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Performance Evaluation of Continuous Production Process by Triangular Fuzzy Process Capability Indices

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KEYWORDS

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Continuous production process,
Process capability analysi,
Fuzzy set theory,
Fuzzy measurements

ABSTRACT

Process capability indices (PCIs) can be used as an effective tool for measuring product quality and process performance. In classic quality control, there are some limitations which prevent a deep and flexible analysis because of the crisp definition of PCA's parameters. Fuzzy set theory can be used to add more flexibility to process capability analyses. In this study, the fuzzy $\bar{X}$ *and* $MR_{\bar{X}}$ *control charts are introduced to monitor continuous production process in triangular fuzzy state. Also, fuzzy PCIs are produced when SLs and measurements are triangular fuzzy numbers (TFN). For this aim, a computer program is coded in Matlab software. The fuzzy control charts are applied to Yazd fiber production plant. The results show that in continuous production processes, the better analysis will be performed by using fuzzy measurements. Also, based on the fuzzy process capability indices (FPCIs), we can have a flexible analysis of the process performance.*

1. Introduction

Control charts and process capability analysis are two applications of statistical process control. Process analysis is a method that is used in many stages of production and is the essential part of quality control plan. If desired features of a product or service are measurable on numerical scales, then variable control charts are used [1]. Process capability analysis (PCA) can be broadly defined as the ability of a process to meet customer expectations which are defined as specification limits (SLs). The measure of process capability summarizes some aspects of a process's ability to meet SLs, and it is a useful approach to define a relationship between the process ability and SLs. The main outputs of PCA are process capability indices (PCIs) which provide a numerical measure of whether a production process is capable of producing items within the specification limits predetermined by the designer or consumer. If the certain minimum values of PCIs have been obtained, the process is classified as ''capable process''. If these minimum values cannot be

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met, the process is classified as ‘‘incapable process”. Therefore, the process capability index can be viewed as an effective and excellent means of measuring product quality and performance. Stability or statistical control of the process is also essential to the correct interpretation of PCIs. Most of the conventional studies for measuring the process capability are based on crisp estimates, wherein the output process measurements are precise. However, it is not common that the measurements of product quality are insufficiently precise [2]. Fuzzy control charts are inevitable to use, when the statistical data in consideration are uncertain or vague; or available information about the process is incomplete or includes human subjectivity [3]. The major contribution of fuzzy set theory lies in its capability of representing vague data. The fuzzy set theory is used to add more information and flexibility to process capability analyses. Fuzzy logic offers a systematic base to deal with situations, which are ambiguous or not well- defined [2].

One of the cases which is necessary to use fuzzy control charts is a continuous production process in which products are continuously produced without interruption. Because in these processes measuring the considered characteristic is not possible in whole production, it would be more optimal to use a fuzzy measure to perform a more precise analysis of the process, instead of selecting a number as the mean. In those processes where selecting multiple observations is not possible, X and MR_X charts are used for the process control. The purpose of this study is to monitor continuous production process, and in this process, selecting an observation is not significant; therefore, to increase precision and better estimation of parameters, several samples are taken from each observation and their means are then calculated. In such a case, the variability of this process is measured via the moving range of mean of two successive observations. Since this method covers a wider range of products, variance can be estimated more accurately and non-random factors demonstrate its effectiveness. Thus, the fuzzy $\overline{X}$ and $MR_{\overline{X}}$ control charts and triangular fuzzy PCIs are introduced. The model used in this study is coded in MATLAB. The output determines the parameters of the process, control charts limits, and triangular fuzzy capability indices.

