International Journal of Industrial Engineering & Production Research (2009) pp.69-75

June & September. 2009, Volume 20, Number 1 & 2,

International Journal of Industrial Engineering & Production Research

Journal Website: http://IJIEPR.iust.ac.ir/

A Chance Constrained Model for a Two Units Series Critical System Suffering From Continuous Deterioration

Kamran Shahanaghi*, Hamid Babaei & Arash Bakhsha

Kamran Shahanaghi, *is with the Department of Industrial engineering, Iran University of Science & Technology,*
Hamid Babaei, *is a Master student at the Department of Industrial engineering, Iran University of Science & Technology.*
Arash Bakhsha, *is a Master student at the Department of Industrial engineering, Iran University of Science & Technology.*

KEYWORDS

Condition Based Maintenance (CBM), Failure, Cost optimization, Monte Carlo Simulation

ABSTRACT

In this paper we focus on a continuously deteriorating two units series equipment which its failure can not be measured by cost criterion. For these types of systems avoiding failure during the actual operation of the system is extremely important. In this paper we determine inspection periods and maintenance policy in such a way that failure probability is limited to a pre-specified value and then optimum policy and inspection period are obtained to minimize long-run cost per time unit. The inspection periods and maintenance policy are found in two phases. Failure probability is limited to a pre-specified value In the first phase, and in the second phase optimum maintenance thresholds and inspection periods are obtained in such a way that minimize long-run expected.

1. Introduction

Reliability is one of the important issues in the assessment of industrial equipment or products. Good product design is of course essential for products with high reliability. However, no matter how good the product design is, products deteriorate over time since they are operating under certain stress or load in the real environment, often involving randomness. Maintenance has, thus, been introduced as an efficient way to assure a satisfactory level of reliability during the useful life of a physical asset.

The main idea behind Condition Based Maintenance (CBM) is to provide decision support for maintenance actions. As such, it is natural to include maintenance policies in the consideration of the machine prognostic process. The aim of CBM is to optimize the maintenance policies according to certain criteria such as risk, cost, reliability and availability. In those CBM models which cost is taken as optimization criterion, the optimum policy is obtained by minimizing the long run cost per time unit (see [2]).

In general CBM models fall into two categories: completely observable systems and partially observable systems. The state of the system in a completely observable system can be completely observed or identified. First of all we discuss completely observable systems. [1] focus on the analytical modeling of a condition based inspection/replacement policy for a stochastically and continuously deteriorating single unit system. They consider both the replacement threshold and the inspection schedule as decision variables for the problem.

They minimize the long run expected cost per unit time by the stationary law for the system state. [11] utilized a Markov chain to describe the CBM model for a deterioration system subject to periodic inspection the optimal inspection frequency and maintenance threshold were found to maximize the system availability. [3] consider a two unit system which can be maintained by good as new preventive or corrective replacements.

They develop a stochastic model based on the semi-regenerative properties of the maintained system state

*Corresponding author. ***Kamran Shahanaghi***
Email: *shahanaghi@iust.ac.ir*
Email: *babaei_eg@yahoo.com*
Email: *a.bakhsha@yahoo.com*
Paper first received March. 15. 2007 ,and in revised form July. 16. 2008.

