



Corrugated Box Production Process Optimization using Dimensional Analysis and Response Surface Method

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ResponseSurfaceMethod(RSM),
Independent Variables,
Response Variable,
Correlation (R),
Root Mean Square Error (RMS)

ABSTRACT

The paper focuses on optimization of the manufacturing process of the corrugated sheet box using Dimensional Analysis (DA) and Response Surface Method (RSM). A mathematical model for five different box sizes in terms of Pi terms using Buckingham's Pi theorem and polynomial equation between independent and response parameters are obtained. RSM is the statistical method useful in the modeling and analysis of problems in which the response variable receives the influence of several independent variables, in order to determine which are the conditions under which should operate these variables to optimize a corrugated box production process. The objective of this study is to minimize the root mean square error (RMS) between the experimental values and computed values of cycle time obtained from mathematical model using RSM. Studies carried out for corrugated sheet box manufacturing industries having man machine system revealed the contribution of many independent parameters like anthropometric data of workers, personal data, machine specification, workplace parameters, product specification, environmental conditions and mechanical properties of corrugated sheet on cycle time. Their effect on response parameter cycle time is totally unknown. The developed model was simulated and optimized with the aid of MATLAB R2011a data cursor tool. The results obtained showed that the correlation R towards linearity, adjusted R^2 upto 95% and RMS error less than 10% were valid..

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1.Introduction

Cardboard packaging is one of the most widely used forms of packaging. The

corrugated cardboard is stiff, strong and light in weight material made up of layers of brown craft paper. These brown craft paper rolls are transported to a corrugation machine where this paper gets crimped and glued to form corrugated cardboard called as SFCB and then this SFCB is cut according required dimension on the cutting machine. According

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to requirement by adding another corrugating medium and a third flat printed liner creates a double wall corrugated board or triple wall corrugated boards on gluing or bonding machine called as 3ply 5ply and 7ply. Then these corrugated boards are transferred to creasing and cutting machine where extra material is removed and creasing operation is performed (i.e., from where the box get folded). The next operation is slotting operation where the strip plate is slotted for stitching and finally with stitching operation corrugated box is manufactured.

2. Literature Review

Montgomery et al. and Jack P. C. et al. suggested RSM is a collection of mathematical and statistical techniques used to determine the optimal levels of the independent variables of a production process, which involves estimating a regression model of first order by the method of least squares, with the coefficients of this model is set search direction by MMSD, subsequently, the step size on the ascent route is chosen until there is no further increase in the response, this method stops [1] [2].

Box et al. in his research says in statistics, response surface methodology (RSM) explores the relationships between several explanatory variables and one or more response variables. The main idea of RSM is to use a sequence of designed experiments to obtain an optimal response using a second degree polynomial model. They acknowledge that this model is only an approximation, but use it because such a model is easy to estimate and apply, even when little is known about the process [3].

Myers et al. fitted a new linear regression model and determined a new path of upward slope. The procedure continues until it fits the regression model of first order. Finally, it was not possible to adjust the regression model of first order, a more detailed design is posed, as the central composite design (CCD), which is the kind of classic design to fit models of second order and find the optimal values of

the independent variables analyzed, using methods of differential calculus [4].

Z. Zahedian-Tejenaki et al. develop a freight transportation model for a railway network considering a hazmat and green transportation issue. A zero-one integer programming model is developed that minimizes the cost of safe transportation and consumed fuel. To make the model closer to the real world, a fuzziness concept in the presented model is considered. This fuzzy model is solved by using a new fuzzy approach, named TH method. A numerical example is considered and the related results are compared with the optimal solution. Finally, the conclusion is provided [5].

A.K. Dubey et al. And S. Raissi et al. response surface methodology is a collection of statistical and mathematical method that is useful for the modeling and analyzing engineering problems. The main objective is to optimize the response surface that is influenced by various process parameters. Response surface methodology also quantifies the relationship between the controllable input parameters and the obtained response surfaces [6] [7].

Christian Gogu et al. response surface approximation (RSA) of temperature was needed in order to reduce optimization computational time. The finite element model used to evaluate the maximum temperature at the design of experiment points involved a total of 15 parameters of interest for the design: 9 thermal material properties and 6 geometric parameters of the ITPS model.

In order to reduce the dimensionality of the response surface approximation, dimensional analysis was utilized. A small number of assumptions simplified the equations of the transient thermal problem allowing easy nondimensionalization using classical techniques. The no dimensional equations together with a global sensitivity analysis showed that the maximum temperature mainly depends on only two no dimensional parameters which were selected to be the design variables for the RSA.

It is important to note that the RSA was still constructed using the accurate finite element

model which does not employ any of the simplifying assumptions used to determine the nondimensional parameters. The two variable RSA was checked for its accuracy in terms of geometric parameters and material properties variables at 855 additional test points using the finite element model. The error in the RSA is not due to the quality of the fit but mainly due to the reduction from 15 to only two variables [11].

Amit Kumar Marwah et al. intend to empirically assess the effects of human metrics on supply chain performance in the context of Indian manufacturing organizations. A rigorous literature review has identified [12] variables. The variables are individually measured and later on reduced in number by factor analysis. As a pilot study, primary data is collected from 100 manufacturing organizations by means of a questionnaire, both offline and online, which is administered across India and a scale is developed. T-test and factor analysis resulted in 3 factors related to human metrics. The outcomes of the research work provide valuable implications for the Indian manufacturing organizations to understand the factors affecting supply chain performance [13].

