Construction projects performance evaluation model based on Choquet Integral, case study (power plant Construction Company in Iran)

Roghayeh Hemmatjou, Nasim Nahavandi†, Behzad Moshiri

Roghayeh Hemmatjou, School of Electrical & Computer Engineering, University of Tehran
Nasim Nahavandi, Department of Industrial Engineering, Tarbiat Modares University
Behzad Moshiri, School of Electrical & Computer Engineering, University of Tehran

KEYWORDS
aggregation operators, non–additive robust ordinal regression, multiple criteria decision making, uniform capacity.

ABSTRACT
Recently, Coquet integral (CI) has been used in multi-criteria decision-analysis (MCDA) problems in which there are non-linear relations between decision criteria. The decision-maker imposes some information, on basis of preferences relations, on decision problem. The coefficients of evaluation method should be defined accurately in order to illustrate the intention of decision maker (DM). Different methods have been proposed in this area. The capacity definition methods based on optimization provide a solution space and pick the coefficients through it. In cases where there is a solution set rather than a unique solution, the coefficients are usually chosen arbitrarily from solution set, damaging the reliability of the result. In addition, solution usually provides more information compared to propositions made by DM. Therefore, DM may not fully interpret the result. So, robust capacity definition methods are proposed to overcome these drawbacks. On the other hand, these methods do not consider evenness (uniformity), which is a major property of capacity. Preferences of DM are made of a subset of alternatives called reference alternatives. When defined capacity sharply focuses on this information, it moves solution to the extreme points of solution space. Therefore, it loses its uniformity or as called evenness, and it is not reliable for evaluating non-reference alternatives. This paper uses an algorithm to define a capacity that is based only on the preference information of DM, and consequently, it becomes of DM’s preferences. Furthermore, it improves evenness of capacity and its reliability in evaluating non–reference alternatives. The algorithm is used to evaluate power plant projects which are huge national projects in Iran. Case–specific criteria are taken into account in addition to general criteria used in project performance evaluation. The evaluation results, obtained from the proposed algorithm, are compared with those of the most representative utility function method.

© 2016 IUST Publication, IJIEPR. Vol. 27, No. 3, All Rights Reserved
1. Introduction

Developing a framework for evaluating construction projects is an ambiguous task, because each project has unique characteristics and is defined to achieve specific objectives. On the other hand, different stakeholders evaluate project success based on the degree of satisfaction of their expectations and achievement of the objectives that they have defined for each project. As a result, they may consider different criteria and key performance indicators (KPIs) for project evaluation, and the same project may have different success levels depending on the evaluators’ point of views [1]. It is worth noting that there is a difference between success criteria and success factors; success criteria are the principles or standards of measuring project success. On the other hand, success factors are circumstances, facts or influences which lead project to success, but they are not considered for judgment purposes [2]. Takim [3] proposed a conceptual model to visualize the relationship between success criteria and factors.

The most common approach in project evaluation is to measure its output results, i.e., cost performance, time performance, etc., without considering the projects execution environment. While external criteria, such as sanction, economic stability, and priority of project, have great impact on the project’s performance, they have rarely been considered in project evaluation. This article considers those criteria in projects evaluation and uses MCDA methods for evaluation purpose. Since the criteria of interest have interaction, 2–additive Choquet integral (CI) is used as the aggregation operator in order to take the interaction between criteria into account. 2–additive Choquet integral is able to take the importance of each criterion and interaction between pairs of criteria into account. The coefficients of model are determined by disaggregation–aggregation approach (indirect method). Different capacity definition methods have been proposed in the literature based on properties of aggregation model that are intended to be considered in the model. For example, maximum entropy and minimum variance methods are proposed to define an even (uniform) capacity in which the coefficients are as close to WAM method as possible. Non–

2. Literature review

2-1. Projects evaluation methods review

Project-oriented companies should evaluate their projects in order to be aware of their objectives’ achievement level and their strengths and weaknesses. The most frequently used criteria for project evaluation are time, cost, and quality; the “iron triangle” as named by Atkinson [4]; these criteria form the basis of project evaluation and are the internal criteria that concentrate on project managers’ evaluation.

In later studies, stakeholders’ satisfaction has been introduced as another success criterion [4-6]. Chan [7] divided project performance indicators into two groups: objective and subjective. He considered the criteria related to stakeholders’ satisfaction in subjective group.

*Corresponding author: Nasim Nahavandi
Email: n_nahavandi@modares.ac.ir
Received 23 July 2014; revised 22 December 2016; accepted 22 January 2017
It is important to terminate the project without major injuries. Health and (&) safety criterion is considered to assess the project’s performance according to the rate of accidents and injuries [2, 3, 6-8]. Construction industry’s product has been often used for a long time, so it is a great concern to consider any potential damage that may be caused by a construction project and consider the environmental impact of project as a major criterion for its evaluation [3, 6, 7].