and the associated cost model is used to optimize the performance of the maintained model. [8] uses Monte-carlo simulation to model the continuously monitored deteriorating systems. They assume that after each maintenance action a random amount of improvement is made on the state of the system which is independent of current system state. Then the optimized thresholds of maintenance are such found that the total expected cost of system be minimized. [10] considers a continuously deteriorating system which is inspected in random times. In this model, they assume that deterioration follows a gamma distribution and system fails if its condition lies upper than a pre-specified threshold. In their model two types of replacement can be done depending on the fact that system is failed or the condition of system exceeds a critical threshold.
Now we consider partially observable systems. [6], proposes a CBM model which assumes that failure rate of the system depend on the variables of the system state and fixed inspection periods. Then the maintenance action is optimized such that the long term costs of maintenance actions and failures are minimized. [5] suggest an approach based on reliability that inspection periods and maintenance thresholds are such estimated that the global cost per unit time is minimized. [4] build the semi-Markov decision process for the maintenance policy optimization of condition based preventive maintenance problems and present the approach for joint optimization of inspection rate and maintenance policy [7] applied a stochastic recursive control model for CBM optimization based on the assumptions that the item monitored follows a two-period failure process with the first period of a normal life and the second one of a potential failure. A stochastic recursive filtering model was used to predict the residual, and then a decision model was established to recommend the optimal maintenance actions. The optimal condition monitoring intervals were determined by a hybrid of simulation and analytical analysis. [9] determines the length of the next condition monitoring interval for a given risk level.
As discussed earlier, in CBM the maintenance policies are optimized according to certain criteria such as risk, cost, reliability and availability. Risk is defined as the combination of probability and consequence. Usually, consequence can be measured by cost. In this case, risk criterion is equivalent to the cost criterion. However, there are some cases, e.g., critical equipments in a power plant, in which consequence cannot be estimated by cost. For some systems, such as aircrafts, submarines, military systems, and nuclear systems, it is extremely important to avoid failure during actual operation because it can be dangerous or disastrous (see [7]). Therefore, failure probability is more important than cost in such systems. In these scenarios, probability or reliability criterion would be more appropriate. The novelty of this work stems from the fact that failure probability of the series system is restricted to a pre-specified value.

2. Notations and Problem Formulation

- X_t^i State of the system i at time t.
- Y_t^i The amount of deterioration occurred in period t on system i.
- ξ_1 Preventive maintenance threshold.
- ξ_2 Preventive replacement threshold.
- C_{pm} The cost incurred by preventive maintenance action.
- C_{pr} The cost incurred by preventive replacement action which is strictly bigger than C_{pm}.
- $100(1-\alpha_t)$ The percent of improvement made on the system due to preventive maintenance action.
- C_∞ The long-run average cost per time unit.
- $\pi(x,y)$ Stationary law of the deterioration process.
- $E_\pi(.)$ Expected value function at steady state.
- $f(x)$ Probability density function of deterioration occurring during one period.
- $\lambda(X_t)$ Failure rate of the system.
- T The time between two successive inspection periods.
- p Maximum allowed Failure probability determined by decision maker group.
- T_{m} Time to preventive maintenance action completion.
- T_{r} Time to preventive replacement completion.
- T_{Ins} Inspection time
- C_{d} System down time cost per time unit.
- C_{pms} Set up cost of preventive maintenance action.
- C_{prs} Set up cost of preventive replacement action.
- $C(\mathrm{t})$ Cumulative cost per unit time till time t.

Many manufacturing equipment suffer increasing wear with usage which are subject to random failures resulting from this deterioration. Examples of these systems are cutting tools, hydraulic structures, brake linings, airplane engine compressor blades, corroding pipelines and rotating equipment (see [1]).
Also we assume that the system is inspected at equidistant times and time to failure follows an exponential distribution and failure rate is a linear increasing function of system condition. For simplification, failures are assumed to occur at the end of a period. This will not affect the policy much as long as intervals are not too long.
At the end of each period a decision is made to initiate either a preventive maintenance or preventive replacement action according to system condition. The preventive maintenance action is initiated when the state of system

exceeds a threshold ξ_1 and the preventive replacement action is initiated when the state of system exceeds a threshold ξ_2 where $\xi_2 > \xi_1$. Figure 1 shows the deterioration evolution of the system for $X_0^i = 0$.