H.R. Navidi et al. proposed a new game theoretic-based approach for multi-response optimization problem. Present research uses the game theory approach via definition of each response as each player and factors as strategies of each player.

This approach can determine the best predictor factor sets in order to obtain the best joint desirability of responses. For this aim, the signal to noise ratio (SN) index for each response have been calculated with considering the joint values of strategies; then obtained SN ratios for each strategy is modeled in the game theory table. Finally, using Nash Equilibrium, the best strategy which is the best values of predictor factors is determined. A real case and a numerical example are given to show the efficiency of the proposed method. In addition, the performance of the proposed method is compared with the VIKOR method [14].

3. Reduction of Independent Variables Using DA

Hilbert Scheck Jr. suggested several quite simple ways in which a given test can be made compact in operating plan without loss in generality or control. The best known and the most powerful of these is dimensional analysis. In the past dimensional analysis was primarily used as an experimental tool whereby several experimental variables could be combined to form one. The field of fluid mechanics and heat transfer were greatly benefited from the application of this tool. Almost every major experiment in this area was planned with its help. Using this principle modern experiments can substantially improve their working techniques and be made shorter requiring less time without loss of control. Deducing the dimensional equation for a phenomenon reduces the number of independent variables in the experiments. The exact mathematical form of this dimensional equation is the targeted model.

This is achieved by applying Buckingham's π theorem. When this theorem is applied to a system involving n independent variables, (n minus number of primary dimensions viz. L, M, T, and π) i.e. ($n-4$) numbers of π terms are formed. When n is large, even by applying this theorem number of π terms will not be reduced significantly than number of all independent variables. Thus much reduction in number of variables is not achieved. It is evident that, if we take the product of the terms it will also be dimensionless number and hence a π term. This property is used to achieve further reduction of the number of variables [9].

In this work, there are 43 independent variables and one response variable. These 43 independent variables are further reduced to 7 independent π terms using Dimensional Analysis as shown in Table 1 to Table 8. The optimization of a cycle time as response variable for the corrugated sheet box manufacturing process is carried out with the aid of RSM and MATLAB 2011a taking the

anthropometric data of operators, personal factors of an operator, workstation machine specification, workplace parameters, specification of the products, environmental conditions and mechanical properties of corrugated sheet boxes as the independent variables and the cycle time as the dependent variable. The goal is to optimize the response variable cycle time, it is assumed that the independent variables are continuous and controllable by experiments with negligible errors. It is required to find a suitable approximation for the true functional relationship between independent variables and the response surface. DA is used to reduce the variables and following Pi terms were evolved out of it.

Tab .1. Π_1 Term Formulation using DA
Pi term relating to Anthropomorphic Data

| | |
|---|----|
| Arm span | As |
| Foot breadth | Fb |
| Height | Ht |
| Arm reach | Ar |
| $\pi_1 = (As \times Fb) \div (Ht \times Ar)$ | |
| In terms of MLT Indices | |
| $\Pi_1 = \frac{(M^0 L^1 T^0 \times M^0 L^1 T^0)}{(M^0 L^1 T^0 \times M^0 L^1 T^0)}$ | |

Tab .2. Π_2 Term Formulation using DA
Pi term relating to Personal factors of an Operator

| | |
|--|-----------|
| Qualification grade | Qgr |
| BMI prime | BMI prime |
| Age | Ag |
| Experience | Exp |
| Sex | S |
| $\pi_2 = (Qgr \times \text{BMI prime} \times Ag \times S) \div (\text{Exp})$ | |
| In terms of MLT Indices | |
| $\Pi_2 = \frac{(M^0 L^0 T^0 \times M^0 L^0 T^0 \times M^0 L^0 T^1 \times M^0 L^0 T^0)}{(M^0 L^0 T^1)}$ | |

Tab .3. Π_3 Term Formulation using DA
Pi term relating to Machine Specification

| | |
|--|-------------------------|
| Power HP | P |
| Stroke/Seconds | Sps |
| Age of Machine | Aom |
| Machine Down Time | Mc_dt |
| Roller Speed | rps |
| Production rate of Machine | P _{rate} of Mc |
| Machine Width | Mc_wth |
| Weight of Machine | Wt |
| $\pi_3 = (P \times P_{\text{rate of Mc}} \times Aom \times Mc_dt) \div (Wt \times Sps \times Mc_wth \times rps)$ | |
| In terms of MLT Indices | |

$$\Pi_3 = \frac{(M^1 L^2 T^3 \times M^0 L^0 T^{-1} \times M^0 L^0 T^{-1} \times M^0 L^0 T^1)}{(M^1 L^0 T^0 \times M^0 L^1 T^1 \times M^0 L^1 T^0 \times M^0 L^0 T^1)}$$