The projects performance is strongly dependent on top management support; actually, the projects priority is not the same and higher-urgency projects have higher priority in resource assignment and top management will try to prepare all requirements in order to speed up these projects [9]. Zare proposed a three-stage algorithm for projects cost and time estimation in fuzzy environment [10]. Cheung [6] suggested eight criteria including people, cost, time, quality, safety and health, environment, client satisfaction, and communication for project performance evaluation. Al–Tmeemy [11] proposed a model that measures project success not only based on project management success and product success, but also by its market share as a strategic success measure of project. Takim [3] proposed a model that considers two groups of measures in project evaluation: efficiency and effectiveness measures. The studies mentioned so far are both aimed to introduce criteria without determining the importance of each criterion and introduce any probable interaction between criteria. Actually, they do not provide a mathematical model to calculate project success. Lauras [12] proposed a multi-dimensional project evaluation method and used MACBETH\(^2\) method [13] to define the importance of criteria and assign an overall score to each project, supposing that there is no interaction between criteria. Ling [14] utilized regression analysis to determine the criteria that have a considerable contribution to project success and define their importance coefficients and probable correlations in project success model. The criteria may have other kinds of interaction rather than correlation, so the regression models may not be a suitable method in such cases. Projects’ risk evaluation has also attracted researchers. Nasirzadeh used fuzzy decision-making method for projects risk evaluation [15]. Ruhparvar used a combination of fuzzy logic and game theory in this area [16], and Büyükozkan used 2–additive Choquet integral to evaluate projects risk [17].

This paper proposes an algorithm based on 2–additive Choquet integral and uses the algorithm to evaluate the success of projects. 2–additive Choquet integral is able to consider importance of each criterion and interaction between pairs of criteria into account. Case-specific criteria, i.e., project operational environment and organizational experience, are also considered in projects evaluation in addition to general criteria.

2-2. Concepts of Choquet integral
Multi-criteria decision analysis (MCDA) problems aim to evaluate a set of finite alternatives \(A = \{a, b, \ldots, m\}\) with respect to a set of finite criteria \(N = \{1, 2, \ldots, n\}\). Each alternative \(a\) is associated with \(n\)-dimensional profile \(\mathbf{x}^a = (x_1^a, x_2^a, \ldots, x_n^a)\) where \(x_i^a\) is a non-decreasing value function that represents the partial score of alternative \(a\) related to criterion \(i\). In these problems, DM should be able to compare any \(x_i^a\) with any \(x_j^b\), where \(i, j \in N\) and \(a, b \in A\); for instance, \(x_i^a \geq x_j^b\) indicates that object \(a\) with respect to the first criterion should have the same satisfaction level as object \(b\) with respect to the second criterion; thus, this is the concept of commensurateness. In order to let the utilities be commensurate, they should be expressed in the same interval scale, most commonly \([0, 1]\) scale [18, 19].

Most often, DM is not able to make a complete preference between alternatives, but he/she can make preferences only between a subset of alternatives called reference alternatives, \(A' \subseteq A\). Then, a suitable overall utility function \(U: \mathbb{R}^n \rightarrow \mathbb{R}\) should be defined in order to aggregate partial scores. This function is called admissible utility function if it has the ability to restore DM’s preferences. For instance, if DM indicates that alternative \(a\) is preferred to alternative \(b\), i.e., \(a \succeq b\), the admissible utility function used for aggregation should assign a value to \(a\) not smaller than the value assigned to \(b\), equation (1). Multi-attribute utility theory (MAUT) is a well-known additive aggregation method which aims to define such a function [20].

\[
a \succeq b \iff U(x_1^a, x_2^a, \ldots, x_n^a) \geq U(x_1^b, x_2^b, \ldots, x_n^b), \quad \forall a, b \in A
\]  \hspace{1cm} (1)

The form of aggregation function depends mostly on the nature of problem at hand; when criteria can be assumed independent, additive models can be used which have a form of weighted arithmetic mean (WAM), according to equation (2).
WAM(\(a\)) = \(\sum_{i=1}^{n} w_i x_i^a\)  \(\text{Eq. (2)}\)

But, in most real situations, the criteria have interaction [21], for example, in evaluating a set of construction projects, the criteria or KPIs can be cost, time, quality, environmental impact, customer satisfaction, etc. In this case, there is often a negative interaction (redundancy) between quality and customer satisfaction, because the project with better quality would make the customers more satisfied, so even if they are important criteria, their importance, when satisfied simultaneously, is less than the sum of their importance when considered separately. On the other hand, there is a positive interaction (synergy) between cost and quality, because executing a project with high quality requires spending more money in most real cases. When criteria have interaction, the weight vector of WAM can be replaced with monotone set function \(\mu\) on \(N\) called capacity or fuzzy measure. By defining a weight for all subsets of criteria, it becomes possible to calculate the importance of each subset of criteria in decision problem, and consequently, to calculate the criteria interaction. A suitable utility function to be used when criteria interact is a generalization of WAM called discrete Choquet Integral with respect to the defined capacity [22-25]. Choquet integral is a powerful fuzzy integral operator; it is a idempotent, continuous and monotonically non-decreasing operator and is also stable under positive linear transformations [22, 25].

CI cannot be used in aggregation process unless its parameters (capacity) have been well defined in advance. Capacity can be defined directly or indirectly. In direct method, the model should be developed in advance, and then it can be used in alternatives’ evaluation (traditional aggregation method). So, DM should define the whole parameters of model which require DM’s full awareness of the problem at hand. Whenever the parameters are defined and the model is developed, it can be used in alternatives’ evaluation. This method has less applicability because DM is not fully aware of preference relations and cannot define all of the coefficients with high reliability. In indirect method, DM is not required to make full preferences; actually, some preference relations are required to be used in learning process in which parameters will be induced by means of ordinal regression methodology (disaggregation approach). Then, the induced parameters can be used in evaluating the whole alternative set (aggregation approach).

The method of defining parameters, when aggregation model is non–additive, is called non–additive ordinal regression [21, 26]. In this method, DM should express his preferences in the form of partial preorders on reference alternatives \(\succ\), preferences on criteria \(\succ\) and sign of interaction indices, and preferences on them \(\succ_{\text{int}}\) [18, 19, 21, 27, 28]. DM could also express intensity of preferences among pairs of reference alternatives \(\succ\), criteria \(\succ\), and interaction indices \(\succ_{\text{int}}\) as defined in [21, 29].