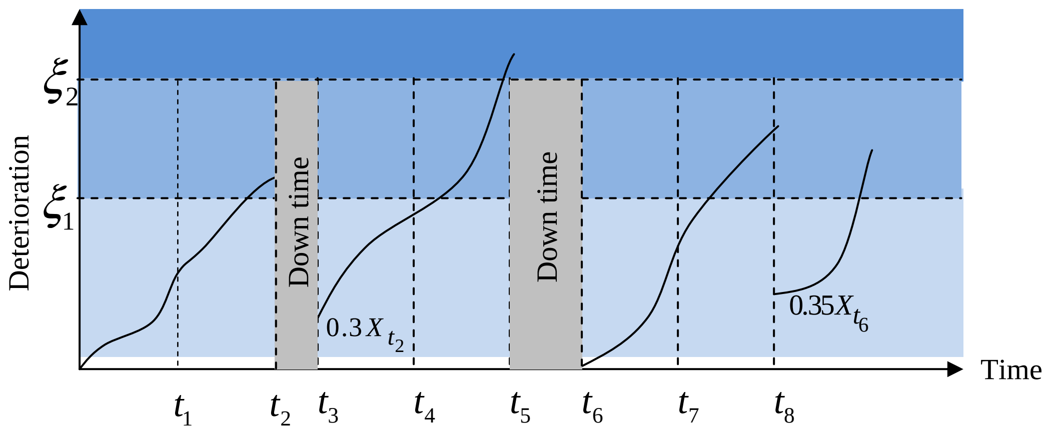

Fig. 1. Simulated behavior of the deteriorating system

For each change of time Δt, the random deteriorations $X_{t+\Delta t}^i - X_t^i$ is assumed to be independent and have the same probability density function. Natural candidates for the associated probability density function can be obtained in the class of infinitely divisible distributions, e.g. gamma distributions. The exponential distribution is a special case of gamma distributions. This distribution is easier to further investigate developments for the evaluation of the maintenance policy. Therefore we assume time to failure follows a non-homogeneous Poisson process, and the failure rate is an increasing function, $\lambda(X_t^i)$, of the variable X_t^i(see [14] for more details). We assume a linear relationship. The reliability of the system is the probability that the system will not fail by the end of time period t. According to Eq.(1), the state of system at the end of period t is $X_{t+1}^i = X_t^i + Y_t^i$, So the reliability of our system at the end of period t is given by:

$$\mathrm{Re} = e^{-\lambda(X_t^i + Y_t^i)T} \tag{1}$$

where T is the time between two successive inspection periods. Formula (1) is identical to the probability that time between failures is greater than T. As discussed in section 1, we also assume that the failure of our critical equipment can not be measured by cost. So in order to avoid failure during operation of the system, the decision maker group constrains failure probability to a maximum allowed value p. Since failure rate depends on the system state, for a large threshold the failure probability would be large. Because in optimum policy the failure probability must not exceed p, first of all we obtain an interval for the thresholds. Then the optimum thresholds are determined by minimizing the cumulative cost per time unit.

These two thresholds are determined such that the cumulative cost of maintenance and failure per time unit is minimized.

If no failure occurs during period t, the condition of system at the beginning of period $t+1$ is:

$$X_{t+1}^i = X_t^i + Y_t^i \tag{2}$$

where Y_t^i is the deterioration occurred in period t.

A stochastic regeneration process is characterized by accumulation of a stochastic input process and an output mechanism that removes all the present quantity whenever it exceeds a critical level. As discussed in section 2, after replacement of the system, it is in the good as new initial state and its future evolution does not depend any more on the past. These replacement times are regeneration points for the system. [12] Shows that for a regeneration process, as time increases the distribution of X_t^i converges to the steady state distribution. The assumption of restoring the system state to X_0^i when system state reaches ξ_2 tells us that after each preventive replacement action the system state is independent of what has happened before.

The long-run average cost per time unit is defined as follows:

$$C_\infty = \lim_{t\to\infty} \frac{E(C(t))}{t} \tag{3}$$

From the regenerative properties of the deterioration process $(X_t^i)_{t\in\mathbb{N}}$ the limit of Eq. (3) at infinity can be changed into a ratio of expectations with respect to the stationary law π over one regeneration cycle at steady state:

$$C_\infty = \frac{E_\pi(C(S))}{E_\pi(S)} \tag{4}$$

where S is the time between two successive regenerative points of the deterioration process at steady state. $E_\pi(S)$ is the expected length of one regenerative cycle at steady state with respect to π. $E_\pi(C(S))$ is the expected cumulative cost incurred by inspections on the first regenerative cycle.

Let (x_1, x_2) and (y_1, y_2) be the system deterioration levels observed at the end of two successive maintenance operations. The possible scenarios are the followings:

Scenario 1: $x_1 < \xi_1,\ x_2 < \xi_1$. The two components are left as they are and the probability density function of amount of occurring deterioration for each unit is $f^{(T)}(y_i - x_i)$.

Scenario 2: $x_1 < \xi_1, \xi_1 < x_2 < \xi_2$ or $x_2 < \xi_1, \xi_1 < x_1 < \xi_2$. The unit for which $\xi_1 < x_i < \xi_2$ is preventively maintained to X_0^i and the other one is left as it is. The probability density function of amount of occurring deterioration will be $f^{(T)}(y_i)$.

