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Dynamic Hub Covering Problem with Flexible Covering Radius

Amir Ebrahimi Zade, Yahia Zare Mehrjerdi *, Hassan Hosseininasab, Alireza Zahmatkesh

Master Student in Department of Industrial Engineering, Yazd University, Yazd Iran . aebrahimyzade@yahoo.com
Associate Professor in Department of Industrial Engineering, Yazd University, Yazd Iran . yazm2000@yahoo.com
Associate Professor in Department of Industrial Engineering, Yazd University, Yazd Iran . h.hosseininasab@yazduni.ac.ir
Master Student in Department of Industrial Engineering, Yazd University, Yazd Iran . alireza_121z@yahoo.com

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ABSTRACT

One of the basic assumptions in hub covering problems is considering the covering radius as an exogenous parameter which cannot be controlled by the decision maker. Practically and in many real world cases with a slight increase in costs, to increase the covering radii, it is possible to save the costs of establishing additional hub nodes. Also change in problem parameters during the planning horizon is one of the key factors causing the results of theoretical models to be impractical in real world situations. To dissolve this problem, in this paper a mathematical model for dynamic single allocation hub covering problem is proposed in which the covering radius of hub nodes is one of the decision variables. Also Due to NP-Hardness of the problem and great computational time required to solve the problem optimally, an effective genetic algorithm with dynamic operators is proposed afterwards. Computational results show the satisfying performance of the proposed genetic algorithm in achieving satisfactory results in a reasonable time.

1. Introduction

The concept of hub nodes arises when there are many origin-destination nodes in a transportation network in order to use the economies of scale. Hubs are nodes in which the flow from various origins is gathered and after reorganization it is dispatched to the destinations. Hub location problem, first introduced by O'Kelly has many applications in cargo delivery systems, airline networks and telecommunication systems [1], he proposed a quadratic model for hub median problems which was the first mathematical model for hub location problems [2]. The objective function in this model is the minimization of total flow costs and in other works such as [3-5] some linearized versions of the problem are proposed. Hub location problems are generally classified in to three sub categories: hub center, hub median and hub covering problems. Considering the point that hub covering problem is investigated in this paper, the interested reader is referred to review papers like [6, 7] for more study on the two remaining sub categories of hub location problem.

* **Corresponding author:** *Yahia Zare Mehrjerdi*
Email: *yazm2000@yahoo.com*

Hub covering problem initially introduced by Campbell [8]. He proposed mathematical models for single and multiple allocation versions of hub set covering and hub maximal covering problems. Campbell also proposed that origin node *i* and destination node *j* can be covered by hubs *k, l* in three ways:

1- Total traveling costs (time or distance) from origin node *i* to destination node *j* (via origin node *i* to hub *k*, destination node *j* to hub *l* and traveling cost between the hub nodes with a discount factorα) does not exceed a specific value.

2- Traveling costs (time or distance) for each of the links in the path from origin node *i* to destination *j* via hubs *k* and *l* do not exceed a specific value.

3- Traveling costs (time or distance) from origin node *i* to hub *k* and from hub *l* to destination node *j* do not exceed a specific value.

All of the existing models in the literature obey the above rules in which it is assumed that the covering radius is a fixed parameter in all cases and the decision maker cannot change its size whereas in many real world applications covering radius of facilities is one of decision variables which the decision maker should decide on the amount [9]. In many cases it is possible to save in costs of establishing extra hub nodes with a slight increase in the covering radius. For example in a transportation system by establishing depots with more capacity and equipping the existing facilities, farther customers can be served, which is equivalent to an increase in the covered area. The capability of an airport to service flights has a direct proportion to the number of runways, facilities and infrastructure which can be increased when necessary. Also in a telecommunication system the area covered by the radio waves depends on the strength of the waves emitted from the transmitter and with reinforcing the emitted waves, larger areas can be covered by the hub node. As specified in the aforementioned examples, with a slight increase in costs, larger covering radius is available and in many cases the increase in covering radius can prevent the superfluous costs of establishing new hubs. In order to capture this situation two types of costs are considered for a hub node, fixed costs of developing hubs and the covering cost which is proportional to the selected covering radius for the hub node.

Facility location decisions are mainly strategic and long term. This results a considerable uncertainty in the parameters of location problems[10]. For example as time goes on, the amount of supply and demand varies in origin and destination nodes, moreover transportation costs among nodes can change due to causes like depreciation of the fleet, increase in fuel cost or using from cheaper facilities in the fleet. Multi-period consideration of the problem provides the capability for the model to establish new facilities, close some of the existing facilities and some variations in the location of current facilities in each period, proportional to changes of the parameters. From a point of view, dynamic facility location problems can be divided in to two categories. The first category consists of problems in which the number of facilities is an exogenous factor. Some of them like [11] specify total number of facilities in the planning horizon and number of facilities in each period should be determined by the model. Some of the authors such as [12, 13] assumed a fixed number of facilities that can be relocated at the end of each period during the planning horizon. The second category consists of the problems in which the number of facilities is an endogenous parameter and their number and location must be determined such that total costs are minimized. Contreras et al. considered the dynamic hub location problem in which the capacity of hub nodes is unlimited and proposed a mixed integer nonlinear model [14]. The authors assumed that establishing hub nodes is costly while their closure contains a profit which is the result of releasing some resources in closed hubs. Also Taghipourian et al. considered virtual hub location problem in dynamic conditions [15]. The authors assumed closed hubs to be costly as well as the established hubs in each period. Hub covering problem discussed here belongs to the second category. Existing models for hub covering problem are static and the dynamic model proposed in this paper integrates the concept of previous dynamic models by considering both cost and benefit for closed hubs. More realistic approach to the problem arises with more scrutiny on the structure of facilities. Facilities in a hub can be categorized in to two types: static and movable facilities. While static facilities remain useless when a hub is closed moving facilities can be transferred to newly developed hubs and cause savings if needed. For example in a hub airport some infrastructure facilities like building, watchtower and runways are static facilities and facilities like airport staff are moving facilities and transferring them can cause savings in employment and education costs. It is assumed that moving facilities released from closed hubs are usable in only one of newly established hubs in the same period. The saving associated to these movements is subtracted from the total costs of the period.

