPLEX solver. Then, by changing some parameters of the model, a sensitivity analysis is provided to illustrate the validity of the model.

4-1. Case study

To validate the model introduced in this paper, Qom’s Monorail Project data are used to solve this model. Considering the density of population around Qom, the city is divided into 15 traffic areas, and the maximum demand of each area is estimated. The center of each area is selected as demand nodes. The data used in this model are based on the ground truth, according to the up-to-date prices and regarding the problem size and assumptions, the necessary changes are applied.

Regarding the distances between cities and assuming an average speed for the ordinary vehicles used on spoke links (40 Km/h) and trains on hub links (70 Km/h), the expected time between each two demand points is derived. Some parameters used to solve the model are summarized as follows. In the following, d_{ij} is the distance between nodes i and j .

Tab. 1. Parameters

P_{ij}	P_{kl}	C_{ij}	C_{kl}	V^m	C^m
740	900	80d _{ij}	2400d _{ij}	1500	150000

The value of attraction coefficients, A_k , is drawn from a uniform distribution in the range (100, 500) as it is proposed by Berman and Krass [32]. There are three capacity levels for each hub, from which only one can be selected by the model. The first level is the tight one, which is shown by Γ_0 ($\Gamma_k^1 = \Gamma_0$). The two other capacity levels can be drawn as follows:

$$\Gamma_k^q = \Gamma_0 (1 + 0.1(q - 1))$$

It should be noted that regarding the capacities, a necessary condition for the feasibility of the problem is that:

$$\sum_k \Gamma_k^{s_k} \geq \sum_i \sum_j w_{ij}$$

where s_k is the capacity levels, which is chosen in a solution.

The fixed cost for each capacity is estimated as follows:

$$\begin{aligned} F_k^1 &= 30000000 \\ F_k^{q=2,3} &= F_k^1 (1 + 0.1k) \end{aligned}$$

The minimum number of passengers for each node is assumed to be one person. The value of γ is assumed to be 0.4 as it is proposed by Torabi and Hassini [31].

The results for a problem with 15 nodes and 4 hubs with 4 hub links are shown in Table 2. In the following tables N , H , and L denote number of nodes, number of hubs, and number of hub links, respectively. HN and NT are hub nodes and number of trains on each hub link, respectively.

Tab. 2. Case study results ($N=15$)

$H \# L$	α	Θ_h	Z_1	$Z_2(\text{min})$	CPU (min)	HN	L	NT
	0.8	0.6,0.4	3.16E+09	301.128	120.356	5,7,8,9	5-7	22
4 # 4							5-8	50
							5-9	30
							7-8	14

4-2. Sensitivity analysis

In this section, the model is solved in various instances to determine the sensitivity of solutions on various values of parameters. The data used to solve the following cases are similar to the data which are discussed in the previous section. As the computational time required for solving the problems with up to 10 nodes is increased considerably, the numerical examples solved in

this section are under a reasonable computational time. The results in Table 3 show that the TH method is not sensitive to the value of Θ_h and in small problems has a low variance. However, in some instances, the CPU time increases. So, the TH method makes reliable and balanced solutions.

Tab. 3. Summary of results for the case study in different situations ($N=5$)

$H \# L$	α	Θ_h	Z_1	Z_2	CPU time	HN	L	NT	
2#1	0.7	0.8,0.2	1.94E+08	48.62	0.00011	2,3	2-3	11	
		0.75,0.25	2.00E+08	48.62	0.00019				
		0.6,0.4	2.00E+08	48.62	0.0002				
	0.8	0.8,0.2	1.99E+08	50.32	0.00014				
		0.75,0.25	1.99E+08	50.32	0.00012				
		0.6,0.4	1.99E+08	50.32	0.00013				
	0.9	0.8,0.2	1.99E+08	52.02	0.00016				
		0.75,0.25	1.99E+08	52.02	0.0002				
		0.6,0.4	1.84E+08	52.02	0.00011				
3#2	0.7	0.8,0.2	1.79E+07	40.596	0.00015	1,2,3	1-2	12	
		0.75,0.25	1.79E+07	40.596	0.00017				
		0.6,0.4	1.79E+07	40.596	0.00015				
	0.8	0.8,0.2	1.76E+07	42.804	0.00017				
		0.75,0.25	1.76E+07	42.804	0.00014				
		0.6,0.4	1.76E+07	42.804	0.00015				
	0.9	0.8,0.2	1.72E+07	45.012	0.00019				
		0.75,0.25	1.72E+07	45.012	0.00014				
		0.6,0.4	1.72E+07	45.012	0.00012				
3#3	0.7	0.8,0.2	8.26E+07	45.393	0.00019	1,2,3	1-2	26	
		0.75,0.25	8.26E+07	45.393	0.00012				
		0.6,0.4	8.26E+07	45.393	0.00012				
	0.8	0.8,0.2	8.23E+07	47.472	0.00011				
		0.75,0.25	8.23E+07	47.472	0.00022				
		0.6,0.4	8.23E+07	47.472	0.00011				
	0.9	0.8,0.2	8.19E+07	49.551	0.00019				
		0.75,0.25	8.19E+07	49.551	0.00016				
		0.6,0.4	8.19E+07	49.551	0.00023				