Scenario 3: $x_1 < \xi_1, \xi_2 < x_2 < \infty$ or $x_2 < \xi_1, \xi_2 < x_1 < \infty$. The unit for which $\xi_2 < x_i < \infty$ is replaced and the new system state is X_0^i. the probability density function of amount of occurring deterioration for this unit is $f^{(T)}(y_i)$ and for the other unit is as in scenario 1.

Scenario 4: $\xi_1 < x_1 < \xi_2, \xi_2 < x_2 < \infty$ or $\xi_1 < x_2 < \xi_2, \xi_2 < x_1 < \infty$. The unit for which $\xi_1 < x_i < \xi_2$ is preventively maintained to X_0^i and the probability density function of amount of occurring deterioration is as scenario 2. The other unit is replaced and the new system state is X_0^i. the probability density function of amount of occurring deterioration for this unit is $f^{(T)}(y_i)$.

Scenario 5: $\xi_1 < x_1 < \xi_2, \xi_1 < x_2 < \xi_2$. The two components are is preventively maintained to X_0^i and the probability density function of amount of occurring deterioration for each unit is $f^{(T)}(y_i)$.

Scenario 6: $\xi_2 < x_1, \xi_2 < x_2$. The two components are replaced and the probability density function of amount of occurring deterioration for each unit is $f^{(T)}(y_i)$.

$$\begin{aligned}
\pi(y_1, y_2) = {} & \underbrace{\left(\int_0^{\xi_1}\int_0^{\xi_1} \pi(x_1, x_2) f^{(T)}(y_1 - x_1) f^{(T)}(y_2 - x_2)\,dx_1 dx_2\right)}_{\text{Scenario 1}} + \\
& \underbrace{\begin{array}{l}\left(\int_0^{\xi_1}\left(\int_{\xi_1}^{\xi_2} \pi(x_1, x_2) f^{(T)}(y_1)\,dx_1\right) f^{(T)}(y_2 - x_2)\,dx_2\right) + \\ \left(\int_0^{\xi_1}\left(\int_{\xi_1}^{\xi_2} \pi(x_1, x_2) f^{(T)}(y_2)\,dx_2\right) f^{(T)}(y_1 - x_1)\,dx_1\right)\end{array}}_{\text{Scenario 2}} \\
+ & \underbrace{\begin{array}{l}\left(\int_0^{\xi_1}\left(\int_{\xi_2}^{\infty} \pi(x_1, x_2) f^{(T)}(y_1)\,dx_1\right) f^{(T)}(y_2 - x_2)\,dx_2\right) + \\ \left(\int_0^{\xi_1}\left(\int_{\xi_2}^{\infty} \pi(x_1, x_2) f^{(T)}(y_2)\,dx_2\right) f^{(T)}(y_1 - x_1)\,dx_1\right) +\end{array}}_{\text{Scenario 3}} \\
+ & \underbrace{\begin{array}{l}\left(\int_{\xi_1}^{\xi_2}\int_{\xi_2}^{\infty} \pi(x_1, x_2) f^{(T)}(y_2) f^{(T)}(y_1)\,dx_1 dx_2\right) + \\ \left(\int_{\xi_1}^{\xi_2}\int_{\xi_2}^{\infty} \pi(x_1, x_2) f^{(T)}(y_1) f^{(T)}(y_2)\,dx_2 dx_1\right)\end{array}}_{\text{Scenario 4}} \\
+ & \underbrace{\left(\int_{\xi_2}^{\infty}\int_{\xi_2}^{\infty} \pi(x_1, x_2)\,dx_1 dx_2\right) f^{(T)}(y_1) f^{(T)}(y_2)}_{\text{Scenario 5}} \\
+ & \underbrace{\left(\int_{\xi_2}^{\infty}\int_{\xi_2}^{\infty} \pi(x_1, x_2)\,dx_1 dx_2\right) f^{(T)}(y_1) f^{(T)}(y_2)}_{\text{Scenario 6}}
\end{aligned} \tag{5}$$

By integration on the whole state space, the description of the different maintenance actions as is shown in Figure 1 can lead to the following expression of the stationary probability density for the deterioration process at inspection times. where $f^{(T)}(x)$ is T th convolution of probability density function of amount of deterioration occuring during two successive periods.