Kara and Tansel introduced a single allocation hub covering model and provided a proof for NP-hardness of the problem [16]. They also proposed three linearized versions for hub set covering problem. Also Wagner proposed a new mathematical model for hub set covering problem [17]. Tan and Kara investigated hub covering problems in cargo delivery systems of turkey and introduced the reputable Turkish data set [18]. Qu and Weng used path relinking approach for solving hub maximal covering problems [19]. Also Calık et al. studied single allocated hub covering problem under the incomplete hub network assumption [20]. They presented an effective heuristic based on the taboo search to solve the problem. Mohammadi et al. considered hub covering problem with congestion in the network and modeled the hubs as M/M/c queue model [21]. The authors proposed an Imperialist Competitive Algorithm to solve the proposed model. Also Mohammadi et al. studied a capacitated single allocation hub covering problem and proposed a mathematical model for the problem. To solve the problem in a reasonable computational time, they proposed a modified GA and a shuffled frog algorithm which perform satisfactorily [22]. Karimi and Bashiri considered hub set covering and maximal covering problems with a different coverage type and proposed two heuristics for the problem [23]. The authors applied the proposed models and the heuristics on the data based on Iranian hub airports and Turkish dataset. Fazel Zarandi et al. considered hub covering problem with backup coverage in which a node will be covered if there are at least Q possible routes to satisfy its demand. Also to enforce dispersion in hub positions, a lower bound is assumed for the distance between the hub nodes [24]. Zarei et al. proposed two mathematical formulations for a hub location problem with multi level capacities in which direct assignment between the non-hub noedes ia allowed [25].

Considering the above explanations, major contributions of the paper to the hub location literature are:

(1) introducing a hub covering problem in which the covering radius of each hub node is a decision variable and in a more realistic approach the covering costs are proportional to the covered area by the hub node; therefore the proposed model balances between the establishment costs of the hub nodes and their covering costs.

(2) Similar to the real world situations, parameters of the problem are allowed to be changed periodically. To calculate the benefits and costs from the closed hubs in each period simultaneously, the equipment in a hub are divided to static and movable facilities.

The proposed dynamic formulation determines established hubs in each period, their covering radius, allocates non-hub nodes to the hubs and determines closed hubs in each period such that total costs are minimized.

The rest of this paper is organized as follows. Initially in section 2.1 we provide a mathematical model for the proposed problem and due to complexity of the aforementioned problem, in section 2.2 a dynamic genetic algorithm is proposed which is capable of achieving appropriate solutions in a reasonable time. Computational results of implementing proposed mathematical model and proposed genetic algorithm on experimental problems are presented in section 3. Finally conclusions and some guidelines for future study are presented in section 4.

2.1- Proposed Mathematical Model

It is supposed that $N = \{1,2,\dots,n\}$ is the set of supply and demand nodes in the network. Each of the nodes is a potential location for establishing hubs. i is the index for supply nodes and j is the index for demand nodes, k and l are indices for hub nodes and t is the index for periods. In addition it is supposed that the costs matrix is symmetric, so$c_{ij} = c_{ji}$. The connection between each pair of the origin-destination (O/D) nodes is available only through the hubs. There is no limitation on the capacity of Hub nodes and they are completely interconnected, hence for connecting each origin to its destination the flow passes through one or two hub nodes. Each node can be connected to only one hub. Each hub node contains two kinds of costs: fixed establishing cost and covering cost. Covering costs of a hub are proportional to its covering radius and the covering radius of the hub equals to the farthest node covered by the hub. Considering the use of special facilities between the hub nodes, discount factor$\alpha(0 \leq \alpha \leq 1)$ is used. For a pair of O/D nodes in a period, total transportation costs equals to sum of transportation costs from origin *i* to hub *k*, hub k to hub *l* considering discount factorα and hub l to destination j. proposed mixed integer model for each period of planning horizon determines established hubs, closed hubs, covering radius of a hub node and allocated nodes to each of the hubs. Although there are simpler formulations for hub covering problem such as the one proposed by Karimi and Bashiri [23], here we use the formulation proposed in [21] for the sake of clarity. Model parameters are:

(1)c_{tij}^{kl} : Is the present value of total transportation cost for travelling from origin *i* to destination j via hubs k and *l* in period *t*. (2)ec_{tk} : Is the present value for fixed cost of establishing a hub in node *k* and in period *t*. (3)fr_{tk} : Is the present value of covering cost of hub at node *k* in period *t*. (4)cc_{tk} : Is the present value for costs of closing a hub at node k in period t including both static and movable facilities. (5)ms_t : Is the present value of the benefits from movable facilities in a closed hub to be used in a newly established hub in period t. (6)d_{ik} : Is the distance from node i to hub *k* and (7) M: a big number.