In recent years, some papers, which have concentrated on different areas of PCIs using fuzzy set theory, have been published. These areas are as follows:

Kaya and Kahraman (2011) used fuzzy set theory to add more information and flexibility to process capability analyses. For this aim, they introduced fuzzy specification limits based on, fuzzy process capability indices [2]. Hsu & Shu (2008) estimated the loss-based fuzzy process capability index (C_{pm}) [4]. Shu and Wu (2011) proposed the fuzzy X and R control charts, whose fuzzy control limits are based on the results of the resolution identity, a well-known theory in the fuzzy set field [5]. Kaya and kahraman (2010) had robust PCIs (RPCIs) for a piston manufacturing company, and the fuzzy set theory is incorporated to increase PCIs’ flexibility and sensitivity by defining specification limits and standard deviation as fuzzy numbers [6]. Kaya and kahraman (2009 & 2008) estimated the indices C_p and C_{pk} when SLs and σ^2 are fuzzy [7, 8]. Parchami, Mashinchi, Yavari, & Maleki (2005) constructed membership functions of PCIs when the SLs are triangular fuzzy numbers (TFN) [9]. Chachi and Taheri (2011) introduced a new approach to construct the two-sided and one-sided fuzzy confidence intervals for the fuzzy parameter based on normal fuzzy random variables [10]. Abdolshah and et al. (2011) measured C_{pmk} process capability indexes in fuzzy and compared with the other fuzzy indicators [11]. Chen, Lin, & Chen (2003) estimated FPCIs and selected the best supplier [12]. Kaya and Khraman (2011) used the fuzzy set theory to add more information and flexibility to process capability indices with asymmetric tolerances [2]. Lee, (2001) calculated the FPCIs when observations are fuzzy numbers [13]. Ghazanfari and et al. (2008) proposed a novel approach based on clustering techniques to estimate Shewhart control chart change-point when a sustained shift occurs in the process mean [14]. ZareBaghiabad and KhademiZare (2015) used triangular fuzzy numbers to develop an efficient three- stage algorithm [15].

2. Traditional Process Capability Indices

Process capability indices measure the potential efficiency of the process. This index does not take into account process concentration and it merely monitors the variability rate of the process. This criterion can be used to determine the minimum rate of wastage, whereas the rate of wastage can be obtained only when the process concentrates exactly on the middle point between the upper and lower SLs. In other words, when the natural tolerance limits $UNTL=\mu+3\sigma$ and $LNTL=\mu-3\sigma$, are placed in the SLs precisely, the process is efficient; and very limited number of defective products are produced. C_p index can be obtained by using equation (1) [1].

$$C_p = \frac{USL - LSL}{6\sigma} \quad (1)$$

Where σ is the standard deviation of the process, USL and LSL are the upper and lower SLs, respectively. As mentioned above, the $\overline{X}$-$MR_{\overline{X}}$ control charts for continuous production process is used in this study. To estimate the value of σ in these charts, the equation (2) is used.

$$\hat{\sigma} = \frac{\overline{MR_{\overline{X}}}}{d_2(n = 2)} \quad (2)$$

As Figure 1 illustrates, this index is not sensitive to process mean shift.

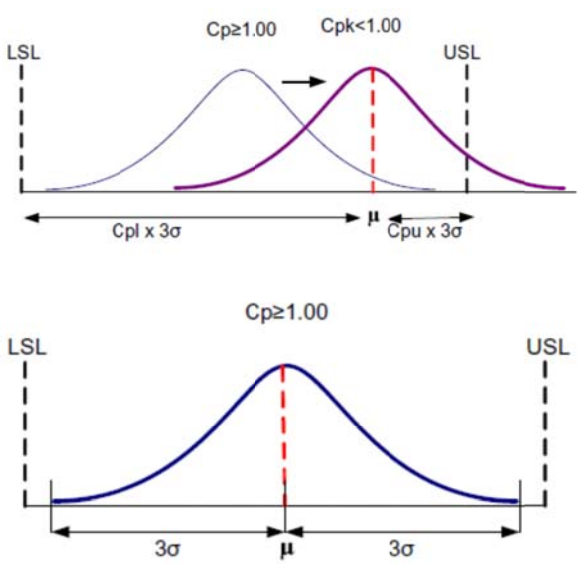

Fig. 1. C_p and C_{pk} indices

If p is defined as follows:

$$P = \left(\frac{1}{C_p}\right)100 \quad (3)$$

Then, P is percent of the gap between the specifications limits, which are used by process [1]. The value of index C_p gives us an opinion about process' performance. For example, if the value of C_p is greater than one, process is efficient. The six quality conditions and the corresponding C_p values are summarized in Table 1 [16].