Tab .4. Π_4 Term Formulation using DA
Pi term relating to Workplace Parameters

| | |
|--|---------|
| Height of stool | Hos |
| Height of work table | Htw |
| Spatial distance between centroid of stool top and work table | Sd1 |
| Area of tabletop | Areatop |
| Spatial distance between centroid of stool top and WIP table | Sd2 |
| $\pi_4 = (Hos \times Htw \times Sd1) \div (\text{Areatop} \times Sd2)$ | |
| In terms of MLT Indices | |
| $\Pi_4 = \frac{(M^0 L^1 T^0 \times M^0 L^1 T^0 \times M^0 L^1 T^0)}{(M^0 L^2 T^0 \times M^0 L^1 T^0)}$ | |

Tab .5. Π_5 Term Formulation using DA
Pi term relating to Specification of the Product

| | |
|--|-----------|
| Thickness | t |
| Length | L |
| Breadth | B |
| Part Weight | Part_Wt |
| Mc_criticality | Mc_criti. |
| Volume | V |
| Bursting Strength | Bs |
| Bursting Factor | Bf |
| $\pi_5 = (Bs \times Vol \times Bf \times t) \div (\text{Part_Wt} \times Mc_criti \times B \times L)$ | |
| In terms of MLT Indices | |
| $\Pi_5 = \frac{(M^1 L^{-2} T^0 \times M^0 L^3 T^0 \times M^0 L^0 T^0 \times M^0 L^1 T^0)}{(M^1 L^0 T^0 \times M^0 L^0 T^0 \times M^0 L^1 T^0 \times M^0 L^1 T^0)}$ | |

Tab .6. Π_6 Term Formulation using DA
Pi term relating to Environmental Condition

| | |
|---|-------|
| Illumination sight range (Average) | Isr |
| Noise level with Operation | dBopr |
| Dry bulb temperature | DBT |
| Illumination at work table | Iwt |
| Noise level without Operation | dB |
| Wet bulb temperature | WBT |
| $\pi_6 = (Isr \times dBopr \times DBT) \div (Iwt \times dB \times WBT)$ | |
| In terms of MLT Indices | |
| $\Pi_6 = \frac{(M^1 L^3 T^{-1} \times M^0 L^0 T^0 \times M^0 L^0 T^0 K^1)}{(M^1 L^3 T^{-1} \times M^0 L^0 T^0 \times M^0 L^0 T^0 K^1)}$ | |

$$(M^1L^3T^{-1} \times M^0L^0T^0 \times M^0L^0T^0K^1)$$

Tab.7. Π_7 Term Formulation using DA

Pi term relating to Mechanical Properties of a Corrugated Box

| | |
|--|-------|
| Caliper | Cal |
| Puncture Resistance Test | PRT |
| Edge Crushing Test | ECT |
| Flat Crushing Test | FCT |
| Cobb | Cob |
| Moisture (%) | Mois. |
| Box Compression Test Peak Load in Kg | PL |
| Box Compression Test Peak Load / Perimeter | PLP |

$$\pi_7 = (Cal \times ECT \times FCT \times Mois \times PL) \div (PRT \times Cob \times PLP)$$

In terms of MLT Indices

$$\Pi_7 = (M^0L^1T^0 \times M^1L^{-1}T^0 \times M^1L^{-2}T^0 \times M^0L^0T^0 \times M^1L^0T^0) / (M^1L^1T^0 \times M^1L^{-2}T^0 \times M^1L^{-1}T^0)$$

Tab .8. Π_8 Term Formulation using DA

Pi term relating to Cycle Time

| | |
|------------------------|-----------|
| Cycle Time | cytime |
| Machine Operation Time | m/coptime |

$$\pi_8 = (cytime) \div (m/coptime)$$

In terms of MLT Indices

$$\Pi_8 = (M^0L^0T^1) / (M^0L^0T^1)$$

4. Response Surface Method (RSM)

Sundaram R.M, RSM is a collection of mathematical and statistical techniques for empirical model building by careful design of experiments, the objective is to optimize a response (output variable) which is influenced by several independent variables (input variables). An experiment is a series of tests, called runs, in which changes are made in the input variables in order to identify the reasons for changes in the output response. Originally, RSM was developed to model experimental responses and then migrated into the modeling of numerical experiments. The difference is in the type of error generated by the response. In physical experiments, inaccuracy can be due, for example, to measurement errors while, in computer experiments, numerical noise is a result of incomplete convergence of iterative processes, round-off errors or the discrete

representation of continuous physical phenomena. In RSM, the errors are assumed to be random. The RSM is practical, economical and relatively easy for use and it was used by lot of researchers for modeling machining processes [8]. Multi dimensional data can be represented in three directions. This is because 3D sketches are visible and makes some sense for us. Multiple inputs contributing for desired outputs are clubbed under two heads. This provides an ease in its graphic representation. Amalgamation of variables forming two orthogonal values is never achieved without sacrifice of precision. The curves so obtained are indicators of broader behaviour but insignificant for true predictions. This combines two polynomial regressions, one along X-axis and another along Y-axis. The product of these two polynomials is treated as the output; often represented along Z-axis. It provides the clement picture of behavioral pattern on the basis of combinational terms. The effect of single variable on output is difficult. It is possible only if all remaining variables in its combinational terms are maintained steady.