The set of decision variables in the model are: (1)x_{tij}^{kl} : A binary decision variable which is one if nodes i, j are connected via hubs k, l in period *t* and otherwise equals 0. (2)y_{tik} : A binary variable which is one if node *i* is connected to hub *k* in period *t* and otherwise it equals 0. (3)r_{tk} : is the covering radius of node *k* in period *t*. (4)p_{tk} : Is a binary variable which is one if a new hub is established in node k in period t and otherwise equals 0. (5)q_{tk} : Is a binary variable which is one if the hub existing in node *k* is closed in period *t* and otherwise equals 0. (6)z_t: Is the minimum of $\sum_k p_{tk}$ and$\sum_k q_{tk}$.

While closing a hub always incurs closure costs, the savings from the closing occur only when there is a possibility to use the released movable facilities in newly established hubs. In order to determine number of hub nodes which facilities can be moved to other hubs, following lemma is proposed. It is assumed that after a hub is closed, there is the possibility to transfer its movable facilities to one of the newly established hubs in the same period and they are not capable of buffering for subsequent periods.

Lemma .Number of possible movements in each period equals to the minimum of total established hubs ($\sum_k p_{tk}$) and total closed hubs in that period ($\sum_k q_{tk}$).

Proof .Generally in each period there are three possible situations. a) Total number of established hubs is greater than the total number of closed hubs ($\sum_k p_{tk} > \sum_k q_{tk}$). In this case it is possible to use the released movable facilities from all of the closed hubs. Hence number of movements will be$\sum_k q_{tk}$. b) Number of established hubs equals to the number of closed hubs ($\sum_k p_{tk} = \sum_k q_{tk}$). In this case movable facilities from each of closed facilities can be allocated to one of the established facilities. Number of movements will be $\sum_k q_{tk}$ or$\sum_k p_{tk}$. c) Number of established hubs is less than the number of closed hubs ($\sum_k p_{tk} < \sum_k q_{tk}$). Despite extra supply, the demand is limiting and it is possible to use moving facilities from $\sum_k p_{tk}$ of closed hubs in newly established hubs. Considering it is impossible to use moving facilities from $\sum_k q_{tk} - \sum_k p_{tk}$ of closed hubs, no saving will be taken in to account.

Considering the above explanations the proposed mathematical model is as follows.

$$Min\ \sum_t \sum_i \sum_k \sum_l \sum_j c_{tij}^{kl} x_{tij}^{kl} + \sum_t \sum_k ec_{tk} p_{tk} + \sum_t \sum_k fr_{tk} r_{tk} + \sum_t \sum_k cc_{tk} q_{tk} - \sum_t ms_t z_t \quad (1)$$

$$\sum_k \sum_l x_{tij}^{kl} = 1 \qquad \forall i, j, t \quad (2)$$

$$2x_{tij}^{kl} \le y_{tjl} + y_{tik} \qquad \forall i, j, t, k, l \quad (3)$$

$$y_{tik} \le y_{tkk} \qquad \forall i, t, k \quad (4)$$

$$\sum_k y_{tik} = 1 \qquad \forall i, t \quad (5)$$

$$r_{tk} \ge d_{ik} y_{tik} \qquad \forall i, t, k \quad (6)$$

$$p_{tk} - q_{tk} = y_{tkk} - y_{t-1kk} \qquad \forall k, t > 1 \quad (7)$$

$$z_t = \min(\sum_k p_{tk}, \sum_k q_{tk}) \qquad \forall t \quad (8)$$

$$x_{tij}^{kl}, y_{tik}, p_{tk}, q_{tk} \in \{0,1\}, r_{tk}, z_t \ge 0\ \&\ integer \qquad \forall i, j, t, k, l \quad (9)$$

Expression (1) is the objective function of the proposed model which is aimed at minimizing total costs. The first part of the objective function considers transportation costs form origin node *i* to destination *j* via hubs *k* and *l*. Second part of the objective function considers hub establishment cost. Covering cost of each hub in each period is the third part of the objective function. Costs associated with closing hub nodes in each period is the fourth part of the objective function and the fifth part is the saving that comes from transferring movable facilities from closed hubs to the established hubs. Constraints (2) guarantee that the connection between each O/D pair is trough one or two hubs. Constraints (3) ensure that in each period, the path from *i* to *j* via hubs *k* and *l* is available if both origin node *i* and destination node *j* are respectively connected to hubs *k* and *l*. constraints (4) ensure that in each period, node *i* can be connected to node *k* if it is set as a hub.

Constraints (5) ensure that each node allocates to only one hub. Covering radius of a hub equals to distance between the hub and the farthest allocated node to the hub which the amount is calculated from equation (6) for each node in each period. With the assistance of constraints (7) for a period, if a hub is newly established in a node, binary variable p_{tk} equals one and binary variable q_{tk} equals zero and if the existing hub in a node is closed, binary variable q_{tk} equals one and binary variable p_{tk} equals zero. Otherwise both of the variables will equal zero. Considering ***lemma 1*** total number of facility movements in each period is calculated from constraints (8). Expression (9) specifies variables $x_{tij}^{kl}, y_{tik}, p_{tk}, q_{tk}$ as binary variables and other variables as integer and nonnegative variables.