Each capacity definition method based on optimization aims to define a capacity that maximizes [19, 27, 30] or minimizes a function [18, 19, 31, 32] with respect to constraints induced from preference relations. The major drawback of almost all methods is that they may not necessarily lead to unique capacity that fulfills DM’s preferential information, and selection of the ultimate solution among all compatible ones is done arbitrarily. Furthermore, some methods lead to uneven capacity which decreases their reliability in predicting the utility of alternatives not included in \(A^\star\) [19]. In order to overcome these potential problems, some methods, such as maximum entropy and minimum variance approaches, have been developed.

All methods, based on optimizing an objective function [18, 27, 30-33], lead to solutions that may provide more information than whatever can be inferred from preferences of DM, so DM gets confused and cannot fully interpret the actual meaning of these maximizations or minimizations. Actually, it is much preferred to define the capacity only with respect to information introduced by DM [34]. In order to overcome the mentioned drawback, Robust Ordinal Regression is proposed which takes into account all sets of parameters (utility functions) compatible with the preferences of DM. This approach uses the primary preferential information of DM to suggest robust comparisons among alternatives, criteria, etc.

The first Robust Ordinal Regression method proposed is UTA\(^\text{GMS}\) [26], which is a generalization of UTA multi–criteria method. The most important generalized points are that UTA\(^\text{GMS}\) requires partial preorder instead of complete preorder required in UTA and considers all additive utility functions compatible with the preferences of DM, while UTA considers only one such function. By considering all compatible utility functions, it becomes possible to use robust ordinal regression method [26].
The preference information used in UTA\textsuperscript{GMS} is a set of pairwise comparisons provided by DM on set $A'$ [26]. Then, by means of linear programming, two binary relations on set $A$ will be defined; necessary preference relation (NPR) which holds for any pair $a, b \in A$ if all compatible utility functions assign a value to $a$ not smaller than the value assigned to $b$, and possible preference relation (PPR) which holds for any pair $a, b \in A$ if at least one compatible utility function assigns a value to $a$ not smaller than the value assigned to $b$ [21, 28]. NPRs are robust with respect to indirect preference information because any pair of alternatives with NPR will be ranked in the same way whatever the compatible utility function. NPR is partial preorder, i.e., reflexive and transitive, and PPR is strongly complete and negatively transitive [26].

Generalized Regression with Intensities of Preference (GRIP) [35] is the generalization of UTA\textsuperscript{GMS} that takes into account not only the preferences on alternatives, but also the intensity of preferences among pairs of alternatives [21, 28, 35]. GRIP is similar to MACBETH method with the exception that MACBETH requires almost complete preference relations, while GRIP has the ability to provide a solution based on the preferences about which the DM is certain [35]. Non–additive robust ordinal regression (NAROR) [21] is a method inspired from UTA\textsuperscript{GMS} and GRIP and can be used when utility function is CI. NAROR uses the same procedure proposed in UTA\textsuperscript{GMS} and GRIP in defining NPRs and PPRs when utility function is CI. The extension of NAROR to bipolar and level-dependent CI has been proposed in [29].

The most representative utility function [28] utilizes NPRs and PPRs to define a utility function that demonstrates in the best way the NPRs and PPRs with the purpose of helping DM to have a better interpretation of results of NAROR method [28]. This paper has improved the algorithm proposed in [28] by considering evenness property when defining a robust capacity which is the representative, see section Error! Reference source not found.. The proposed algorithm has been used in MAPNA Special Projects Construction & Development Co.’s (MD–3) terminated projects evaluation, section 0.

In this paper, note that for the simplicity reasons, the cardinality of subsets $S, T, ...$ will be shown by lower case letters $s, t, ...$; the brackets $\{} $ are omitted for small cardinality subsets. For example, $(N \setminus i)$ is used instead of $(N \setminus \{i\})$, $\mathcal{P}(N)$ indicates power set of $N$, $(1_s, 0_{N \setminus A})$ represents a binary alternative, i.e., an alternative that has complete satisfaction level in the criteria included in $A$ and complete non–satisfaction level in remaining criteria. $E(.)$ represents the expected value.

3. Basic definitions of Choquet integral

3.1. Choquet integral as an aggregation function

**Definition 1** – A capacity on $N$ is set function $\mu: \mathcal{P}(N) \to [0, 1]$ with $\mu(\emptyset) = 0, \mu(N) = 1$ (boundary condition), and if $S, T \subseteq N, S \subseteq T \Rightarrow \mu(S) \leq \mu(T)$ (monotonicity condition); Monotonicity means that the weight of subset of criteria can only increase when one introduces a new criterion to the subset [20, 22, 24]. $\mu(S)$ is the weight or importance of coalition $S$ of criteria [20, 22, 24]. Capacity $\mu$ on $N$ is additive if $\mu(S \cup T) = \mu(S) + \mu(T)$ for any $S, T \subseteq N, S \cap T = \emptyset$. When fuzzy measure is additive, utility function can be defined by $n$ parameters, whereas general capacity requires defining $2^n - 2$ parameters, knowing that $\mu(\emptyset) = 0, \mu(N) = 1$.