Tab. 1. Quality conditions and C_p values

Quality condition	C_p value
Super excellent	$2 \le C_p$
Excellent	$1.67 \le C_p \le 2$
Satisfactory	$1.33 \le C_p \le 1.67$
Capable	$1 \le C_p \le 1.33$
Inadequate	$0.67 \le C_p \le 1$
Poor	$C_p < 0.67$

C_p focuses on the dispersion of the process, and thus gives no indication of the actual process performance. Kane introduced the index C_{pk} to overcome this problem [17]. The index C_{pk} is used to provide an indication of the variability associated with a process. It shows how a process confirms its specifications. The index is usually used to relate the 'natural tolerances (3σ)' to the SLs. C_{pk} describes how well the process fits within the SLs, taking into account the location of the process mean [18], [1].
When a process only has one of the upper or lower SLs, C_{pu} or C_{pl} index is used, that is calculated Equation (4).
When the process has the upper SLs.

$$C_{pu} = \frac{USL - \mu}{3\sigma} \quad (4)$$

When the process has the lower SLs, C_{pl} is calculated in equation (5).

$$C_{pl} = \frac{\mu - LSL}{3\sigma} \quad (5)$$

If the process mean is not at the center of the SLs, C_{pk} index can be used in equation (6) [1].

$$C_{pk} = \min\{C_{pu}, C_{pl}\} = \min\left\{\frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right\} \quad (6)$$

C_{pk} index is computed using both location and dispersion information about the process. C_{pk} index is the shorter standardized distance from the center of the process to either LSL or USL [19].
The indices C_p and C_{pk} are appropriate measures of progress for quality improvement situations when reduction of variability is the guiding factor, and process yield is the primary measure of a success [20].
Montgomery examined several cases, which can explain the relationship between C_p and C_{pk}, are given below [1]:

- If $C_{pk} = C_p$, the process is centred at the midpoint of the SLs.
- If $C_{pk}, < C_p$, the process is off-centered. This can be accepted as the lower capability than the case at which the process is centered. The reason is that it is not operating at the midpoint of the interval between the SLs.
- If $C_{pk} = 0$, the process mean is exactly equal to one of the SLs.
- If $C_{pk} < 0$, the process mean lies outside the SLs, that is for or $\mu >$ USL or $\mu <$ LSL, $C_{pk} < 0$.

If $C_{pk} < -1$, the entire process lies outside the SLs. It should be noted that some authors define C_{pk} to be non-negative, so that values less than zero are defined as zero.

3. Fuzzy Process Capability Indices and Control Charts

The main aim of a control charting method is to detect quickly undesired faults in the process [21]. Fuzzy control charts are inevitable to use when the statistical data in consideration are uncertain or vague. One of the cases for which it is necessary to use fuzzy control charts and calculate FPCIs is continuous production processes such as steel manufacturing industries, metal wires, yarn production, and other related products. Nowadays, to measure the considered characteristic and process control in such industries, a number is used as a mean throughout the whole production process. For instance, in yarn production, because measuring strength of all parts of a yarn spindle is difficult, the strength of several points is measured, and then the mean of these points will be applied to process analysis. Clearly, this type of measure is not accurate enough to analyze the continuous production process properly, and the exact values of the mean and variance cannot be determined. As a result, the classical control charts are not fit for this process as they lack precision. Fuzzy techniques help us to measure several points along the yarn in terms of triangular fuzzy numbers (TFNs), while these numbers can be used in process analysis and control charts. This technique analyzes the process more accurately by relying on linguistic terms, and the process will not go out -of control when only one point falls out of range.