Considering nonlinearity of equation (8), following set of constraints is proposed. Equation (10) introduces virtual variable v_t as subtraction of$\sum_k q_{tk}$ from$\sum_k p_{tk}$. Considering equation (15), if the desired minimum is$\sum_k q_{tk}$, hence $v_t$is nonnegative, we require v'_t being equal to zero but if the desired minimum is$\sum_k p_{tk}$, hence $v_t$is negative, we require v'_tbeing equal tov_t. With the aid of constraints (11) and (12) binary variable w_t will be one if v_t is negative and if $v_t$is nonnegative w_t equals zero. Constraints (13) and (14) together provide situation in which if$w_t = 1$then $v'_t \leq v_t$ and if$w_t = 0$ then$v'_t \leq 0$. With the assistance of these constraints an upper bound for variable z_t is obtained and considering the utility of the maximum amount of z_t in objective function, these variables will attain their upper bound.

$$v_t = \sum_k p_{tk} - \sum_k q_{tk} \qquad \forall t \qquad (10)$$

$$v_t \geq -Mw_t \qquad \forall t \qquad (11)$$

$$v_t < M(1 - w_t) \qquad \forall t \qquad (12)$$

$$v' - M(1 - w_t) \leq v_t \qquad \forall t \qquad (13)$$

$$v'_t \leq Mw_t \qquad \forall t \qquad (14)$$

$$z_t = v'_t + \sum_k p_{tk} - v_t \qquad \forall t \qquad (15)$$

$$w_t \in \{0,1\}, v_t, v'_t \geq 0 \qquad \forall t \qquad (16)$$

In order to linearize the model it is possible to use constraints (10) to (16) instead of (8).

2.2- Proposed Genetic Algorithm

Kara and Tansel proved the NP-Hardness of hub covering problems [16]; hence our problem which is a more complex form of hub covering problem will be NP-Hard respectively. Due to the complexity of the problem, high computational time is needed to attain optimal solution. In order to get suitable solutions in a reasonable computational time a Genetic Algorithm (GA) is proposed for the investigated problem. GA is a metaheuristic algorithm based on Darwinians theory of evolution first introduced by Holland [26]. GA transmits a set of solutions for consecutive iterations, called population, in each iteration some new individuals are added to the population and some individuals with lower utility will be eliminated from the population. This goes on until a specific criterion is met which is called stopping criteria.

Compared with the classic GA, this paper proposes a schema for the problem, develops genetic operators based on the chromosome structure, introduces a dynamic immigration operator and two stopping criterions for the algorithm. The following subsections describe the main features for the proposed genetic algorithm.

2.2.1- Chromosome Structure

One of the most important specifications of the GA which has a great effect on the effectiveness of algorithm is the chromosome structure and this structure should capture all the features of the problem. Proposed structure is presented in Fig.1. The status of the nodes should be specified in all periods of the planning horizon, hence total number of gens in a chromosome structure equals to the multiplication of the number of nodes to the number of periods. The first stratum of the proposed structure determines number of the hub which the node is allocated; so if the container of a gene equals to its order it is set as a hub. Considering the difficulty of implementing the GA operators on first stratum, second stratum is introduced in which hub nodes are determined.

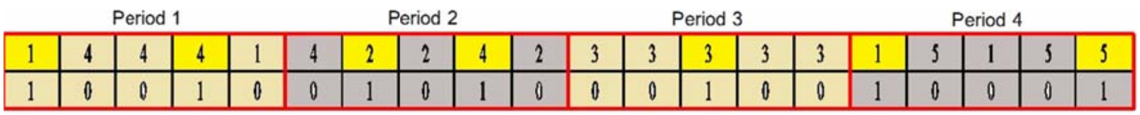

Fig.1-Chromosome Structure

2.2.2- Initial Population

GA is a population based algorithm and permanently transmits a set of individuals to different iterations. Firstly it is necessary to generate a set of solutions as initial population. To emphasize the importance of the population size it is noticeable that extra-large size of the initial population results in trashy increase of computational time and small size of the population will cause the algorithm not achieving to global optima. Population size is chosen as 200. In order to create initial population in each period, some of the nodes are selected as hubs and the remaining nodes are allocated to nearest hub.

2.2.3- Selection Strategies

There are different methods to select parents for implementing crossover and mutation. Here roulette wheel method is used which was first introduced by Goldberg [27]. In this method after sorting all population members according to their fitness, each one will be allocated a selection probability which is proportional to its rank. In this situation all the members of the population have the chance to be selected, although the chromosome with better fitness will have more chance to be selected.

2.2.4- Genetic Operators

The operators in GA are tools for better search in solution space. Reproduction of the individuals in each iteration causes to add new individuals to the population which the characteristics are their parent's patrimony. This phenomenon is presented in crossover operator. Scarcely and due to tribulation in structure, paucity of chromosomes have salient differences with the others. Similar to the illustrious role of mutation in the human evolution this operator also is very important for the genetic algorithm in salvation from local optima. A phenomenon that many human populations are faced with is the entrance of some individuals to the population from other populations which is called the immigration. Proposed algorithm considers this social phenomenon as another GA operator.