As it is obvious from results of Tables 3 to 5, increasing the value of Θ_h causes an increase in Z_1 and increase in Z_2 . Because increasing the Θ_h means that the importance of the first objective (i.e., total profit) is increased, so the model tries to find better results for the first objective, so the value of the first objective increases and the value

of the second objective (i.e., total time) increases and becomes worse.

Table 6, Figs. 1 and 2 show the effect of adding hub links in a problem with 7 nodes and 4 hubs.

Tab. 4. Summary of results for the case study in different situations (N=7)

<i>H # L</i>	<i>α</i>	Θ_h	Z_1	Z_2	CPU time	<i>HN</i>	<i>L</i>	<i>NT</i>
3#2	0.7	0.8,0.2	2.58E+08	85.056	0.00114	1,2,3	1-2	11
		0.75,0.25	2.58E+08	85.056	0.00078		1-3	14
		0.6,0.4	2.58E+08	85.056	0.00089			
	0.8	0.8,0.2	2.56E+08	92.1	0.00114			
		0.75,0.25	2.49E+08	87.864	0.00188			
		0.6,0.4	2.49E+08	87.864	0.00131			
	0.9	0.8,0.2	2.54E+08	95.58	0.00137			
		0.75,0.25	2.47E+08	90.672	0.00165			
		0.6,0.4	2.47E+08	90.672	0.00113			
4#3	0.7	0.8,0.2	1.14E+08	88.188	0.00086	1,2,3,4	1-2	11
		0.75,0.25	1.14E+08	88.188	0.0009		1-3	14
		0.6,0.4	1.14E+08	88.188	0.0009		1-4	34
	0.8	0.8,0.2	1.19E+08	97.908	0.00116			
		0.75,0.25	1.19E+08	97.908	0.00088			
		0.6,0.4	1.12E+08	93.672	0.00168			
	0.9	0.8,0.2	1.10E+08	99.156	0.00111			
		0.75,0.25	1.10E+08	99.156	0.00131			
		0.6,0.4	1.10E+08	99.156	0.00091			
4#4	0.7	0.8,0.2	9.15E+07	89.288	0.00444	1,2,3,4	1-2	28
		0.75,0.25	8.42E+07	85.724	0.0031		1-3	4
		0.6,0.4	8.42E+07	85.724	0.00272		1-4	24
	0.8	0.8,0.2	9.90E+07	116.546	0.00413		3-4	10
		0.75,0.25	9.99E+07	116.546	0.00372			
		0.6,0.4	8.99E+07	95.092	0.00254			
	0.9	0.8,0.2	9.74E+07	123.738	0.00415			
		0.75,0.25	9.83E+07	123.738	0.00481			
		0.6,0.4	8.83E+07	100.896	0.00304			

Tab. 5. Summary of results for the case study in different situations (N=10)

<i>H # L</i>	<i>α</i>	Θ_h	Z_1	Z_2	CPU time	<i>HN</i>	<i>L</i>	<i>NT</i>
3#2	0.7	0.8,0.2	3.88E+08	138.744	13.0182	1,2,3	1-2	15
		0.75,0.25	3.88E+08	138.744	15.3107		1-3	18
		0.6,0.4	3.82E+08	138.744	11.2614			
	0.8	0.8,0.2	3.88E+08	142.956	12.1414			