In this paper, $\overline{X}$-$MR_{\overline{X}}$ fuzzy control charts in continuous production processes are introduced to control the mean and process variability, while the relationships used in this paper are based on those formulated by Kaya and Kahraman [2], which is written on the basis of $\overline{X}$-$MR_{\overline{X}}$ control charts.

The equations (7) and (8) estimates the value of moving range and σ, respectively.

$$MR_i = |\overline{X_i} - \overline{X_{i-1}}| \tag{7}$$

$$\hat{\sigma} = \frac{\overline{MR_{\overline{X}}}}{d_2(n=2)} \tag{8}$$

Control limits for introduced charts are calculated by the following equations [1]:

For $\overline{X}$ control chart,

$$UCL = \overline{\overline{X}} + 3\frac{\overline{MR_{\overline{X}}}}{d_2(n=2)} \tag{9}$$

$$CL = \overline{\overline{X}} \tag{10}$$

$$LCL = \overline{\overline{X}} - 3\frac{\overline{MR_{\overline{X}}}}{d_2(n=2)} \tag{11}$$

For $MR_{\overline{X}}$ chart,

$$UCL = D_4\overline{MR_{\overline{X}}} \tag{12}$$

$$CL = \overline{MR_{\overline{X}}} \tag{13}$$

$$LCL = D_3\overline{MR_{\overline{X}}} \tag{14}$$

Where d_2, D_3, D_4 are constants, and n is the sample size (n=2).

$\tilde{\overline{X}}$ - $\widetilde{MR_{\overline{X}}}$ fuzzy control charts, which are designed based on triangular fuzzy measurements, are introduced in the next section.

4. Fuzzy Control Charts for TFN Case

If a quality characteristic is defined in terms of a triangular fuzzy number (a, b, c), and n is the sample size, then the moving range is calculated in equations (15) and (16):

$$\tilde{\overline{X}}_j = \left(\frac{\sum_{i=1}^{n} a_i}{n}, \frac{\sum_{i=1}^{n} b_i}{n}, \frac{\sum_{i=1}^{n} c_i}{n}\right) = TFN(o_1, o_2, o_3)\ , j = 1,2,\dots,m \tag{15}$$

$$MR_j = \left|\tilde{\overline{X}}_j - \widetilde{\overline{X}_{j-1}}\right| = TFN(g_1, g_2, g_3)\ , j = 1,2,\dots,m \tag{16}$$

After m samples are checked, $\tilde{\overline{\overline{X}}}$ and $\widetilde{\overline{MR_{\overline{X}}}}$ can be calculated in equations (17) and (18):

$$\tilde{\overline{\overline{X}}} = \left(\frac{\sum_{i=1}^{m} o_{1i}}{m}, \frac{\sum_{i=1}^{m} o_{2i}}{m}, \frac{\sum_{i=1}^{m} o_{3i}}{m}\right) = TFN(\mu_1, \mu_2, \mu_3) \tag{17}$$

$$\widetilde{\overline{MR_{\overline{X}}}} = \left(\frac{\sum_{i=1}^{m} g_{1i}}{m}, \frac{\sum_{i=1}^{m} g_{2i}}{m}, \frac{\sum_{i=1}^{m} g_{3i}}{m}\right) = TFN(mr_1, mr_2, mr_3) \tag{18}$$