There are multiple methods for implementing crossover. Here a modified version of single point crossover is proposed. Considering the problem is multi-period, implementing single-point crossover might not be so suitable. For more proficiency in each period single point crossover is implemented randomly. Second stratum of the chromosome is used to do crossover. Fig.2 shows the implementation of crossover on the second stratum of a chromosome with four periods.

The second operator of GA is mutation. If the best solution found by the algorithm remains the same for consecutive iterations the algorithm might be trapped in local optima. To escape this situation mutation operator is instrumental. Like crossover, second stratum of the chromosome is selected for implementing mutation. Some of the gens in the chromosome are selected randomly, if the selected node is a hub it will be changed to a non-hub node and if it is a non-hub node it will be changed to a hub. Fig.3 demonstrates implementation of a mutation with seven changes on a problem with four periods. One of the possible occurrences is to make a period without any hubs like the third period in Fig.3; in this case the same numbers of the genes are selected randomly and will be changed to hub nodes.

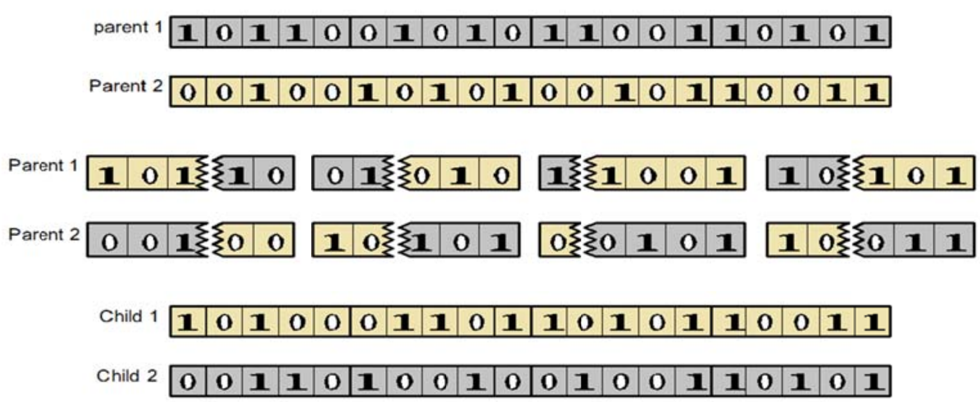

Fig. 2- crossover on the second stratum of chromosome with four periods

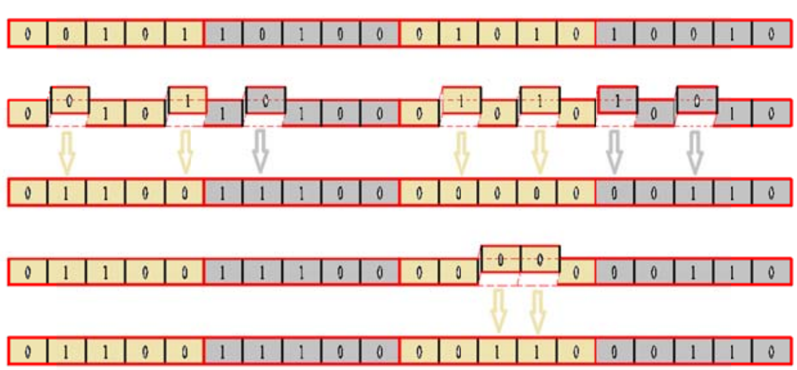

Fig. 3 - mutation operator

Although increasing the number of changes in a selected chromosome for mutation causes the increment of computational time, more changes provide better search in solution space. Proposed dynamic mutation operator increases the number of changes in a selected chromosome for mutation along with increasing the number of iterative solutions. Number of genes that are remodeled in a mutant is presented in (17).

$$nm = \begin{cases} \left\lceil \frac{nnode}{5} \right\rceil & if\ Is < \frac{maxit}{3} \\ \left\lceil \frac{nnode}{3} \right\rceil & if\ IS < \frac{maxit}{2} \\ \left\lceil \frac{nnode}{2} \right\rceil & if\ IS > \frac{maxit}{2} \end{cases} \tag{17}$$

In this equation $nvar$ is the number of supply and demand nodes, Is is the number of iterative solutions in which the best solution found by the algorithm remains unchanged and $maxit$ is the maximum number of iterations.

The third operator introduced here is called immigration. It is supposed that there are some immigrants to the society in each period. In real world situation alongside with the economic and scientific growth of a society, general tendency of the people from other populations increases to immigrate to the society, the designed dynamic immigration operator similarly increases rate of immigration to the society with increasing the probability of achieving the global optima, which the sign is remaining the best solution ever found unchanged for consecutive iterations. Increasing the rate of immigrants to the society like mutation operator increases the capability of algorithm to escape from local optima. Similar to initial population, immigrants are produced randomly and the rate of immigration is presented in equation (18).

$$IM_p = \frac{IS}{n_{POP}} \tag{18}$$

In this equation IS is the number of consecutive iterations in which the best solution remains unchanged, $n_{POP}$is the number of individuals in the population and IM_p is the rate of immigration.