The evaluation of π is tricky and requires to solve a one sided integral equation. Hence due to complexity of Eq.(5) we do not use stationary distribution of the deterioration process to minimize cumulative cost per time unit.

These implications seem enough to use simulation in minimizing total cumulative cost per time unit.

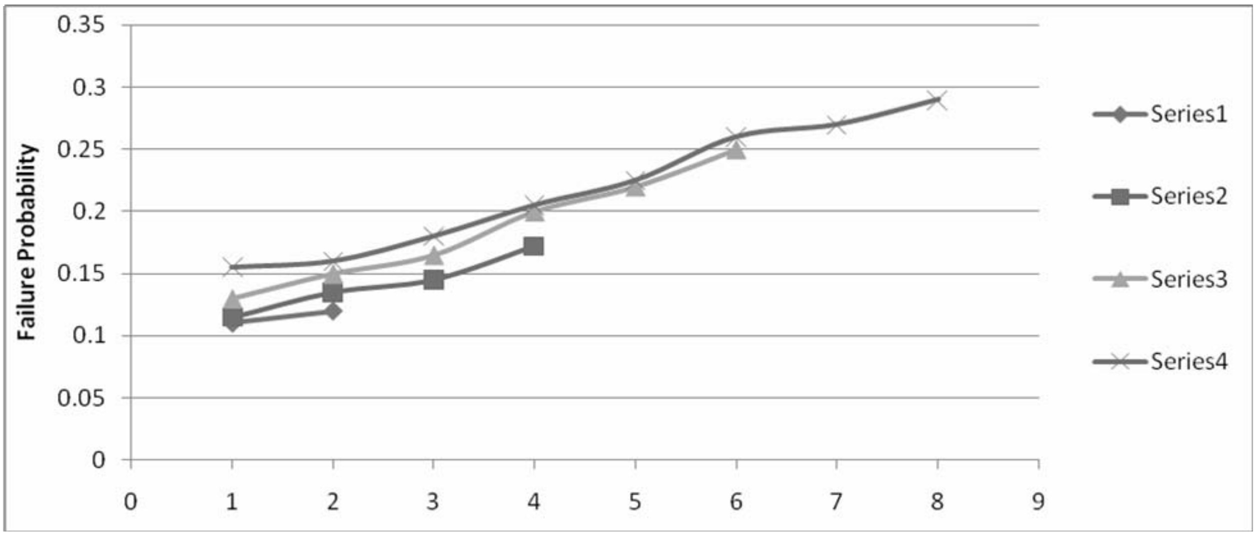

Fig. 2-1. Failure probability of different maintenance policies

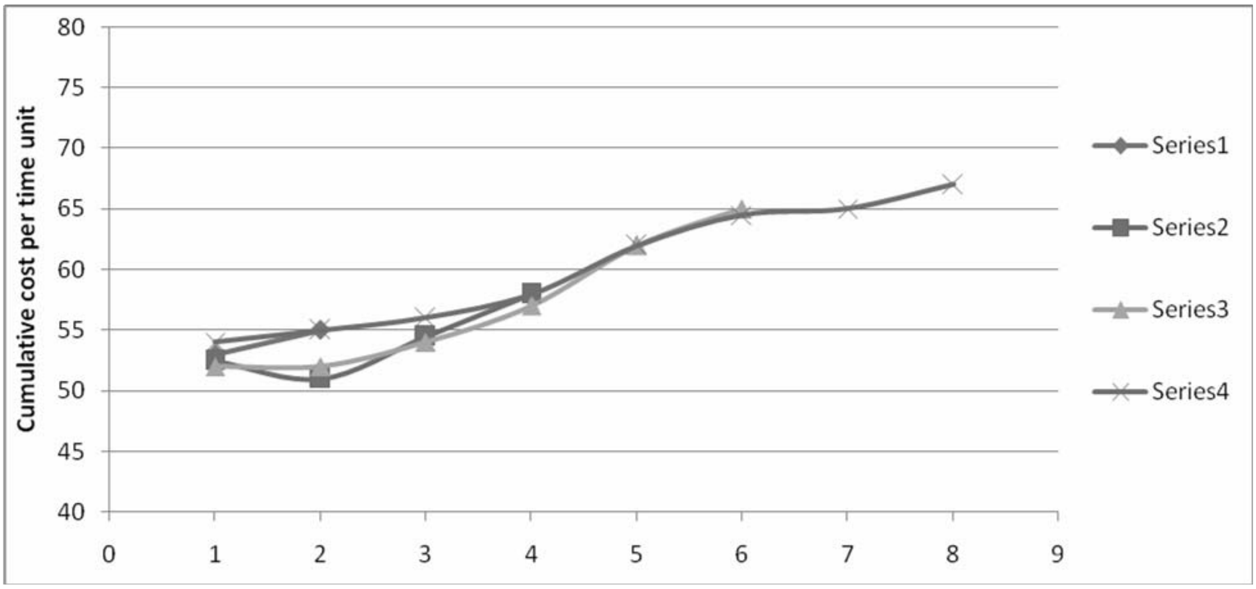

Fig. 2-2. Optimum results for the constrained failure probability

3. Optimization Procedure

In order to simulate continuous deterioration process we discretize the state space. We assume that between exponentially distributed times there exists a small amount of deterioration Δ. Therefore we have $X_{t+1} = X_t + N\Delta$.

As discussed earlier the probability of failure during period t is given by:

$$1 - e^{-\lambda(X_t + Y_t)T} \tag{6}$$

Let triple (ξ_1, ξ_2, T) denote a combination of thresholds and inspection period for our critical deteriorating system. First of all we split feasible space into different thresholds and inspection periods and select several points (ξ_1, ξ_2, T) from the feasible space for simulation.

Then failure probability (see Equation (1)) of the system at infinity is calculated as discussed earlier. Those triples for which failure probability does not exceeds p are selected for the optimization in the next period. In order to obtain more precise solutions, this search can be performed finer.

First of all ξ_1, ξ_2 and T are determined in such a way that the mean of failure probabilities defined by (6) at the end of each period does not exceed p. Taking mean smoothes the influence of outlier failure probability.

In the next stage the optimum value of ξ_1, ξ_2 and T are obtained by minimizing cumulative cost per unit time. Then in the second phase of the algorithm, optimum value of these variables are determined from these intervals.

At the end of inspection period t if the amount of deterioration is bigger than ξ_2 the preventive replacement action is initiated which takes T_{r} units of time to complete. At the beginning of the next inspection period the system state is X_t^i. In this case the cumulative cost per unit time is