		0.75,0.25	3.88E+08	142.956	18.2994			
		0.6,0.4	3.88E+08	142.956	14.0612			
	0.9	0.8,0.2	3.88E+08	147.168	34.0642			
		0.75,0.25	3.88E+08	147.168	43.0532			
		0.6,0.4	3.88E+08	147.168	27.0915			
4#3	0.7	0.8,0.2	2.52E+08	151.242	11.1936	1,2,3,4	1-2	15
		0.75,0.25	2.52E+08	151.242	17.2666		1-3	18
		0.6,0.4	2.52E+08	151.242	6.57558		1-4	34
	0.8	0.8,0.2	2.52E+08	159.468	12.2324			
		0.75,0.25	2.52E+08	159.468	20.2324			
		0.6,0.4	2.46E+08	159.468	6.59222			
	0.9	0.8,0.2	2.52E+08	167.694	11.541			
		0.75,0.25	2.52E+08	167.694	16.541			
		0.6,0.4	2.11E+08	156.54	8.22614			
4#4	0.7	0.8,0.2	2.50E+08	135.824	15.5098	5,6,8,9	5-6	22
		0.75,0.25	2.50E+08	135.824	26.5098		5-8	55
		0.6,0.4	2.56E+08	135.824	9.12981		5-9	23
	0.8	0.8,0.2	2.48E+08	143.056	15.2043		8-9	19
		0.75,0.25	2.48E+08	143.056	27.2043			
		0.6,0.4	2.46E+08	159.468	6.52957			
	0.9	0.8,0.2	2.43E+08	150.288	13.3636			
		0.75,0.25	2.43E+08	150.288	25.3378			
		0.6,0.4	2.38E+08	176.958	7.33302			

It should be noted that the number of hub links should be at least equal to the number of links in a tree shaped network to create a connected hub network. Therefore, the number of hub links to be

used is at least the total number of hubs minus one.

$$\sum_k \sum_{l>k} y_{kl} \geq \sum_k \sum_q h_k^q - 1$$

Tab. 6. Results of changing the number of hub links

No. of hub links	Z ₁	Z ₂	Hub links
3	1.12E+08	93.672	1-2,1-3,1-4
4	8.99E+07	95.092	1-2,1-3,1-4,3-4
5	8.88E+07	104.476	1-2,1-3,1-5,2-3,5-3
6	7.63E+07	104.876	1-2,1-3,1-5,2-3,5-3,2-5

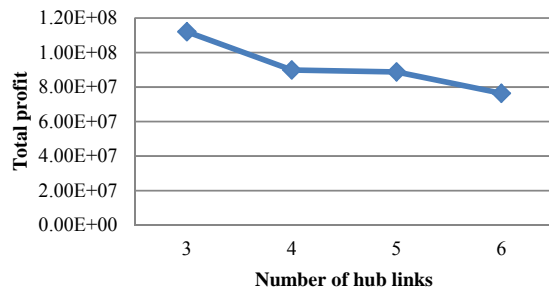


Fig. 1. Effects of a number of hub links on the total profit

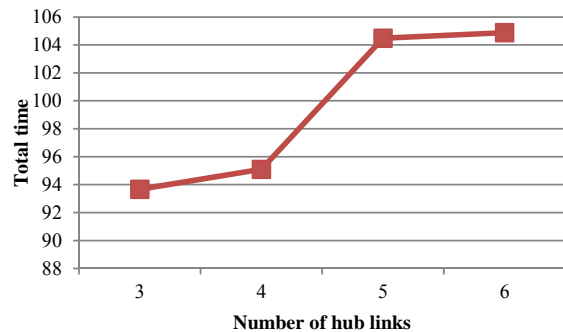


Fig. 2. Effects of the number of hub links on the total time

As it is obvious in Figs. 1 and 2, by adding more hub links due to high construction cost of subway, the total cost increases, so the total profit decreases. Furthermore, by adding more than tree shaped network links, the total transportation time increases because passengers should pass through more links to reach their destination. So, the best hub network structure in this problem is a tree shaped structure that can be made by the following hub link number.

$$\sum_{k > l} y_{kl} = \sum_k \sum_q h_k^q - 1$$

Table 7 and Fig. 3 show that by increasing the capacity level, the total profit increases while the

total time decreases. The reason is that by increasing the hub capacity, the hub can cover all the nodes which are close to it, and it is not necessary to allocate demand nodes to further hubs due to low capacity level, so the transportation cost decreases; consequently, the profit increases. The transportation time also decreases due to allocation to the nearer hubs. As it is shown in Fig. 3, the total profit and time becomes constant from capacity level 130000. The results show that the total profit is more sensitive on the capacity level.