2.2.5- Stopping criteria

Various criteria to stop GA have been introduced heretofore. Maximum number of iterations is one the most widely used stopping criteria. Due to the solution space of the problem in some cases, the algorithm reaches the optimum solution in primal iterations and remains unchanged until the last iteration. There are two possibilities; first the algorithm is trapped in a locally optimal solution. In this case, as described in 2.2.4, the designed algorithm will try to escape local optima with increasing the severity of search in solution space with the aid of intensifying the number of permutations in a mutant chromosome and also increasing the rate of migration to the population. Second possibility is that the algorithm has reached optimum solution; in this case it is ideal to stop the algorithm immediately. With the aim of parsimony in computational time for the problems in which the optimal solution is achieved rapidly, another stopping criterion is utilized alongside with maximum iterations. Provided that the best solution remains unchanged for$maximum\ iterations/2$, the algorithm will be terminated.

To aggregate the above explanations pseudo codes for the proposed GA is presented in Fig.4.

```
Genetic Algorithm pseudo code
1.  parameters setting
✓   Set parameters: Maximum number of iterations (Maxit), Population
    size (nPop),Crossover percentage (pc), Mutation percentage (pm)
1.  Initialization
✓   Create nPop individuals and evaluate theme.
2.  iterations
    While Maxit iterations or Maxit/2 of iterative solutions are not met do:
✓   Crossover
o   Select 2(pc × nPop/2) chromosomes using roulette wheel selection.
o   apply single point crossover for each period of the
    Chromosome separately and evaluate the childes.
✓   Mutation
o   Select nPop × pm chromosomes randomly.
o   Calculate number of mutations (nm) using equation 17.
o   Apply nm random changes in each chromosome and evaluate the
    mutant.
✓   Migration
o   Calculate migration rate (mr) using equation 18.
o   Create mr × nPop chromosomes randomly and evaluate the
    immigrants.
✓   aggregate childes, mutants and the immigrants
            and select the nPop superior individuals.
✓   Store the best individual as solution.
3.  Show result
✓   Exhibit the best solution ever found as the final solution.
```

Fig. 4- pseudo code for the proposed GA

3. Computational Results

3.1- Effects of the Flexible Covering Radius

To analyze effects of the proposed flexible covering radius on the costs of the designed network; some numerical examples are extracted from the Turkish data set. The designed problems are single – period with 10, 15, 20, 25 nodes and in all of them discount factor α is set 0.5. Computational results in table 1 compare the designed hub network when there is a fixed covering radius versus the proposed flexible covering radius. For the fixed radius case two types of covering radius are considered: loose covering radii (LCR) in which the radii is equal to the average distance among the nodes and tight covering radii (TCR) which is assumed to be (0.5LCR).

In the networks with a fixed covering radius LCR case is expected to have less costs than the TCR because of a greater covering radius and consequently fewer established hubs. Considering the computational results in table 1, the designed networks with flexible covering radius have less cost and fewer established hubs compared to the classical models in both TCR and LCR cases. In the designed networks with a flexible covering radius, each of the established hubs has a particular covering radius proportional to the covered area. For example in the problem with 25 nodes, covering radius for the hub established in node 19 is 94 while it is 551 for the hub in node 25. Fewer hub nodes and larger radii in the problems with flexible covering radius versus TCR and LCR cases results from the balance between the hubs establishment costs and covering costs.

Tab. 1. Effects of the flexible covering radius on the designed network

Fixed covering radii					Flexible covering radii		
n	Type	radii	Hub nodes	Cost	Max–radii	Hub nodes	Cost
10	TCR	243	1,3,6,8,10	88261	701	6	77721
10	LCR	487	1,3,6,8	81479			
15	TCR	247	1,3,6,8,10,12	124857	671	1,3,6,10	120614
15	LCR	495	1,3,5,6,12	121681			
20	TCR	239	1,3,4,5,6,10,12,16,20	181796	744	1,3,6,11,19,20	163857
20	LCR	477	1,3,4,6,12,16,19,20	180457			
25	TCR	252	1,3,5,6,7,13,16,17,20,25	276792	551	3,6,16,19,20,25	247022
25	LCR	504	1,3,6,7,17,19,20,21,25	266122			

3.2- Experimental Problems

To compare the performance of the proposed GA with results of the mathematical model some numerical examples are required. In the designed problems, fixed cost of establishing hubs, follows uniform distribution in range [7000, 12000] and the saving that comes from the closure of hubs has a uniform distribution in range [4000, 7000], considering that the costs are assumed to be dynamic, these costs change with rate $1 + \alpha$ periodically in which α has a uniform distribution in the interval [-0.05, 0.15]. Closure costs are uniformly distributed in the range of [1000, 4000]. Covering costs are a function of the distances between the nodes. Distances have uniform distribution in range [33, 99] and the covering cost initially is the distance multiplied to 10. Similar to the transportation costs these costs change periodically with the rate of$1 + \gamma$ in which $\gamma$is uniformly distributed in [-0.2, 0.8]. Numerical experiments have 5, 10, 15, 20, 30, 40, 50, 60, 70, 80, 100 nodes. For each size of the problem discount factor α equals to 0.3, 0.6, 0.9 and number of periods (t) changes between 2, 3 and 4. Parameters tuning in the proposed GA results in: population size is set 200, maximum number of iterations 250, crossover percentage 0.85 and mutation percentage 0.1. The rest of the required parameters will be tuned by the algorithm as illustrated before. The experiments were done on a system with 4 GB of RAM and core i5 CPU. Optimal solutions to the problems were found by GAMS 22.2 using CPLEX solver and the proposed GA was coded and ran with MATLAB R2011b.