$$C(\mathrm{t}) = \frac{(t-1)C(\mathrm{t}\text{-}1) + C_{\mathrm{pr}} + C_{\mathrm{prs}} + T_{\mathrm{r}}C_{\mathrm{d}}}{t} \tag{7}$$

here T_{r} is assumed to follows a lognormal distribution. The preventive replacement and preventive maintenance times can be estimated from the replacement and maintenance time data.

If the amount of deterioration is less than ξ_2 and bigger than ξ_1 the maintenance action is initiated which takes T_{m} units of time to complete and at the beginning of the next inspection period the system state is X_0. In this case the cumulative cost per unit time is

$$C(\mathrm{t}) = \frac{(t-1)C(t\text{-}1) + C_{pm} + C_{pms} + T_m C_d}{t} \tag{8}$$

if the system state is less than ξ_1 the cumulative cost per inspection period is

$$C\,(\mathrm{t}) = \frac{(t-1)C\,(\mathrm{t}\text{-}1) + T_{\mathrm{Ins}}C_d}{t} \tag{9}$$

4. Numerical Results

We simulated this deterioration process for $\Delta = 0.2,\ \mu = 10,\ \lambda = 0.3, C_{\mathrm{pm}} = 1,\ C_{\mathrm{pr}} = 4,\ \ C_{\mathrm{d}} = 16,$ $C_{\mathrm{pms}} = 0.2,\ \ C_{\mathrm{prs}} = 0.5,\ p = 0.2$

Figure 2 shows the obtained results for the objective failure probability for $T = 1$. There are four plots corresponding to $\xi_2 = 2,\ \xi_2 = 4,\ \xi_2 = 6,\ \xi_2 = 8$. It is obvious from Figure 2 that if we intend to limit failure probability to 20%, then the optimum results for (ξ_1, ξ_2, T) are as follows:

$$\begin{aligned}&(1, 1, 1), (1, 2, 1), (1, 2, 2), (1, 4, 1),\\ &(2, 4, 1), (2, 4, 1), (1, 4, 2), (2, 4, 2),\\ &(1, 6, 1), (2, 6, 1), (3, 6, 1), (4, 6, 1),\\ &(1, 6, 2), (2, 6, 2), (1, 8, 1), (2, 8, 1),\\ &(3, 8, 1), (1, 8, 2),\end{aligned} \tag{10}$$

We simulated the deterioration process for 1000 periods in each of these different triples in (10) and choose the optimum policy. The obtained results are shown in figure 3. As is obvious from figure 3 the optimum result is $(2, 4, 1)$. According to figure 2, the failure probability for this policy is nearly 0.12.