Then, control limits for $\overline{X}$ - $\widetilde{\overline{MR}_{\overline{X}}}$ control charts can be calculated as the following equations:
For $\overline{X}$ control chart:

$$\widetilde{UCL_x} = \widetilde{\overline{\overline{X}}} + 3\frac{\widetilde{\overline{MR}_{\overline{X}}}}{d_2(n=2)} = (\mu_1 + 3mr_1, \mu_2 + 3mr_2, \mu_3 + 3mr_3) = TFN(UCL_{x_1}, UCL_{x_2}, UCL_{x_3}) \quad (19)$$

$$\widetilde{CL_x} = \widetilde{\overline{\overline{X}}} = (\mu_1, \mu_2, \mu_3) = TFN(CL_{x_1}, CL_{x_2}, CL_{x_3}) \quad (20)$$

$$\widetilde{LCL_x} = \widetilde{\overline{\overline{X}}} - 3\frac{\widetilde{\overline{MR}_{\overline{X}}}}{d_2(n=2)} = (\mu_1 - 3mr_3, \mu_2 - 3mr_2, \mu_3 - 3mr_1) = TFN(LCL_{x_1}, LCL_{x_2}, LCL_{x_3}) \quad (21)$$

$$(\widetilde{LCL_{x_1}}, \widetilde{LCL_{x_2}}, \widetilde{LCL_{x_3}}) = \begin{cases} LCL_{x_1} = \begin{cases} \mu_1 - 3mr_3 & \text{if } (\mu_1 - 3mr_3) \geq 0 \\ 0 & \text{if} (\mu_1 - 3mr_3) < 0 \end{cases} \\ LCL_{x_2} = \begin{cases} \mu_2 - 3mr_2 & \text{if } (\mu_2 - 3mr_2) \geq 0 \\ 0 & \text{if} (\mu_2 - 3mr_2) < 0 \end{cases} \\ LCL_{x_3} = \begin{cases} \mu_3 - 3mr_1 & \text{if } (\mu_3 - 3mr_1) \geq 0 \\ 0 & \text{if} (\mu_3 - 3mr_1) < 0 \end{cases} \end{cases} \quad (22)$$

For $MR_{\overline{X}}$ chart:

$$UCL = D_4\widetilde{\overline{MR}_{\overline{X}}} = (D_4mr_1, D_4mr_2, D_4mr_3) = TFN(UCL_{mr_1}, UCL_{mr_2}, UCL_{mr_3}) \quad (23)$$

$$CL = \widetilde{\overline{MR}_{\overline{X}}} = (mr_1, mr_2, mr_3) = TFN((CL_{mr_1}, CL_{mr_2}, CL_{mr_3}) \quad (24)$$

$$LCL = D_3\widetilde{\overline{MR}_{\overline{X}}} = (D_3mr_1, D_3mr_2, D_3mr_3) = TFN(LCL_{mr_1}, LCL_{mr_2}, LCL_{mr_3}) \quad (25)$$

After the process control limits are determined, the next step is to calculate the FPCIs.

5. Fuzzy Process Capability Indices for TFN Case

FPCIs have considerable amount of advantages and remarkable capabilities than their crisp types. They provide more information, they are more sensitive and flexible, and also more appropriate for implementation to real life cases as they can successfully illustrate human judgment [6]. For cases in which crisp numbers cannot be appropriate for defining SLs, fuzzy numbers can be used to represent SLs. In this study, FPCIs are obtained using TFN for defining the upper and lower SLs. It is further assumed that SLs and measurements of the considered quality characteristics are defined in terms of linguistic variables.
SLs can be defined as in the following equations:

$$\widetilde{USL} = TFN(u_1, u_2, u_3) \quad (26)$$

$$\widetilde{LSL} = TFN(l_1, l_2, l_3) \quad (27)$$

It is known that the process mean and variance are two critical parameters to calculate PCIs. Their fuzzy estimations give us a chance to produce FPCIs. Fuzzy process mean ($\tilde{\mu}$) and standard deviation ($\tilde{\sigma}$) can be calculated in equations (28):

$$\tilde{\mu} = \widetilde{\overline{\overline{X}}} = TFN(\mu_1, \mu_2, \mu_3) \quad (28)$$

$$\tilde{\sigma} = \frac{\widetilde{\overline{MR}_{\overline{X}}}}{d_2(n=2)} = \left(\frac{mr_1}{d_2}, \frac{mr_2}{d_2}, \frac{mr_3}{d_2}\right) = TFN(s_1, s_2, s_3) \quad (29)$$