Tab. 7. Results of changing the capacity level of hubs

Scenario number	Γ_0	Z_1	Z_2	Hub*capacity level
1	106000	-2.4E+08	144.632	4*2,5*3,6*2,7*1
2	110000	-2.2E+08	144.318	4*1,5*1,6*3,7*1
3	114000	-2.1E+08	124.632	4*1,5*1,6*2,7*1
4	118000	-4566620	121.068	1*1,2*1,3*2,4*1
5	122000	6029107	120.416	1*1,2*1,3*3,4*1
6	126000	6029079	120.416	1*1,2*1,3*3,4*1
7	130000	20089610	117.864	1*3,2*2,4*1,5*1
8	134000	20355700	116.546	1*3,2*1,3*1,4*1
9	138000	20355700	116.546	1*3,2*1,3*1,4*1
10	142000	20355700	116.546	1*3,2*1,3*1,4*1

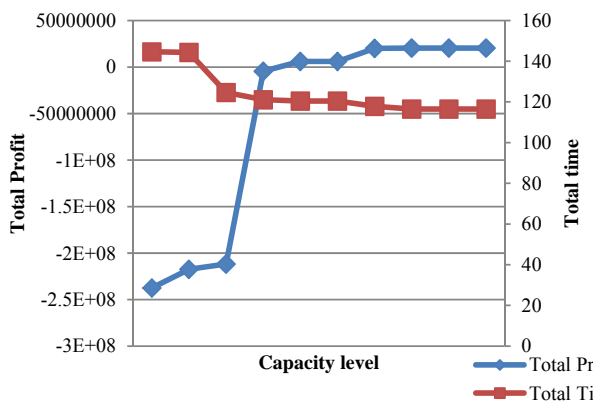


Fig. 3. Effects of the capacity level on total profit and total time

5. Conclusion and Future Work

In this study, a capacitated bi-objective hub arc location problem for designing a metro network is proposed. The hub network of the proposed model is incomplete to comply with reality and

the demand considered is elastic. Each hub has three potential hub capacities from which only one can be chosen as the final capacity of each hub. The capacity of hub links is also limited. Furthermore, the number of each vehicle on each link is calculated by the model. The multi-objective model is converted to a single objective model using TH model. In addition, the proposed model is tested using the real transportation data of Qom city and sensitivity analysis is done by changing some of the parameters. The results in Figs. 1 and 2 show that in the proposed model, by adding more hub links due to high construction cost of subway, the total cost increases; therefore, the total profit decreases. Moreover, Table 7 and Fig. 3 show that increasing the capacity level results in increasing the total profit while decreasing the total time. For future work, the model can be extended considering uncertainty of the model parameters. The model can be solved by heuristics and meta-heuristics for large-sized data as future studies.

References

- [1] O’Kelly, M.E., “The Location of Interacting Hub Facilities”, *Transportation Science*, Vol. 20, (1986), pp. 92-106.
- [2] O’Kelly, M.E., “A quadratic integer program for the location of interacting hub facilities”, *European Journal of Operational Research*, Vol. 32, (1987), pp. 393-404.
- [3] Campbell, J.F., “Integer programming formulations of discrete hub location problems”, *European Journal of Operational Research*, Vol. 72, (1994), pp. 387-405.
- [4] Karimi, H., Setak, M., “Proprietor and customer costs in the incomplete hub location-routing network topology”, *Applied Mathematical Modelling*, Vol. 38, No. 3, (2014), pp. 1011-1023.
- [5] Contreras, I., Fernández, E., Marín, A., “The Tree of Hubs Location Problem”, *European Journal of Operational Research*, Vol. 202, (2010), pp. 390–400.
- [6] Xu, J., Chiu, S.Y., Glover, F., “Using tabu search to solve the Steiner tree-star problem in telecommunications network design”, *Telecommunication Systems*, Vol. 6, (1996), pp. 117-125.
- [7] Martins de Sá, E., Saraiva de Camargo, R, de Miranda, G., “An improved Benders decomposition algorithm for the tree of hubs location problem”, *European Journal of Operational Research*, Vol. 226, (2013), pp. 185–202.
- [8] Papadakos, N., “Practical enhancements to the Magnanti Wong method”, *Operations Research Letters*, Vol. 36, (2008), pp. 444–449.
- [9] Chiu, S., Lee, Y., Ryan, J., “A Steiner ring star problem in designing survivable telecommunication networks”, INFORMS National Meeting, New Orleans, (1995).
- [10] Wang, Z., Lin, C., Chan, C.K., “Demonstration of a single-fiber self-healing CWDM metro access ring network with unidirectional OADM”, *IEEE Photonics Technology Letters*, Vol. 18, (2006), pp. 163–165.
- [11] Alumur, S.A., Kara, B.Y., Karasan, O.E., “The design of single allocation incomplete hub networks”, *Transportation Research Part B*, Vol. 43, (2009), pp. 936–951.
- [12] Yoon, M.-G., Current, J., “The hub location and network design problem with fixed and variable arc costs: Formulation and dual-based solution heuristic”, *Journal of the Operational Research Society*, Vol. 59, No. 1, (2006), pp. 80-89.
- [13] Campbell, J.F., Ernst, A.T., Krishnamoorthy, M., ‘Hub arc location problems: part I introduction and results’, *Management Science*, Vol. 51, No. 10, (2005a), pp.1540–1555.
- [14] Campbell, J.F., Ernst, A.T., Krishnamoorthy, M., ‘Hub arc location problems: Part II formulations and optimal algorithms’, *Management Science*, Vol. 51, No. 10, (2005b) pp.1556–1571.
- [15] Sasaki, M., Campbell, J.F., Ernst, A.T, Krishnamoorthy, M, “Hub Arc Location with Competition”, *Technical Report of the Nanzan Academic Society Information Sciences and Engineering*, -Vol. 2, (2009), pp.1-42.
- [16] Campbell, J.F., “Hub location for time definite transportation”, *Computers and Operations Research*, Vol. 36, (2009), pp. 3107–16.
- [17] Yaman, H., “The hierarchical hub median problem with single assignment”, *Transportation Research Part B*, Vol. 43, (2009), pp. 643–658.
- [18] Alumur, S.A., Yaman, H., Karab, B.Y., “Hierarchical multimodal hub location problem with time-definite deliveries”, *Transportation Research Part E*, Vol. 48, No. 6, (2012), pp. 1107–1120.
- [19] Sedehzadeh, S., Tavakkoli-Moghaddam, R., “Using a fuzzy approach for a new bi-objective model for a multi-modal tree p-hub median location problem”, *Int. J. Fuzzy Computation and Modelling*, Vol. 1, No. 2, (2014).