5. Model Formulation

A mathematical model, relating the relationships among the process dependent variable and the independent variables in a second-order equation is developed. The regression analysis was performed to estimate the response function as a second order polynomial.

$$Y_k = \beta_0 + \sum_{i=1}^n \beta_i \pi_i + \sum_{i=1}^n \beta_{ii} \pi_i^2 + \sum_{i=1}^{n-1} \sum_{j=i+1}^n \beta_{ij} \pi_i \pi_j + e \tag{1}$$

here $Y_k = \pi_8$ is the predicted response, k =cycle time, β_0 , β_i , β_{ii} , β_{ij} are constant coefficients estimated from regression and e is random error. They represent the linear, quadratic and interactive effects of π_i , π_i^2 , $\pi_i \pi_j$ on response variable π_8 . Optimizing the response variable Y_k , it is assumed that the independent variables are continuous and controllable by the experimenter with negligible error. The response or the

dependent variable is assumed to be a random variable. In corrugated sheet box manufacturing process, it is necessary to find a suitable combination of Pi terms X (Product of π_2, π_3, π_5 pi term) and Y (Product of π_1, π_4, π_6 and π_7). The observed Response Z as a function of the X and Y can be written as

$$Y_k = f(X; Y) + e \tag{2}$$

The quality of fit of the second order equation was expressed by the coefficient of determination R^2 . Usually a best fit polynomial is fitted. The parameters of the polynomials are estimated by the method of least squares. A statistical software package Matlab2011a is used for regression analysis of the data obtained and to estimate the coefficient of the regression equation. The equations were validated by the statistical tests called the ANOVA analysis. Design-based experimental data were matched according to the second order polynomial equation. The independent variables were fitted to the second order model equation and examined for the goodness of fit. The quality of fit of the second order equation was expressed by the coefficient of determination R^2 , and its statistical significance was determined by *F*-test. The significance of each term in the equation is to estimate the goodness of fit in each case. The proposed relationship between the cycle time responses and independent variables for five products and one for overall is represented by the following:

Linear model Polynomial21:
 $f(x, y) = p_{00} + p_{10}x + p_{01}y + p_{20}x^2 + p_{11}xy$
 Where x is normalized by mean 2.743 and std 2.361
 And y is normalized by mean 3.329 and std 8.045

Tab.9. Coefficients for Model RSM1OCT

| Coefficients (with 95% confidence bounds) | | |
|---|---------|--------------------|
| p00 | 2.238 | (2.223, 2.253) |
| p10 | -0.2215 | (-0.2455, -0.1976) |
| p01 | -0.2309 | (-0.2481, -0.2137) |

| | |
|-----|--------------------------|
| p20 | 0.09086 (0.08069, 0.101) |
| p11 | 0.03548 (0.023, 0.04796) |

Putting the values of coefficients in above equation, we get

$$Y_k = \pi_8 = 2.238 - 0.2215 * (\pi_2 * \pi_3 * \pi_5) - 0.2309 * (\pi_1 * \pi_4 * \pi_6 * \pi_7) + 0.09086 * (\pi_2 * \pi_3 * \pi_5)^2 + 0.03548 * (\pi_2 * \pi_3 * \pi_5) * (\pi_1 * \pi_4 * \pi_6 * \pi_7) \tag{3}$$

$$= 2.238 - 0.2215 * 2.244721502 - 0.2309 * 0.020738463 + 0.09086 * (2.244721502)^2 + 0.03548 * (2.244721502) * (0.020738463)$$

$$Y_k = \pi_8 = 2.19548$$

Tab.10. Result Analysis Model RSM1OCT

| Analysis of RSM Model and Goodness of fit | |
|---|---------|
| SSE | 1.896 |
| R-square | 0.9595 |
| Adjusted R-square | 0.9591 |
| RMSE | 0.06759 |

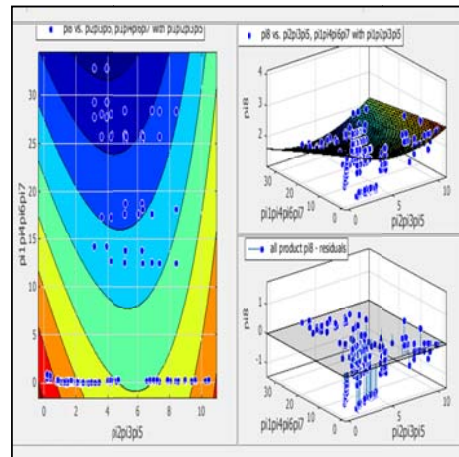


Fig.1. Contour Plot, 3D Response Surface Plot and Residual Plot showing interactive effect of $\pi_2\pi_3\pi_5$ along X, $\pi_1\pi_4\pi_6\pi_7$ along Y on π_8 (Response Variable) along Z for Model RSM1OCT.