The fuzzy estimations of $\tilde{\mu}$ and $\tilde{\sigma}$ clearly include more information than the crisp values. Notice that the crisp values belong to the fuzzy estimations with a membership value of 1. Based on these definitions, FPCIs can be calculated in the following equations:

$$\widetilde{C_p} = \frac{\widetilde{USL} - \widetilde{LSL}}{6\tilde{\sigma}} = TFN\left(\frac{u_1 - l_3}{6s_3}, \frac{u_2 - l_2}{6s_2}, \frac{u_3 - l_1}{6s_1}\right) \quad (30)$$

$$\widetilde{C_{pu}} = \frac{\widetilde{USL} - \tilde{\mu}}{3\tilde{\sigma}} = TFN\left(\frac{u_1 - \mu_3}{3s_3}, \frac{u_2 - \mu_2}{3s_2}, \frac{u_3 - \mu_1}{3s_1}\right) \quad (31)$$

$$\widetilde{C_{pl}} = \frac{\tilde{\mu} - \widetilde{LSL}}{3\tilde{\sigma}} = TFN\left(\frac{\mu_1 - l_3}{3s_3}, \frac{\mu_2 - l_2}{3s_2}, \frac{\mu_3 - l_1}{3s_1}\right) \quad (32)$$

Index $\widetilde{C_{pk}}$ can be derived by equation (33). It is a necessity to use a ranking method, since these indices are expressed as fuzzy numbers.

$$\widetilde{C_{pk}} = \min\{\widetilde{C_{pu}}, \widetilde{C_{pl}}\} \quad (33)$$

If $\tilde{\mu}$ is at the center of the SLs, $\widetilde{C_p}$ and $\widetilde{C_{pk}}$ indices are equal. If $\tilde{\mu}$ is equal to upper or lower of the specification limit, $\widetilde{C_{pk}}$ is zero and if the process mean is out of the SLs, $\widetilde{C_{pk}}$ is negative.

6. Case Study

This paper introduces the fuzzy control charts and PCIs in continuous production processes. To test the charts in practice, a study was conducted in Yazd Fibers Plant, Yazd, Iran. The company was established in 2000 and is known as one of the modern plants to product bulked continuous filament (BCF) fibres in Iran. One hundred employees presently work in the company, and it won the Standard Quality Award in 2007 and 2012. The statistical population under study included the colored carpet yarn products manufactured in the plant. Process engineers of the company decided to draw on FPCIs to increase not only the flexibility of the process, but also the sensitiveness of the results for a characteristic of carpet yarn. Yarn has several measurable characteristics, such as yarn strength, yarn grade, and yarn elongation. Because yarn strength is an important quality characteristic to quality experts, yarn strength is the characteristic researched in this study. Because the length of a wrapped yarn around a spindle is long, yarn strength is measured by the machine in terms of TFN from different points with a regular 100-meter interval along the length of the yarns. A computer program is coded in MATLAB to evaluate the process performance.

The output determines the fuzzy control limits and parameters of $\overline{X}$- $MR_{\overline{X}}$ control charts, and calculates the triangular FPCIs. The triangular fuzzy data related to the yarn strength characteristics are collected over 19 consecutive days, as 28 samples (n = 3) as shown in Table 2.