**Definition 2** – The Möbius representation of capacity is defined by set function $m: 2^{N} \to \mathbb{R}$

Capacity could be defined in terms of Möbius representation:

$$\mu(R) = \sum_{T \subseteq R} m(T), \forall R \subseteq N. \quad (3)$$

The Möbius representation can be obtained from the capacity by:

$$m(R) = \sum_{T \subseteq R} (-1)^{|T|} \mu(T), \forall R \subseteq N \quad (4)$$

Boundary and monotonicity conditions could be defined in terms of Möbius representation by $m(\emptyset) = 0, \sum_{T \subseteq N} m(T) = 1$ and $\sum_{T \subseteq R} m(T) \geq 0, \forall R \subseteq N, \forall i \in R$, respectively.

**Definition 3** – The Choquet integral [36] of $x$ with respect to capacity $\mu$ is defined by [20, 23, 37]:

$$C_\mu(x) = \sum_{i=1}^{n} x_i \mu(A(i)) - \mu(A(i+1)) = \sum_{i=1}^{n} \mu(A(i)) [x_i - x_{i-1}] \quad (5)$$

Where $\cdot$ indicates a permutation on $N$, such that $0 \leq x_{(1)} \leq x_{(2)} \leq ... \leq x_{(n)} \leq 1$ and $A(i) = \{i, \ldots, (n)\}$ for all $i \in \{1, \ldots, n\}$ and $A(i+1) = \emptyset$ and $x_{(0)} = 0$. For instance, if $x_2 \leq x_3 \leq x_1$, we have:

$$C_\mu(x_1, x_2, x_3) = x_2 [\mu([2,3,1]) - \mu([3,1])]
+ x_3 [\mu([3,1]) - \mu(1)]
+ x_1 [\mu(1) - \mu(0)]$$

CI can be defined in terms of Möbius representation by:

$$C_\mu(x) = \sum_{T \subseteq N} m(T) A_{\subseteq T} x_i \quad (6)$$

3.2. Behavioral analysis of Choquet integral

In order to have a better comprehension of behavioral properties of Choquet integral, some indices have been introduced. The most
important ones are importance index or Shapley value [38], interaction index or Murofushi–Soneda interaction index [39] entropy [40, 41] and variance [18], which are defined as follows:

**Definition 4**—The Shapley value of criterion \(i \in N\) with respect to capacity \(\mu\) is commonly used as its importance index:

\[
\vartheta_i = \sum_{T \subseteq N\setminus i} \binom{n-t-1}{n-1} \left[ \mu(T \cup i) - \mu(T) \right],
\]

\(\forall i \in N\) \hspace{1cm} (7)

The importance of each criterion \(i \in N\) should be measured considering its global effects on decision problem, i.e., the effect of criterion when considered alone \(\mu(i)\) and its effect when combined with other coalitions of criteria not containing \(i\), \(\mu(T \cup i), T \subseteq N\setminus i\). So, the Shapley value can be interpreted as a weight average value of the marginal contribution \(\mu(T \cup i) - \mu(T)\) of element \(i\) in all combinations. This index fulfills the following properties [25]:

1. \(\vartheta_i \geq 0\), \(\forall i \in N\)
2. \(\sum_{i=1}^{n} \vartheta_i = 1\)

The equivalent formula for importance index in terms of Möbius representation is:

\[
\vartheta_i = \sum_{T \subseteq N\setminus i} \frac{m(T \cup i)}{t+1} \hspace{1cm} (8)
\]

**Definition 5**—Interaction index between criteria \(i, j \in N\) with respect to the value \(\mu(T)\) is measured by Murofushi–Soneda interaction index:

\[
I(ij) = \sum_{T \subseteq N\setminus \{i, j\}} \binom{n-t-2}{n-1} \left[ \mu(T \cup i \cup j) - \mu(T \cup i) - \mu(T \cup j) + \mu(T) \right]
\]

or equivalently in terms of Möbius representation:

\[
I(ij) = \sum_{T \subseteq N\setminus \{i, j\}} \frac{m(T \cup i \cup j)}{t+1} \hspace{1cm} (10)
\]

measures the difference between marginal contribution of criterion \(j\) in the presence of \(i\) and its marginal contribution in the absence of \(i\). If these criteria are positively correlated or competitive (resp. negatively correlated or complementary), the sign of this expression will be \(\leq 0\) (resp. \(\geq 0\)). Interaction index of \(i, j\) is the mean value of this marginal interaction in the presence of any subset of criteria excluding \(i, j\) [25].

**Definition 6**—Entropy is a measure of uniformity or evenness of capacity. Some indices have been introduced to measure the entropy of capacity; Marichal entropy \(H_M(\mu)\) [41] and Havrda and Charvat entropy of order \(\beta\) [42] are two (cases) of such measures. The entropy measure calculates the average value of contribution of partial scores to calculate aggregated value \(C(\mu)\) [40]:

1. If the entropy is close to its maximum value, then all partial scores contribute almost equally to aggregated value, so the aggregation function behaves like WAM.
2. If the entropy is close to its minimum value, then one of partial scores contributes much more than the others to aggregated value, so aggregation function has disjunctive or conjunctive behavior.

Since most capacity definition models are based on preference information given in reference alternatives, defining its parameters as close to WAM as possible increases its reliability in evaluating the alternatives that have not contributed to capacity definition (non-reference alternatives).