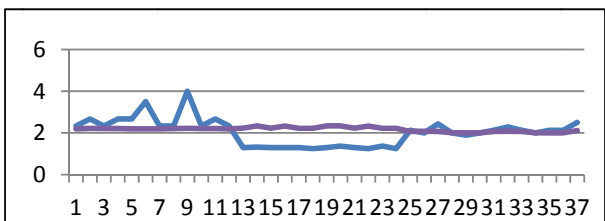


Fig.2. Best Fit between $Y_{\text{experimental}}$ Vs Y_{computed} for Model RSM1OCT

After fitting data with model RSM1OCT as shown in fig.1 3D response surface plot, the goodness of fit is evaluated. The results show that the linear model polynomial 21 fit coefficients are accurately known (bounds are small). As expected, the fit results for poly 21 are reasonable because the generated data follows a second order polynomial curve. The 95% confidence bounds on the fitted coefficients as shown in Tab.9 indicate that they are acceptably precise. The adjusted R-square statistic is generally the best indicator of the fit quality when you compare two models. The adjusted R-square statistic can take on any value less than or equal to 1, with a value closer to 1 indicating a better fit. Negative values can occur when the model contains terms that do not help to predict the response. Tab.10. shows the adjusted R² as 0.9591 which is closer to 1 indicating a better fit between independent and response variables. A graphical display of the residuals for a second degree polynomial fit is shown in fig.1. The residual plot in fig.1 shows that the residuals are calculated as the vertical distance from the data point to the fitted curve. The residuals appear randomly scattered around zero indicating that the model describes the data well.

5.2 Model 2 (RSM11CT)

Linear model Polynomial23:
 $f(x,y) = p00 + p10*x + p01*y + p20*x^2 + p11*x*y + p02*y^2 + p21*x^2*y + p12*x*y^2 + p03*y^3$
 Where x is normalized by mean 3.192 and Std. 7.76 and y is normalized by mean 3.137 and std 2.53

Putting the values of coefficients in above equation, we get

$$Y_k = \pi 8 = 2.25 - 0.04913 * (\pi 2 * \pi 3 * \pi 5) - 0.6638 * (\pi 1 * \pi 4 * \pi 6 * \pi 7) - 0.03283 * (\pi 2 * \pi 3 * \pi 5)^2 + 0.1666 * (\pi 2 * \pi 3 * \pi 5) * (\pi 1 * \pi 4 * \pi 6 * \pi 7) - 0.3794 * (\pi 1 * \pi 4 * \pi 6 * \pi 7)^2 + 0.04074 * (\pi 2 * \pi 3 * \pi 5)^2 * (\pi 1 * \pi 4 * \pi 6 * \pi 7) - 0.131 * (\pi 2 * \pi 3 * \pi 5) * (\pi 1 * \pi 4 * \pi 6 * \pi 7)^2 +$$

$$0.3787 * (\pi 1 * \pi 4 * \pi 6 * \pi 7)^3 \tag{4}$$

Tab.11. Result Analysis Model RSM11CT

| Analysis of RSM Model and Goodness of fit | |
|---|--------|
| SSE | 1.542 |
| R-square | 0.8266 |
| Adjusted R-square | 0.8081 |
| RMSE | 0.1434 |

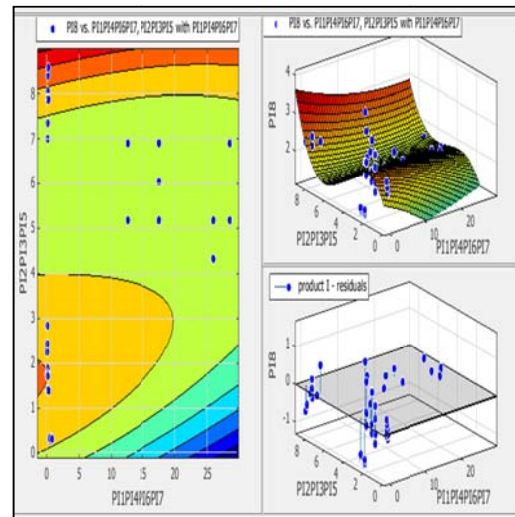


Fig3. Contour Plot, 3D Response Surface Plot and Residual Plot for Model RSM11CT

5.3 Model 3 (RSM12CT)

Linear model Polynomial22:
 $f(x,y) = p00 + p10*x + p01*y + p20*x^2 + p11*x*y + p02*y^2$
 Where x is normalized by mean 3.164 and std 7.68
 And y is normalized by mean 2.004 and std 1.58
 Putting the values of coefficients in above equation, we get

$$Y_k = \pi 8 = 2.921 - 0.9933 * (\pi 2 * \pi 3 * \pi 5) + 0.4021 * (\pi 1 * \pi 4 * \pi 6 * \pi 7) + 0.1217 * (\pi 2 * \pi 3 * \pi 5)^2 + 0.2438 * (\pi 2 * \pi 3 * \pi 5) * (\pi 1 * \pi 4 * \pi 6 * \pi 7) - 0.5708 * (\pi 1 * \pi 4 * \pi 6 * \pi 7)^2 \tag{5}$$

Tab.12. Result Analysis Model RSM12CT

| Analysis of RSM Model and Goodness of fit | |
|---|--------|
| SSE | 1.54 |
| R-square | 0.8969 |
| Adjusted R-square | 0.8903 |
| RMSE | 0.1405 |

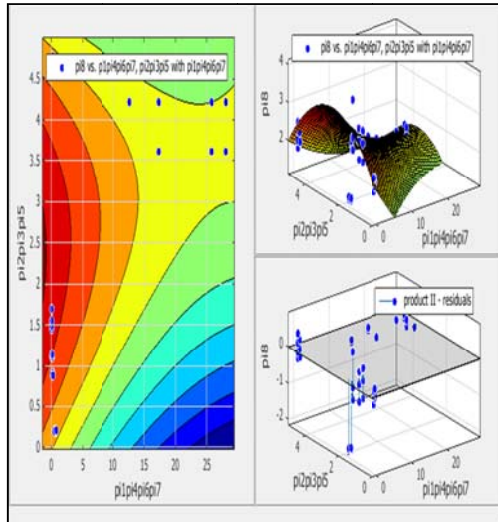


Fig.4 . Contour Plot, 3D Response Surface Plot and Residual Plot for Model RSM12CT

5.4 Model 4 (RSM13CT)

Linear model Polynomial23:

$$f(x,y) = p00 + p10*x + p01*y + p20*x^2 + p11*x*y + p02*y^2 + p21*x^2*y + p12*x*y^2 + p03*y^3$$

Where x is normalized by mean 3.167 and Std. 7.56 and y is normalized by mean 2.963 and Std 2.236.