Tab. 2. Triangular fuzzy numbers for yarn strength measurement

Sample	x_1	x_2	x_3
1	(21.56,21.61,21.91)	(22.27,22.39,22.39)	(21.79,21.85,22.14)
2	(20.7,20.98,21.33)	(21.79,21.9,22.33)	(22.44,22.59,22.61)
3	(20.59,20.64,20.72)	(21.31,21.81,21.85)	(20.86,21.03,21.28)
4	(21.49,21.77,21.94)	(21.32,21.34,21.73)	(20.38,20.8,20.83)
5	(20.44,20.54,20.67)	(21.36,21.42,21.56)	(20.63,21.1,21.21)
6	(20.95,21.24,21.45)	(21.21,21.43,21.54)	(20.64,20.93,21.13)
7	(22.07,22.11,22.31)	(21.7,21.75,21.8)	(20.6,20.96,21.13)
8	(20.07,20.23,20.45)	(20.61,21.2,21.56)	(20.5,20.79,20.83)
9	(21.33,21.37,21.48)	(20.89,20.97,21.17)	(19.91,19.95,20.02)
10	(21.48,21.96,22.01)	(20.36,20.43,21.07)	(20.22,20.78,20.81)
11	(20.74,20.88,21.14)	(21.11,21.45,21.65)	(21.17,21.23,21.37)
12	(20.34,20.44,20.48)	(20,20.3,20.38)	(20.53,20.58,20.87)
13	(20.9,21.57,22.04)	(21.65,21.65,21.72)	(19.32,19.66,20.08)
14	(21.39,21.43,21.65)	(21.71,22.06,22.09)	(21.74,21.74,21.9)
15	(19.98,20.63,20.93)	(20.72,20.87,21.41)	(20.57,20.59,20.83)
16	(20.79,21.5,21.71)	(21.57,21.95,22.08)	(21.34,21.53,21.64)
17	(21.81,22.05,22.36)	(22.21,22.62,22.64)	(21.55,21.61,21.73)
18	(20.8,21.06,21.09)	(21.28,21.41,21.66)	(20.68,20.69,20.9)
19	(20.16,20.62,20.71)	(21.38,21.59,21.82)	(19.94,20.06,20.49)
20	(21.63,21.75,21.78)	(21.62,21.72,22.08)	(20.49,20.57,20.86)
21	(20.99,21,21)	(21.34,21.47,21.59)	(21.04,21.26,21.3)
22	(21.64,21.7,21.71)	(21.24,21.26,21.31)	(21.29,21.35,21.6)
23	(20.22,20.36,20.74)	(20.41,21.11,21.12)	(20.95,21,21.27)
24	(20.53,21.77,22.07)	(21.55,21.65,21.72)	(20.58,20.61,20.67)
25	(20.05,20.33,20.37)	(20.46,20.66,20.82)	(20.61,21.41,21.55)
26	(20.13,20.2,20.28)	(20.74,20.79,21.03)	(20.15,20.39,20.67)
27	(21.35,21.36,21.39)	(20.39,20.67,20.79)	(20,20.18,20.97)
28	(20.03,20.45,20.52)	(20,21.02,21.09)	(21.31,21.51,21.82)

To calculate the fuzzy process parameters and fuzzy control limits of $\overline{X}$-$MR_{\overline{X}}$ charts, the data of Table 2 are imported into the software, and the results are shown in Tables 3 and 4.

Tab. 3. Process parameters

Process Mean=(20.9225, 21.1538, 21.3387)
Average Moving Range=(0.4586, 0.5342, 0.5768)
Standard Deviation=(0.4066, 0.4736, 0.5113)

Tab. 4. Control limits of the fuzzy $\overline{X}$-$MR_{\overline{X}}$ control charts

$\overline{X}$ control chart	$MR_{\overline{X}}$ control chart
UCL=(22.1423, 22.5745, 22.8727)	UCL=(1.4984, 1.7452, 1.8844)
CL=(20.9225, 21.1538, 21.3387)	CL=(0.4586, 0.5342, 0.5768)
LCL=(19.3885, 19.7331, 20.1189)	LCL=(0 , 0 , 0)

After the control charts limits are determined, the next step is to calculate the FPCIs. Because the designer determined only the lower specification limit for yarn strength, LSL = (19, 19.1, 19.2), only C_{pl} index is calculated and the results are illustrated in Table 5.