Havrda and Charvat entropy of order \(\beta\) is a generalization of Shannon entropy. The extension of this entropy to capacity is given by [18]:

\[
\overline{H}_MC^\beta(\mu) = \frac{1}{1-\beta} \left[ \sum_{i \in N} \sum_{S \subseteq N\setminus i} \gamma_S(n) \mid \mu(S \cup i) - \mu(S) \mid^{1-\beta} \right], \hspace{1cm} (11)
\]

**Definition 7**—Variance of capacity can be defined by equation (12) and its formula in terms of Möbius representation by equation (13) [18, 25].

\[
\overline{V}(\mu) = \frac{1}{n} \sum_{i \in N} \sum_{S \subseteq N\setminus i} \gamma_S(n) \left[ \mu(S \cup i) - \mu(S) - \frac{1}{n} \right]^2, \hspace{1cm} \gamma_S(n) = \frac{(n-s-1)!}{n!} \hspace{1cm} (12)
\]

\[
\overline{V}(m) = \sum_{S \subseteq N} m(S)m(T) \frac{s+t-|SUT|}{(s+1)(t+1)(|SUT|+2)}, \hspace{1cm} (13)
\]

It is obvious that for any capacity \(\mu\) on \(N\), there is a linear relationship between Havrda and Charvat entropy of order 2 and variance of capacity [18]:

\[
\overline{H}_MC^2(\mu) = \frac{n-1}{n} - n \overline{V}(\mu) \hspace{1cm} (14)
\]

So, maximizing \(\overline{H}_MC^2\) is equivalent to minimizing \(\overline{V}(\mu)\). In this paper, variance of capacity is used as the measure of uniformity, and the capacity with less variance value is much uniform.

Defining \(2^n - 2\) parameters involves time and space complexity, so in most cases, \(k\)–additive capacity [23] is used which takes into account the interactions between up to \(k\) criteria \(k \in \{1, ..., n\}\) and decreases the number of parameters to be defined as \(\sum_{i=1}^{k} \binom{n}{i}\).

**Definition 10**—A capacity is said to be \(k\)–additive if:
Suppose that \( E^\varepsilon \) is a set of constraints including the constraints inferred from preference relations and boundary and monotonicity conditions as follows [21]:

1. \( a \succeq b \Leftrightarrow C_\mu(a) \preceq C_\mu(b) + \varepsilon \), with \( a, b \in A \)
2. \( (a, b) \succeq^* (c, d) \Leftrightarrow C_\mu(a) - C_\mu(b) \preceq C_\mu(c) - C_\mu(d) + \varepsilon \), with \( a, b, c, d \in A \)
3. \( i \preceq j \Leftrightarrow \theta(i) \preceq \theta(j) + \varepsilon \), with \( i, j \in N \)
4. \( (i, j) \succeq^* (l, k) \Leftrightarrow \theta(i) - \theta(j) \preceq \theta(l) - \theta(k) + \varepsilon \), with \( i, j, l, k \in N \)
5. \( i_{ij} \leq -\varepsilon \) or \( l_{ij} \geq \varepsilon \), with \( i, j \in N \)
6. \( (i, j) \geq_{\text{int}} (l, k) \Leftrightarrow l_{ij} \geq l_{ik} + \varepsilon \), with \( i, j, k, l \in N \)
7. \( [(i, j), (l, k)] \geq_{\text{int}} [(r, s), (t, w)] \Leftrightarrow |l_{ij} - l_{ik}| \geq |l_{rw} - l_{nw}| + \varepsilon \), with \( i, j, k, l, r, s, t, w \in N \)
8. \( m(\emptyset) = 0, \sum_{i \in N} m(i) + \sum_{(i, j) \in N} m(i, j) = 1 \)
9. \( m(i) \geq 0, \forall i \in N, m(i) + \sum_{j \neq i} m(i, j) \geq 0, \forall i \in N, \forall T \subseteq N \setminus \{i\} \)

The preferences of DM are given by partial preorder \( \succeq^* \) which can be decomposed to its asymmetric \( > \) and symmetric part \( \sim \) whose semantics are respectively [21, 28]:

\[ a \succ b \iff a \text{ is preferred to } b \text{ with } a, b \in A \]

\[ a \sim b \iff a \text{ is indifferent to } b \text{ with } a, b \in A \]

If \( E^\varepsilon \) be a consistent system, it could lead to a set of compatible fuzzy measures. Thus, by establishing two linear programs (LPs), i.e., P1 and P2, NPR \( (\leq^\varepsilon) \) and PPR \( (\succeq^\varepsilon) \) on alternatives can be defined as follows [21, 28]:

1) Necessary preference relation NPR: \( x \leq^\varepsilon y \), \( x, y \in A \) if for all compatible fuzzy measures \( x \) is preferred to \( y \), i.e., \( C_\mu(x) \geq C_\mu(y) \).

2) Possible preference relation PPR: \( x \succeq^\varepsilon y \), \( x, y \in A \) if and only if for at least one compatible fuzzy measure \( x \) is preferred to \( y \), i.e., \( C_\mu(x) \geq C_\mu(y) \).

The LPs are [21]:

P1: \[ \max \varepsilon \]

s.t. \( E^\varepsilon \) plus constraint \( C_\mu(y) \geq C_\mu(x) + \varepsilon \)

P2: \[ \max \varepsilon \]

s.t. \( E^\varepsilon \) plus constraint \( C_\mu(x) \geq C_\mu(y) \)

NPRs and PPRs could be defined with respect to the sign of \( \varepsilon \) in P1 and P2, respectively. If constraints of \( E^\varepsilon \) constitute a consistent system, the following elicitations can be inferred [21]:

1. Non–positive value for \( \varepsilon \) in P1 means that there is no compatible fuzzy measure for which \( y \leq^\varepsilon x \), so \( x \leq^\varepsilon y \). This elicitation is derived from the property of NPR and PPR defined in [26] which indicates that for all \( a, b \in A \) either \( a \preceq^\varepsilon b \) or \( b \preceq^\varepsilon a \).