5. Conclusions

In this paper we considered a two units series critical system suffering from continuous deterioration. Since failure of this system can be disastrous it had better to find a policy which constrains failure chance to a pre-specified value. The failure probability was obtained for different policies, and then optimum policy based on long-run cost per time unit was found. Adopting this policy guarantees a chance of failure less than 0.2 with the most possible low cost. The complexity of the system motivated us to use simulation to optimize maintenance policy. In order to obtain an optimum result with a high degree of confidence, probability density function of the deterioration process at infinity can be computed and expected cost of maintenance according to this probability density function must be calculated to minimize maintenance costs. This stationary distribution of deterioration process can be obtained from numerical analysis methods as discussed in the paper.

References

[1] Grall, A., Berenguer, C., Dieulle, L., "*A Condition Based Maintenance Policy for Stochastically Deteriorating Systems*" Reliability Engineering and System Safety 76, 2002, pp.167-180.

[2] Jardine, A.K.S., Daming Lin, Dragan Banjevic "*A Review on Machinery Diagnostics and Prognostics Implementing Condition Based Maintenance*" Mechanical Systems and Signal Processing 20, 2006, pp.1483-1510.

[3] Castanier, B., Grall, A., Berenguer, C., "*A Condition Based Maintenance Policy with Non-Periodic Inspections for a Two Unit Series System*", Reliability Engineering and System Safety 87, 2005, pp.109-120.

[4] Chen, D., Kishor, S., Trivedi "*Optimization for Condition Based Maintenance with Semi-Markov Decision Process*" Reliability Engineering and System Safety 90, 2005, pp. 25-29.

[5] Kumar, D., Westberg, U., *Maintenance Scheduling Under Age Replacement Policy Using Proportional Hazards model and TTTplotting,* European Journal of Operational Research 99, 1997, pp. 507 –515.

[6] Barbera, F., Schneider, H., Kelle, P., *A Condition Based Maintenance Model with Exponential Failures and Fixed Inspection Intervals,* Journal of the Operational Research Society 47, 1996, pp. 1037–1045.

[7] Hongzhou Wang, *A Survey of Maintenance Policies of Deteriorating Systems*, European Journal of Operational Research 139, 2002, pp.469–489.

[8] Barata, J., Soares, C.G., Marseguerra, M., Zio, E., *Simulation Modelling of Repairable Multi-Component Deteriorating Systems for 'On Condition' Maintenance Optimisation*, Reliability Engineering and System Safety 76, 2002, pp. 255 –264.

[9] Goode, K.B., Roylance, B.J., Moore, J., *Development of Model to Predict Condition Monitoring Interval Times*, Iron making and Steelmaking 27, 2000, pp. 63–68.

[10] Dieulle, L., Berenguer, C., Grall, A., Roussignol, M., "*Sequential Condition-Based Maintenance Scheduling for a Deteriorating System*" European Journal of Operational Research 150, 2003, pp. 451–461.

[11] Amari, S.V., McLaughlin, L., *"Optimal Design of a Condition-Based Maintenance Model"*, in: Proceedings of the Annual Reliability and Maintainability Symposium, Los Angeles, CA, USA, 2004, pp. 528 -533.

[12] Ross, S.M., Stochastic Processes. Wiley, New York, 1983.

[13] Wang, W.B., *A Stochastic Control Model for on Line Condition Based Maintenance Decision Support, in: Sixth World Multiconference on Systematics*, Cybernetics and Informatics, vol. VI, Proceedings-Industrial Systems and Engineering I, Orlando, 2002, pp. 370 –374.

[14] Feller, W., *An Introduction to Probability Theory and its Applications.* Wiley series in probability andp[]\936.0 mathematical statistics. 2nd edition. New York, 1971