Tab. 5. PCI for yarn strength

$\widetilde{C_{pl}} = (1.1229, 1.4456, 1.9173)$

Fuzzy PCA takes into account all possible index values together with their degree of membership as it can be seen from Figure 2 clearly. This method provides a more detailed and flexible analysis to increase process quality. According to the PCI value and Table 1, it can be said that the process status is satisfactory. Since the first membership value is close to 1, it is recommended to decrease the variation of yarn strength and to increase the basic point of the process. Also, quality experts can change the process mean to improve the performance and increase the process capability.

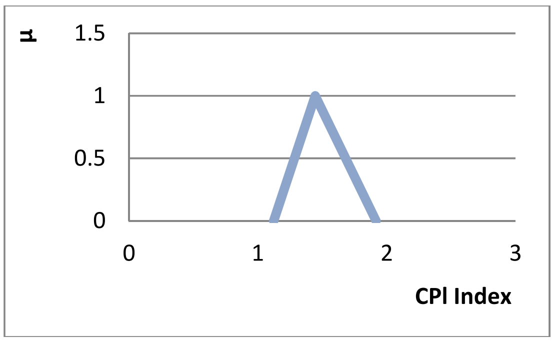

Fig. 2. The membership function of the index $\widetilde{C_{pl}}$

7. Conclusion

Process capability analysis has been widely used in manufacturing industry to provide numerical measures of process performance. The results of this analysis are used to improve the process performance. PCA can be broadly defined as the ability of a process to meet customer expectations, which are defined as SLs. Process capability indices measure the potential efficiency of the process. This index does not take into account process concentration, and it merely monitors the variability rate of process. The measure of process capability summarizes some aspects of a process's ability to meet SLs, and it is a very useful approach to define a relationship between the process ability and SLs. Fuzzy control charts are inevitable to use when the statistical data in consideration are uncertain or vague.

One of the cases for which it is necessary to use fuzzy quality control is continuous production process. Because in these a processes, measuring the considered characteristics in whole production is not possible, the beginning parts of the products are measured, and then a number is determined as a mean. Fuzzy control charts seem to provide a more functional method regulating process control. In this paper, the fuzzy $\overline{X}$ and $MR_{\overline{X}}$ control charts and fuzzy (PCIs) are used to evaluate the performance of the continuous production process. The model is coded in MATLAB, and it is noteworthy that this computer program is applicable to any measurable characteristic with any sample and sample size, that is, output determines the control charts limits and PCIs

In terms of TFN. Finally, a case study was conducted in Yazd Fibers Plant on 28 samples of carpet yarns (n = 3). Then, strength measurement was performed according to triangular fuzzy, and the data are collected from different points with a regular 100- meter interval along the length of the yarns. The process parameters and fuzzy control limits relevant to $\overline{X}$-$MR_{\overline{X}}$ charts were calculated in MATLAB. Because only the lower specification limit for yarn strength was determined by the designer, only C_{pl} index was calculated. The results show that in continuous production processes, fuzzy measurements can yield a more accurate analysis. Furthermore, the values of PCIs indicate that the process is satisfactory, although it needs some improvements to decrease process variation. The FPCIs include the classical value with a membership value of 1.00 and show all possible values of PCIs. Also, the fuzzy estimations of PCIs include more information and flexibility to evaluate the process performance, when it is compared with the crisp case.

For further research, fuzzy $\overline{X}$ and $MR_{\overline{X}}$ control charts can be implemented for homogeneous processes in which the product is fluid (either gas or liquid), and the repeated sampling is not meaningfully applicable. Also, the other fuzzy PCIs such as $\widetilde{C_{pm}}$, $\widetilde{C_{pmk}}$, and $\widetilde{C_a}$ can be reconsidered by taking into account fuzzy SLs and the fuzzy estimations of μ and σ^2.

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