2. Positive value for \( \varepsilon \) in P2 concludes that \( x \succeq^\varepsilon y \).

3.4. The most representative utility function

The most representative utility function [28] uses NPRs and PPRs to define a utility function that demonstrates in the best way the NPRs and PPRs with the purpose of helping DM to have a better interpretation of results of NAROR method. This utility function is obtained by maximizing the difference between the values assigned by CI to pairs of alternatives for which there is NPR and minimizing the difference between scores of pairs for which there is no such relation [28]. The algorithm for calculating the most representative capacity definition is as follows [28]:

4. The proposed algorithm

An algorithm is proposed which uses NPRs and PPRs to find a representative capacity with evenness property among all compatible ones. Variance of capacity is used as a measure of evenness. As \( \varepsilon \) belongs to unit interval in constraints 3–7 of \( E^\varepsilon \), the partial scores should be translated to \([0, 1]\) interval in order to let \( \varepsilon \) belong to unit interval in all constraints, so the
results will be more reasonable. The steps of algorithm are as follows:
1. Establish the necessary and possible preference relations on set A of alternatives.
2. Add the set of constraints \( E^o \) to constraints \( C_i(x) \geq C_i(y) + Y \) for all pairs \((x, y) \in A \times A\), such that \( x \succeq y \) and \( y \succeq x \), i.e., \( x \succeq y \).
3. Compute \( \max Y \).
4. Let \( Y \) obtained in the previous point, be equal to \( Y^* \) and add constraint \( Y = Y^* \) to the set of constraints of step 2.
5. For all pairs of alternatives \((x, y) \in A \times A\) such that \( y \succeq x \) and \( x \succeq y \), which are the pairs of alternatives such that \( x \succeq y \) and \( y \succeq x \), add the constraints \( C_i(x) \geq C_i(y) + \delta \) and \( C_i(y) \geq C_i(x) + \delta \) to the set of constraints of point 4.
6. Compute \( \min \delta \).

The following algorithm can be enriched by considering the following points:
- The constraints inferred from necessary preference relations represent the preferences regarding the alternatives, so while adding these constraints to the system, there is no need to keep the primary preferences of DM on alternatives preferences.
- If the primary preferences of DM be consistent, then we should maximize the difference of preference not only on the alternatives which are necessarily preferred to each other, but also on the criteria, interaction indices, and the intensity of preferences. One can declare that the necessary preferences are derived from primary preferences of DM; this is a true point, but how can one define the contribution of each constraint to the necessary preference inference? Actually, all the primary preferences of DM should have the same importance in capacity definition problem, and the difference between all preferences should be maximized either as the primary or secondary preferences.

In most cases, adding all sets of constraints derived from the possible preferences of DM of type, i.e., \( C_i(x) \geq C_i(y) + \delta \) and \( C_i(y) \geq C_i(x) + \delta \), empties the solution space.
1. Get the preference information of DM as described in 3-3
2. Establish the system of constraints based on DM’s preference relations and boundary and monotonicity conditions.
3. Check the consistency of constraints; if the system is consistent, go to step 6, else go to step 4.
4. Use the method proposed in [43] or any other method to determine all subsets of constraints with the smallest cardinality causing inconsistency and ask DM to select the subset with the least importance to be revised or deleted in the next step.
5. Ask DM to revise the preference relations, then go to step 3.
6. Define the NPRs and PPRs on the whole alternative set A, as defined in 3-3.
7. If the NPRs and PPRs are acceptable, go to step 8, else go to step 5.
8. Add the set of constraints \( C_i(x) \geq C_i(y) + Y \) for all couples \((x, y) \in A \times A\) to constraint \( E^b \), such that \( x \succeq y \) and \( y \succeq x \), i.e., \( x \succeq y \).
9. Establish two LP problems Pr1 and Pr2 with objectives \( Z1 = \max Y \) and \( Z2 = \max \varepsilon + Y \), respectively, subjected to the constraints of step 8. Calculate the variances of capacity related to these systems; \( V1 \), \( V2 \), respectively.
10. Define \( V = \min\{V1, V2\} \). Let \( \varepsilon^* \) and \( Y^* \) be the values of \( \varepsilon \) and \( Y \), respectively, in the system that lead to \( V^* \).
11. Add constraints \( \varepsilon \geq \varepsilon^* \) and \( Y \geq Y^* \) to the set of constraints of step 8.
12. Add constraints \( C_i(x) \geq C_i(y) + \delta \) and \( C_i(y) \geq C_i(x) + \delta \) for each pair \((x, y) \in A \times A\), such that \( y \succeq x \) and \( x \succeq y \), i.e., \( x \succeq y \).
13. Compute \( \min \delta \).

The algorithm’s flowchart is given in Figure 1. This algorithm is a modification of algorithm proposed in [28]. After defining the necessary and possible preference relations, the primary algorithm solves only problem Pr1 which aims to maximize \( Y \) which is a variable defined when necessary preference relations are imposed on the problem in step 8 of algorithm. Therefore, maximizing \( Y \) prioritizes the contribution of constraints related to the necessary preference relations over the contribution of the previous constraints in defining capacity. Consequently, the results are expected to be less even. Our algorithm solves Pr2 as well as Pr1. Pr2 maximizes \( \varepsilon + Y \). In this way, the whole set of constraints contributes to solve the problem, and higher evenness is expected when capacity is defined in this way. This property will concentrate on the edge points of linear programming around the central part of solution space, and subsequently, the entropy will be higher by escaping from extreme points. When
both Pr1 and Pr2 are solved, the results’ evenness will be compared. As it is shown in next sections, Pr2 leads to more even results and its results are used in making constraints for the next steps of algorithm, which are the basis for defining an even capacity. This is the process of defining a representative capacity which has also evenness property to some extent.