- [20] Mohammadi, M., Jolai, J., Tavakkoli-Moghaddam, R., "Solving a new stochastic multi-mode p-hub covering location problem considering risk by a novel multi-objective algorithm", *Applied Mathematical Modelling*, Vol. 37, (2013), pp.10053–10073.
- [21] Correia, I., Nickel, S., Saldanha-da-Gama, F., "Single-assignment hub location problems with multiple capacity levels", *Transportation Research Part B*, Vol. 44, (2010), pp. 1047–1066.
- [22] Correia, I., Nickel, S., Saldanha-da-Gama, F., "The capacitated single-allocation hub location problem revisited: A note on a classical formulation", *European Journal of Operational Research*, Vol. 207, (2010), pp. 92–96.
- [23] Correia, I., Nickel, S., Saldanha-da-Gama, F., "Hub and spoke network design with single-assignment, capacity decisions and balancing requirements", *Applied Mathematical Modelling*, Vol. 35, (2011), pp. 4841–4851.
- [24] Redondo, J.L., Ferná'ndez, J., Arrondo, A.G., Garcí'a, I., Ortigosa, P.M., "Fixed or variable demand? Does it matter when locating a facility?", *Omega*, Vol. 40, (2012), pp. 9–20.
- [25] Khosravi, Sh, Akbari Jokar, M.R, "hub location problem considering an elastic demand", CIE44 & IMSS'14 Proceedings, 14-16 October, Istanbul / Turkey, (2014), pp. 581-588.
- [26] Nickel, S., Schobel, A., Sonneborn, T., "Hub location problems in urban traffic networks", *Mathematical Methods on Optimization in Transportation Systems*, Kluwer Academic Publishers, Dordrecht, The Netherlands, pp. 95-107, 2001.
- [27] Gelareh, S., Nickel, S., "Hub location problems in transportation networks", *Transportation Research Part E: Logistics and Transportation Review*, Vol. 47, No. 6, (2011), pp. 1092-1111.
- [28] Setak, M., Karimi, H., Rastani, S., "Designing incomplete hub location-routing network in urban transportation problem", *International Journal of Engineering - Transactions C*, Vol. 26, No. 9, (2013), pp. 997-1006.
- [29] Karimi, H., Setak, M., "Flow shipment scheduling in an incomplete hub location-routing network design problem", *Computational and Applied Mathematics*, (2016), pp. 1-33.
- [30] Djenić, A., Radojičić, N., Marić, M., Mladenović, M., "Parallel VNS for Bus Terminal Location Problem", *Applied Soft Computing*, Vol. 42, (2016), pp. 448–458.
- [31] Torabi, S.A., Hassini, E., "An interactive possibilistic programming approach for multiple objective supply chain master planning", *Fuzzy Sets and Systems*, Vol. 159, (2008), pp. 193 – 214.
- [32] Berman, O., Krass, D., "Locating multiple competitive facilities: spatial interaction models with variable expenditures", *Annals of Operations Research*, Vol. 111, (2002), pp. 197–225.

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