Putting the values of coefficients in above equation, we get

$$Y_k = \pi8 = 2.289 + 0.0272 * (\pi2*\pi3*\pi5) - 0.674 * (\pi1*\pi4*\pi6*\pi7) - 0.1173 * (\pi2*\pi3*\pi5)^2 + 0.2452 * (\pi2*\pi3*\pi5) * (\pi1*\pi4*\pi6*\pi7) - 0.4285 * (\pi1*\pi4*\pi6*\pi7)^2 + 0.116 * (\pi2*\pi3*\pi5)^2 * (\pi1*\pi4*\pi6*\pi7) - 0.2691 * (\pi2*\pi3*\pi5) * (\pi1*\pi4*\pi6*\pi7)^2 + 0.4514 * (\pi1*\pi4*\pi6*\pi7)^3 \tag{6}$$

Tab.13. Result Analysis Model RSM13CT

| Analysis of RSM Model and Goodness of fit | |
|---|--------|
| SSE | 2.116 |
| R-square | 0.695 |
| Adjusted R-square | 0.6625 |
| RMSE | 0.168 |

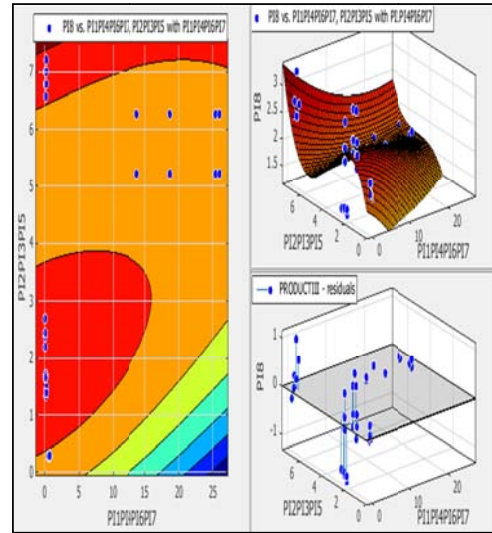


Fig.5. Contour Plot, 3D Response Surface Plot and Residual Plot for Model RSM13CT

5.5 Model 5 (RSM14CT)

Linear model Polynomial21:

$$f(x,y) = p00 + p10*x + p01*y + p20*x^2 + p11*x*y$$

Where x is normalized by mean 3.197 and std 7.764

And y is normalized by mean 3.821 and std 3.07

Putting the values of coefficients in above equation, we get

$$Y_k = \pi8 = 2.091 - 0.5197 * (\pi2*\pi3*\pi5) + 0.07856 * (\pi1*\pi4*\pi6*\pi7) + 0.1304 * (\pi2*\pi3*\pi5)^2 - 0.0242 * (\pi2*\pi3*\pi5) * (\pi1*\pi4*\pi6*\pi7) \tag{7}$$

Tab.14. Result Analysis Model RSM14CT Analysis of RSM Model and Goodness of fit

| | |
|-------------------|---------|
| SSE | 0.6404 |
| R-square | 0.9204 |
| Adjusted R-square | 0.9164 |
| RMSE | 0.09004 |

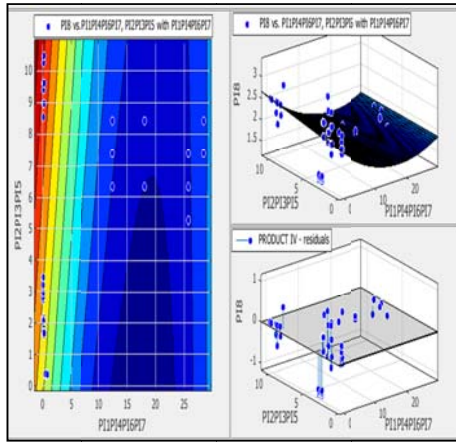


Fig.6. Contour Plot, 3D Response Surface Plot and Residual Plot for Model RSM14CT

5.6 Model 6 (RSM15CT)

Linear model Polynomial22:

$$f(x,y) = p00 + p10*x + p01*y + p20*x^2 + p11*x*y + p02*y^2$$

Putting the values of coefficients in above equation, we get

$$Y_k = \pi8 = 1.777 - 0.1935 * (\pi2*\pi3*\pi5) + 1.344 * (\pi1*\pi4*\pi6*\pi7) + 0.001609 * (\pi2*\pi3*\pi5)^2 + 0.03109 * (\pi2*\pi3*\pi5) * (\pi1*\pi4*\pi6*\pi7) - 0.3173 * (\pi1*\pi4*\pi6*\pi7)^2 \quad (8)$$