![Algorithm’s flowchart](image)

**Fig. 1. Algorithm’s flowchart**

5. Application of the Proposed Approach

Project performance evaluation calculates the satisfaction level of the projects’ objectives and presents an opportunity to become aware of strengths and weaknesses in performance. MAPNA Special Projects Construction & Development Company (MD–3) has evaluated its six terminated power plant projects. MD–3 is a project–based company operating mostly in management and execution of power plant, power and steam, and utility construction projects.

5-1. Determining the Criteria

Many criteria have been used to evaluate the projects based on evaluator’s expectation. This article introduces the general and case-specific criteria to be considered in this company’s projects evaluation. The criteria are classified in two groups: objective and subjective. These criteria are: cost (C), time (T), quality (Q), Customer Satisfaction (CS), and health, safety and environment (HSE). This paper introduces the criteria that are derived from the conditions in which the project is executed and cannot be controlled by project-oriented organization: Priority (P), Project Operational Environment (POE), and Organizational Experience (OE). More detailed explanations are given in the following subsections.

5-1-1. Objective criteria

The objective criteria (cost, time, quality, HSE, and Customer Satisfaction) are frequently used in project evaluation as described in the literature review section. These criteria can be measured based on their associated formulas that the company’s specialists have recently developed. The formulas provide the partial score in [0,100] scale that have been divided by 100 and are given in Table 1.

Time, cost, and quality are common success criteria in almost all project success models. The cost criterion shows how much the project is successful in execution under the approved budget. The time criterion represents how much the project is successful in achieving the key milestones on time. The quality criterion defines the quality of the project performance in fulfilling the project quality requirements. HSE criterion represents the rate of fatal and non-fatal injuries in project execution process. As the case study projects come from construction industry, it is obvious that a weak performance in HSE could violate the iron triangle performance. Finally, customer satisfaction is a critical criterion which helps the performing organization to undertake future projects with the same client.

5-1-2. Subjective Criteria

The partial scores in subjective criteria (priority, project operational environment, and organizational experience) have been calculated based on questionnaire survey; the experts that participated in the project during its execution were asked to define the projects’ rating on each
criterion based on five-point Likert scale. Specific numerical value is associated with each linguistic term. The partial scores were calculated as the average value of the responses. The basic definitions of these criteria are given as follows: Priority assessment actually measures the priority of each project when assigning organizational resources. Project Operational Environment illustrates the economic environment of project, i.e., the sanction and market price variations. Whenever prices are much stable and sanction is ignorable, the partial score will be higher. Organizational Experience defines the experience and knowledge of organization in execution of the previous projects. Table 1 contains the partial scores in [0,1] scale. The algorithm proposed in this paper and the most representative utility function proposed in literature are used as aggregation operators and the projects scores with respect to aggregation operators are summarized in Table 1.

<table>
<thead>
<tr>
<th>Tab. 1. Partial and aggregated scores</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Partial scores</strong></td>
</tr>
<tr>
<td><strong>Objective criteria</strong></td>
</tr>
<tr>
<td>Project_1</td>
</tr>
<tr>
<td>Project_2</td>
</tr>
<tr>
<td>Project_3</td>
</tr>
<tr>
<td>Project_4</td>
</tr>
<tr>
<td>Project_5</td>
</tr>
<tr>
<td>Project_6</td>
</tr>
<tr>
<td><strong>Avg.</strong></td>
</tr>
<tr>
<td><strong>Std.</strong></td>
</tr>
</tbody>
</table>

5-2. Determining the Preference Information
Two well–experienced experts of company were invited to participate in an interview in order to define the preference information. The basic concepts of model have been explained to them and they provide the following preference relations:

5-2-1. Preference on Criteria
Objective criteria are the five most important criteria in MD-3’s projects evaluation according to the results of the previous research done (executed) in this company [44]. The experts have judged that objective criteria are more important than subjective ones and keep the same ranking as mentioned in [44], i.e., the ranking from most to least important is cost (C), time (T), quality (Q), client satisfaction (CS), and HSE. In recent interviews, they have ranked the subjective criteria from most to least important as: priority (P), project operational environment (POE), and organizational experience (OE). So, the complete ranking would be as follows: C ≻ T ≻ Q ≻ CS ≻ HSE ≻ P ≻ POE ≻ OE.

5-2-2. Preferences on alternatives
- Project_4 has better performance than that of project_5 according to quality, client satisfaction, and HSE. They have the same performance in time and almost the same performance in cost, so project_4 ≻ project_5.
- Project_5 has better performance than that of project_6 according to the first four criteria, so project_5 ≻ project_6.
- Project_4 dominates project_3 in the first five important criteria, so project_4 ≻ project_3.
- Project_2 dominates project_3 in all criteria, so project_2 ≻ project_3.