3.3- Performance of the Proposed GA

The amount of objective function and the computational time for mixed integer model along with the proposed GA is presented in Table 2. For the problems with 20 nodes and more, the optimal solution was not obtainable even after 6000 seconds. As shown in Table 2 for the problems of size 5, computational time to solve the problem optimally is less than the required time for the proposed GA,

Although by increasing the problem size, computational time for optimal solution has an exponentially growth. Fig.5 delineates the growth in average computational time for the proposed mixed integer model versus the proposed GA in each of the experimental problems. This figure corroborates better performance of the proposed GA for medium and large size problems in the context of computational time.

It is obvious from the table that for each size of the problem as the number of periods (t) increases computational time for GAMS and the proposed GA increases which is the result of increasing the complexity of the problem. Considering Table 2 discount factor α does not have a meaningful effect on the computational time whereas the amount of objective function increases with increasing the discount factor in most cases. Also the proposed GA shows a salient performance in the case of quality of the solution.

For problems in which the optimal solution was found in a reasonable time, proposed GA reaches the optimum solution in all cases. As mentioned in 2.2.5, the proposed GA decides on some of the problem parameters such as the number of iterations alongside with its progress in consecutive generations. The last column of Table 2 shows the number of iterations (It) in which the proposed GA reached the solution.

As the number of nodes (n) increases, a slight increase in the number of iterations is sensible which results from the more complex structure of the problem. For the case of problems with 100 nodes the algorithm approaches the upper bound of 200 iterations whereas for smaller problems, number of required iterations for the algorithm has a meaningful decrease which results in considerable saving in computational time.

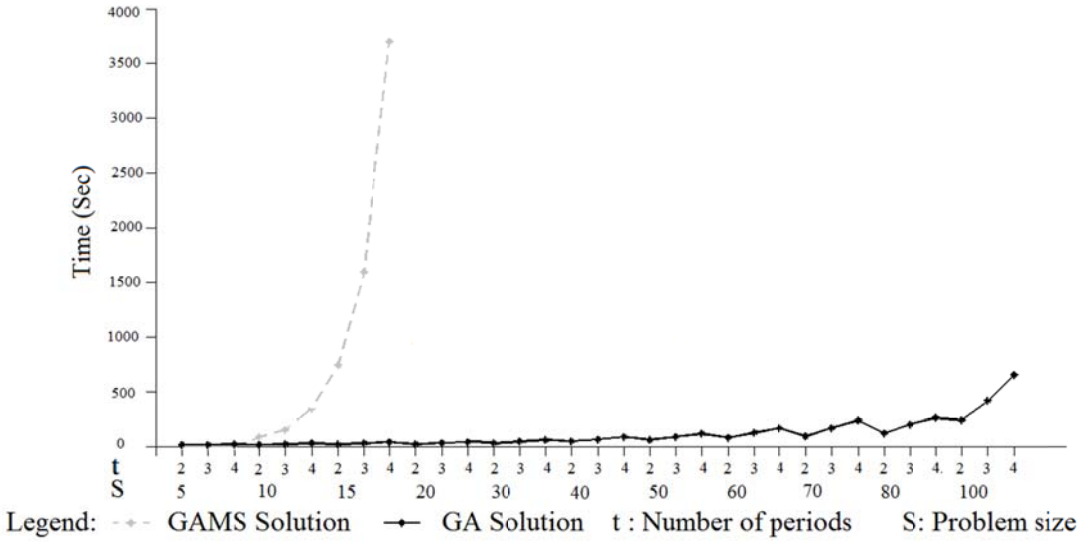

Fig. 5. Computational time growth

Tab. 2. Computational results

			GAMS		GA						GAMS		GA		
n[1]	t[2]	α[3]	Objective value	[4]CPU(Sec)	Objective value	CPU(Sec)	[5]It.	n	t	α	Objective value	CPU(Sec)	Objective value	CPU(Sec)	It.
5	2	0.3	10232	0.97	10232	12.75	121			0.6	-	-	475376	60.45	130
		0.6	10520	0.94	10520	14.18	135			0.9	-	-	551486	58.28	127
		0.9	10808	0.99	10808	12.82	123		4	0.3	-	-	545107	87.54	143
	3	0.3	19212	1.03	19212	14.66	100			0.6	-	-	665146	78.29	129
		0.6	19212	1.09	19212	14.56	100			0.9	-	-	733251	78.52	133
		0.9	19212	0.97	19212	14.59	100	50	2	0.3	-	-	434556	57.12	135
	4	0.3	23047	1.52	23047	18.53	100			0.6	-	-	503656	54.29	130
		0.6	23047	1.51	23047	18.30	100			0.9	-	-	540452	46.96	113
		0.9	22326	1.53	22326	21.04	103		3	0.3	-	-	636508	85.47	144
10	2	0.3	29249	82.84	29249	14.61	100			0.6	-	-	738361	77.35	130
		0.6	29249	82.31	29249	14.48	100			0.9	-	-	822362	75.45	129
		0.9	29249	82.40	29249	14.31	100		4	0.3	-	-	813389	116.85	152
	3	0.3	41457	150.29	41457	20.05	100			0.6	-	-	968432	103.70	135
		0.6	41457	150.40	41457	20.37	102			0.9	-	-	1068770	89.33	119
		0.9	41457	150.39	41457	20.79	104	60	2	0.3	-	-	602728	80.14	156
	4	0.3	47137	345.95	47137	27.76	108			0.6	-	-	716929	68.75	134
		0.6	51320	346.04	51320	27.29	107			0.9	-	-	810410	55.75	109
		0.9	47137	346.09	47137	28.64	112		3	0.3	-	-	874574	124.46	167
15	2	0.3	55751	737.18	55751	18.36	101			0.6	-	-	1035040	104.47	143
		0.6	55751	738.28	55751	18.24	102			0.9	-	-	1173152	92.90	128
		0.9	55751	739.38	55751	18.37	107		4	0.3	-	-	1027456	168.38	171
	3	0.3	80985	1593.28	80985	27.15	109			0.6	-	-	1371700	153.92	160