Tab.15. Result Analysis Model RSM15CT

| Analysis of RSM Model and Goodness of fit | |
|---|--------|
| SSE | 1.424 |
| R-square | 0.8914 |
| Adjusted R-square | 0.8845 |
| RMSE | 0.1351 |

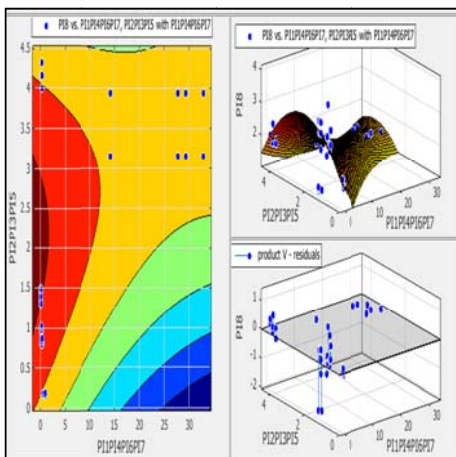


Fig.7. Contour Plot, 3D Response Surface Plot and Residual Plot for Model RSM15CT

6- Conclusion

The present paper gave an illustration of how dimensional analysis (DA) can be applied to significantly reduce the number of independent variables used to optimize the cycle time as response variable using response surface method (RSM). Using DA 43 independent variables has been reduced to 07 dimensionless Pi terms. This can greatly help in constructing a response surface approximation function of fewer variables. These 7 Pi terms are further grouped in two X and Y and along Z the response variable Pi8 is placed as input in RSM. The second or cubic order response surface model for seven Pi terms and one response variables is developed from the experimental data gathered during experimentation. Six models equations were developed using RSM in MATLAB 2011a software. To test whether the data are well fitted in the model or not, the values of SSE, R, R², Adjusted R² and RMSE are observed. In general, the more appropriate Regression model is the higher the values of R² (R is correlation coefficient) and the smaller the values of RMSE (Root Mean Square Error). From the developed models, calculated RMSE value of the regression analysis on cycle time is well within 10% limits, which are smaller and R value for response variable (cycle time) are around 85-97% respectively. The closer the value of R (correlation coefficient) to 1, the better is the correlation between the experimental and predicted values. Here the value of R² being close to 1 indicate a close agreement between the experimental results and the theoretical values predicted by the model equation. This implies that the prediction of experimental data is quite satisfactory.

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Appendix: Prediction of Cycle Time as a Response Variable Using DA & RSM