5-2-3. Interaction between criteria
- There is a negative interaction between priority and time performance.
- There is a negative interaction between organizational experience and quality of project.
- There is a negative interaction between project operational environment and cost of project.
- There is a positive interaction between quality and cost performance.
The constraints inferred from preference relations are as follows:
1. project_4>project_5 ⇒ C_µ(4) ≥ C_µ(5) + \varepsilon
2. project_5>project_6 ⇒ C_µ(5) ≥ C_µ(6) + \varepsilon
3. project_4>project_3 ⇒ C_µ(4) ≥ C_µ(3) + \varepsilon
4. project_2>project_3 ⇒ C_µ(2) ≥ C_µ(3) + \varepsilon
5. project_1>project_2 ⇒ C_µ(1) ≥ C_µ(2) + \varepsilon
6. C ∈ T ⇒ \theta(C) ≥ \theta(T) + \varepsilon
7. T ≥ Q ⇒ \theta(T) ≥ \theta(Q) + \varepsilon
8. Q ≥ CS ⇒ \theta(Q) ≥ \theta(CS) + \varepsilon
9. CS ≥ HSE ⇒ \theta(CS) ≥ \theta(HSE) + \varepsilon
10. HSE ≥ P ⇒ \theta(HSE) ≥ \theta(P) + \varepsilon
11. P ≥ POE ⇒ \theta(P) ≥ \theta(POE) + \varepsilon
12. POE ≥ OE ⇒ \theta(POE) ≥ \theta(OE) + \varepsilon
13. l_P T ≤ −\varepsilon
14. l_OE Q ≤ −\varepsilon
15. l_POE C ≤ −\varepsilon
16. l_Q C ≥ \varepsilon

These relations together with the boundary and monotonicity conditions form a consistent system of constraints, E^A. Using MATLAB programming software, problems P1 and P2 were solved for any pair of projects in order to define NPRs and PPRs. DM approved all the NPRs and PPRs, so the related constraints were added to the problem according to step 8. Pr1 and Pr2 were solved and their provided solutions’ evenness was measured using equation 13.

The capacities related to Pr1 and Pr2, in terms of Möbius representation, are shown in Figure 2; dashed line represents the capacity defined by Pr1, and continuous line represents the one defined by Pr2. The points are connected with line in order to make the visual comparisons of capacities’ evenness easier. The first eight coefficients represent m(i) for i=1,...,8, and the next ones represent m(i,j) for i = 1,...,8, j = i + 1,...,8 or equal interactions between pairs of criteria. For example, m(1,3) is a negative value in figure and shows a negative interaction between the first and third criteria. This relation either belongs to primary preferences of decision maker or inferred from them through NPR set and added to problem in step 8. Pr1 focused on this relation more strongly and provided a more negative interaction to this pair of criteria in comparison with Pr2. The same illustration can be made about m(1), m(3), and m(2,4). In general, according to the figure, dashed line navigates through points closer to extreme points and provides less even capacity compared to continuous line. Pr1 and Pr2 were solved, and results are summarized in table 2. The variance of capacity defined by Pr2 equals 0.0166 which is much smaller than that of Pr1, i.e., 0.0253. So, the values of ε’ and Y’ were picked from the results of Pr2, and consequently, constraints ε ≥ 0.0269, Y ≥ 0.1183 were added to the problem to ensure achieving at least the same evenness in the next steps. Then, problem min δ was solved in order to impose the effect of possible preferences on capacity definition. This problem could not find a feasible solution. Ultimately, the algorithm terminated with the capacity calculated by Pr2 as the best solution provided. According to the aforementioned explanation, if the basic algorithm was used for capacity definition problem, it would not fulfill the evenness property.

The capacity is used for all projects’ evaluation (both reference and non-reference projects), and the results are given in table 1. The results obtained by primary algorithm (Pr1) are also given in table 1 for a better comparison. Comparing the scores of projects by our algorithm and primary algorithm, the preferences of decision-makers about the projects scores are fulfilled in both methods. But, our algorithm provides more even results. For example about constraint project_5>project_6 ⇒ C_µ(5) ≥ C_µ(6) + \varepsilon, the difference of scores between these two projects is 0.119, while it equals 0.177 in previous method because it focuses on constraints more sharply and provides results closer to extreme points. This conclusion works on all constraints related to comparisons of projects, i.e., projects pairs (4,3), (2,3), and (1,2), except (4,5). And finally, standard deviation of the projects’ scores obtained by our algorithm equals 0.066, while it is 0.083 by using primary algorithm. This is also another evidence for evenness of our results.
6. Conclusion

Some methods have been proposed to define a capacity based on preference information of decision maker. Two main approaches in capacity definition area are methods based on robustness and evenness considerations. Since only a limited number of alternatives are present in preference information, in capacity definition, the methods based on evenness aim to improve the capacity’s reliability in evaluating the alternatives that do not belong to preference information. On the other hand, the methods based on robustness considerations lead to a capacity defined solely based on the preference information of DM. Therefore, their solution is comprehensible by DM. The most representative capacity definition method defines a robust capacity that represents the preferences of DM in the best way.

This paper has proposed an algorithm to integrate the advantages of both approaches in capacity definition. It made changes in most repetitive capacity definition method in order to add evenness to its robust results.

Provided capacity is used in evaluating six terminated projects of MAPNA Special Projects Construction & Development Co (MD-3). Two groups of criteria were used in projects evaluation: the objective criteria which include some general criteria selected from the previous studies and the subjective criteria which include case-specific criteria driven from projects environment. Based on the results depicted in figures and tables of the paper, our algorithm has a great ability in obtaining an even capacity which is basically robust. The projects of MD-3 were evaluated with both of the proposed algorithm and primary one. The scores of projects were compared according to decision-maker’s preference information and they provide additional evidence on the evenness of our results.

The proposed algorithm can be used in similar performance evaluation problems in which criteria have interaction. Also, it can be extended to other generalizations of Choquet integral.

Acknowledgement

The authors appreciate the supports of MAPNA Special Projects Construction & Development Co (MD–3) company in particular Mr. Hamidreza Afshari.

References


Follow This Article at The Following Site