			GAMS		GA						GAMS		GA		
n[1]	t[2]	α[3]	Objective value	[4]CPU(Sec)	Objective value	CPU(Sec)	[5]It.	n	t	α	Objective value	CPU(Sec)	Objective value	CPU(Sec)	It.
		0.6	84009	1592.03	84009	33.37	128			0.9	-	-	1540899	123.31	133
		0.9	85651	1593.49	85651	28.63	112	70	2	0.3	-	-	816489	91.79	144
	4	0.3	108187	3698.35	108187	36.53	115			0.6	-	-	948789	90.34	144
		0.6	107708	3699.50	107708	37.36	119			0.9	-	-	1048865	79.05	127
		0.9	107708	3700.05	107708	33.79	109		3	0.3	-	-	1085829	166.54	180
20	2	0.3	-	-	82997	19.23	108			0.6	-	-	1383974	137.66	154
		0.6	-	-	91847	18.52	106			0.9	-	-	1557740	110.27	126
		0.9	-	-	94303	19.82	114		4	0.3	-	-	1125533	236.42	198
	3	0.3	-	-	123816	29.08	117			0.6	-	-	1533438	221.29	184
		0.6	-	-	138235	26.17	106			0.9	-	-	2078191	161.50	144
		0.9	-	-	143138	28.71	116	80	2	0.3	-	-	1019915	120.68	157
	4	0.3	-	-	149148	37.48	117			0.6	-	-	1205921	110.89	149
		0.6	-	-	164548	38.14	120			0.9	-	-	1349050	93.82	128
		0.9	-	-	180081	36.47	116		3	0.3	-	-	1255795	201.48	180
30	2	0.3	-	-	177775	27.71	112			0.6	-	-	1611591	208.37	186
		0.6	-	-	194652	27.40	110			0.9	-	-	2046308	161.96	153
		0.9	-	-	202176	27.29	110		4	0.3	-	-	1432306	261.06	180
	3	0.3	-	-	263619	43.48	120			0.6	-	-	1905039	288.90	199
		0.6	-	-	280087	43.25	122			0.9	-	-	2413498	289.33	200
		0.9	-	-	303519	43.75	124	100	2	0.3	-	-	1401816	240.66	186
	4	0.3	-	-	346700	57.09	125			0.6	-	-	1776351	243.82	184
		0.6	-	-	392356	56.08	125			0.9	-	-	2082965	170.72	139
		0.9	-	-	407596	53.39	119		3	0.3	-	-	1730317	413.00	199
40	2	0.3	-	-	282178	44.91	133			0.6	-	-	2308048	417.78	198
		0.6	-	-	322915	38.96	119			0.9	-	-	2829832	381.54	198
		0.9	-	-	351033	37.60	115		4	0.3	-	-	2129941	650.85	194
	3	0.3	-	-	422745	63.59	134			0.6	-	-	2657675	517.93	199
										0.9	-	-	3607594	557.12	197

[1]n: Number of nodes, [2]t: Number of periods, [3] α: Discount factor, [4]CPU: Computational time, [5]It: Number of iterations for the proposed GA

4. Conclusion and Further Research Area

In this paper we proposed a mathematical model for dynamic hub covering problem with flexible covering radius. The proposed model assumes that each hub node comprises of fixed and variable costs which the latter case is a function of the hubs' covering radius. Furthermore in order to increase the flexibility of the model so as to exert the changes in problem parameters, a dynamic model is proposed. With a more exquisite consideration, facilities in each hub are divided to movable and static and the saving that comes from the allocation of the released movable facilities to the needful established hubs in each period, is computed. Considering the computational complexity of the problem, in which the optimal solution of medium and large size problems was not attainable in a reasonable time, afterwards an effective GA is proposed to solve the problem. The proposed GA introduces the concept of dynamic migration to the population which alongside with the dynamic mutation operator improved the quality of the attained solutions. According to the computational results, the performance of proposed GA is salient and in all of the designed experiments in which the optimal solution was attainable in a reasonable time, GA's solution equates to the optimal solution.

The dynamic model proposed in this paper, considers the saving that results from the movable facilities to be the same for all of the hubs in a period. More interesting problem arises when the debated savings are variable for different hubs due to their size and other features. Flexible covering radius proposed in this paper like other existing models in the literature consider the covering radius to be rigid although in some real world cases like wireless communication networks as the consumer recedes from the transmitter, signal strength weakens slowly and at last the connection will be lost. The authors are working on this case in order to approximate the problem to real world situation.

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