| Using DA Independent Pi Terms | | | | | | | Z | X | Y | Y _{computed} From Model Y _k |
|-------------------------------|----------------|----------------|----------------|----------------|----------------|----------------|-----------------------------------|--|---|--|
| Π ₁ | Π ₂ | Π ₃ | Π ₄ | Π ₅ | Π ₆ | Π ₇ | Π ₈ =Y _{expt} | Π ₂ Π ₃ Π ₅ | Π ₁ Π ₄ Π ₆ Π ₇ | |
| 0.116107 | 31.08000 | 0.001846 | 0.069998 | 39.12133 | 1.382881 | 1.845239 | 2.333333 | 2.244721502 | 0.020738463 | 2.19548 |
| 0.116107 | 31.08000 | 0.001938 | 0.069998 | 39.12133 | 1.382881 | 1.845239 | 2.666667 | 2.356957578 | 0.020738463 | 2.21763 |
| 0.116107 | 31.08000 | 0.001892 | 0.069998 | 39.12133 | 1.382881 | 1.845239 | 2.333333 | 2.30083954 | 0.020738463 | 2.206269 |
| 0.116107 | 31.08000 | 0.001892 | 0.069998 | 39.12133 | 1.342271 | 2.867886 | 2.666667 | 2.30083954 | 0.031285387 | 2.204695 |
| 0.116107 | 31.08000 | 0.001846 | 0.069998 | 39.12133 | 1.342271 | 2.867886 | 2.666667 | 2.244721502 | 0.031285387 | 2.193885 |
| 0.116107 | 31.08000 | 0.001846 | 0.069998 | 39.12133 | 1.342271 | 2.867886 | 3.500000 | 2.244721502 | 0.031285387 | 2.193885 |
| 0.116107 | 31.08000 | 0.001846 | 0.069998 | 39.12133 | 1.367723 | 1.702743 | 2.333333 | 2.244721502 | 0.018927208 | 2.195754 |
| 0.116107 | 31.08000 | 0.001892 | 0.069998 | 39.12133 | 1.367723 | 1.702743 | 2.333333 | 2.30083954 | 0.018927208 | 2.206539 |
| 0.116107 | 31.08000 | 0.001938 | 0.069998 | 39.12133 | 1.367723 | 1.702743 | 4.000000 | 2.356957578 | 0.018927208 | 2.217896 |
| 0.116107 | 31.08000 | 0.001892 | 0.069998 | 39.12133 | 1.410234 | 3.120132 | 2.333333 | 2.30083954 | 0.035760496 | 2.204027 |
| 0.116107 | 31.08000 | 0.001892 | 0.069998 | 39.12133 | 1.410234 | 3.120132 | 2.666667 | 2.30083954 | 0.035760496 | 2.204027 |
| 0.116107 | 31.08000 | 0.001846 | 0.069998 | 39.12133 | 1.410234 | 3.120132 | 2.333333 | 2.244721502 | 0.035760496 | 2.193208 |
| 0.129645 | 24.75000 | 0.000358 | 0.144643 | 273.8493 | 1.328125 | 1.845239 | 1.300000 | 2.427741865 | 0.045956361 | 2.229125 |
| 0.129645 | 24.75000 | 0.000418 | 0.144643 | 273.8493 | 1.328125 | 1.845239 | 1.315789 | 2.832365509 | 0.045956361 | 2.333544 |
| 0.129645 | 24.75000 | 0.000358 | 0.144643 | 273.8493 | 1.328125 | 1.845239 | 1.300000 | 2.427741865 | 0.045956361 | 2.229125 |
| 0.129645 | 24.75000 | 0.000418 | 0.144643 | 273.8493 | 1.400000 | 2.867886 | 1.300000 | 2.832365509 | 0.075291179 | 2.329718 |
| 0.129645 | 24.75000 | 0.000358 | 0.144643 | 273.8493 | 1.400000 | 2.867886 | 1.300000 | 2.427741865 | 0.075291179 | 2.224878 |
| 0.129645 | 24.75000 | 0.000358 | 0.144643 | 273.8493 | 1.400000 | 2.867886 | 1.250000 | 2.427741865 | 0.075291179 | 2.224878 |
| 0.129645 | 24.75000 | 0.000418 | 0.144643 | 273.8493 | 1.194500 | 1.702743 | 1.300000 | 2.832365509 | 0.03814077 | 2.334563 |
| 0.129645 | 24.75000 | 0.000418 | 0.144643 | 273.8493 | 1.194500 | 1.702743 | 1.368421 | 2.832365509 | 0.03814077 | 2.334563 |
| 0.129645 | 24.75000 | 0.000358 | 0.144643 | 273.8493 | 1.194500 | 1.702743 | 1.300000 | 2.427741865 | 0.03814077 | 2.230256 |
| 0.129645 | 24.75000 | 0.000418 | 0.144643 | 273.8493 | 1.346154 | 3.120132 | 1.250000 | 2.832365509 | 0.078762911 | 2.329265 |
| 0.129645 | 24.75000 | 0.000358 | 0.144643 | 273.8493 | 1.346154 | 3.120132 | 1.368421 | 2.427741865 | 0.078762911 | 2.224376 |
| 0.129645 | 24.75000 | 0.000358 | 0.144643 | 273.8493 | 1.346154 | 3.120132 | 1.250000 | 2.427741865 | 0.078762911 | 2.224376 |
| 0.132669 | 68.26667 | 3.43E-05 | 0.886824 | 136.9246 | 2.172414 | 1.845239 | 2.125000 | 0.320555173 | 0.471630133 | 2.072798 |
| 0.132669 | 68.26667 | 3.29E-05 | 0.886824 | 136.9246 | 2.172414 | 1.845239 | 2.000000 | 0.307732966 | 0.471630133 | 2.074692 |
| 0.132669 | 68.26667 | 3.43E-05 | 0.886824 | 136.9246 | 2.172414 | 1.845239 | 2.428571 | 0.320555173 | 0.471630133 | 2.072798 |
| 0.132669 | 68.26667 | 3.02E-05 | 0.886824 | 136.9246 | 2.274784 | 2.867886 | 2.000000 | 0.282088552 | 0.767553301 | 2.013201 |
| 0.132669 | 68.26667 | 3.43E-05 | 0.886824 | 136.9246 | 2.274784 | 2.867886 | 1.888889 | 0.320555173 | 0.767553301 | 2.007835 |
| 0.132669 | 68.26667 | 3.43E-05 | 0.886824 | 136.9246 | 2.274784 | 2.867886 | 2.000000 | 0.320555173 | 0.767553301 | 2.007835 |
| 0.132669 | 68.26667 | 3.43E-05 | 0.886824 | 136.9246 | 2.434411 | 1.702743 | 2.125000 | 0.320555173 | 0.487696351 | 2.069271 |
| 0.132669 | 68.26667 | 3.02E-05 | 0.886824 | 136.9246 | 2.434411 | 1.702743 | 2.285714 | 0.282088552 | 0.487696351 | 2.075019 |
| 0.132669 | 68.26667 | 3.43E-05 | 0.886824 | 136.9246 | 2.434411 | 1.702743 | 2.125000 | 0.320555173 | 0.487696351 | 2.069271 |
| 0.132669 | 68.26667 | 3.29E-05 | 0.886824 | 136.9246 | 2.153846 | 3.120132 | 2.000000 | 0.307732966 | 0.790667797 | 2.004509 |
| 0.132669 | 68.26667 | 3.43E-05 | 0.886824 | 136.9246 | 2.153846 | 3.120132 | 2.125000 | 0.320555173 | 0.790667797 | 2.002761 |
| 0.132669 | 68.26667 | 3.29E-05 | 0.886824 | 136.9246 | 2.153846 | 3.120132 | 2.125000 | 0.307732966 | 0.790667797 | 2.004509 |
| 0.136734 | 41.65000 | 0.000449 | 0.266709 | 91.28309 | 1.067479 | 1.845239 | 2.500000 | 1.707538356 | 0.071833579 | 2.112465 |