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A Multi-Level Capacity Approach to the Hub and Spoke Network

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ABSTRACT

The existing works considering the flow-based discount factor in the hub and spoke problems, assume that increasing the amount of flow passing through each edge of network continuously decreases the unit flow transportation cost. Although a higher volume of flow allows for using wider links and consequently cheaper transportation, but the unit of flow enjoys more discounts, quite like replacing the current link by a cheaper link type (i.e., increasing the volume of flow without changing the link type would not affects the unit flow transportation cost). Here, we take a new approach, introducing multi-level capacities to design hub and spoke networks, while alternative links with known capacities, installation costs and discount factors are available to be installed on each network edge. The flow transportation cost and link installation cost are calculated according to the type of links installed on the network edges; thus, not only the correct optimum hub location and spoke allocation is determined, but also the appropriate link type to be installed on the network edges are specified. The capacitated multiple allocation p-hub median problem (CMApHMP) using the multi-level capacity approach is then formulated as a mixed-integer linear program (MILP). We also present a new MILP for the hub location problem using a similar approach in order to restrict the amount of flow transmitting through the hubs. Defining diseconomies of scale for each hub type, the model is to present congestion at the hubs and balance the transmitting flow between the hubs. Two new formulations are presented for both the p-hub median and the hub location problems which requiring a flow between two non-hub nodes to be transferred directly, when a direct link between the nodes is available. These models are useful for the general cost structure where the costs are not required to satisfy the triangular inequality. Direct links between non-hub nodes are allowed in all the proposed formulations.

1. Introduction

Due to the growth in the volume of interaction between origin and destination nodes and increase in the number of interacting origin-destination pairs, design of efficient communication networks is increasingly important. Denote a network by $N=(V,E)$, where $|V|=n$ and E are respectively the sets of nodes and arcs providing the required links in the network. For each pair of nodes $i, j \in V$, denote the amount of the flow (e.g., passenger or commodity) from node i to node j by w_{ij}. The goal is to design an efficient structure to transfer the flow among the interacting nodes. The simplest way is to connect all interacting

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nodes i and j ($w_{ij}\neq 0$) by a direct link. Such a network may be costly or even impossible.

In the specific structure, namely hub-and-spoke network, instead of making a fully connected network, some nodes are selected as hubs to collect, transmit and distribute the flow. So, the flow from an origin is transferred from a path that passes through one or two hubs and then is delivered to the destination node. The main advantage of hub and spoke structure is to remove the redundant links and concentrate the flow in the remaining links. Thus, the unit of flow is enjoying the economies of scale stem from agglomeration of flow.

Indirect transmission of the unit of flow from origin i to destination j via hubs k and m in the hub and spoke structure includes three types of costs, namely collection cost (from node i to hub k), transmission cost (from hub k to hub m) and distribution cost (from hub m to j).

Denoting the unit flow transportation cost from node i to node j by C_{ij}, the cost of sending a unit flow indirectly from node i to node j via hubs k and m is $C_{ikmj}=\alpha_1 C_{ik}+\alpha C_{km}+\alpha_2 C_{mj}$, where α_1, α and α_2 are fixed discount factors for collection, transmission and distribution of flow, respectively. Usually, the relations $0 \leq \alpha \leq \alpha_1, \alpha_2 \leq 1$ are assumed for the discount factors.

Since the economy of scale depends on the volume of flow, discounting the transportation cost on the links with different volumes of flow by a fixed rate might miscalculate the network transportation costs and subsequently adversely affects the correct optimum hub location and spoke allocation. Here, we present a approach, namely multi-level capacity to correctly accounts the network costs. According to multi-level capacity approach, alternative link types with known capacities, installation costs and discount factors are assumed to be available; wider links impose larger fixed installation costs, but provide smaller transportation costs. We let the network edges to select the appropriate link type. Thus, the transportation costs and link installation costs are determined according to the type of links installed on the network edges; thereupon, not only the correct optimum hub location and spoke allocation is determined, but also the appropriate link types to be installed on the network edges are specified. Similar approach is also used for classifying the hubs and avoiding congestion at the hub facilities.

The remaining of our work is organized as follows. A literature review and motivation of multi-level capacity approach are presented in Section 2. The MILP formulations for multi-level capacity multiple allocation p-hub median problem (MCMApHMP) and multi-level capacity multiple allocation hub location problem (MCMAHLP) are presented in Section 3 and 4, respectively. Section 5 provides the computational results. Section 6 points out strengths and weaknesses of the proposed models as well as some directions for further research.

2. Literature Review

We are concerned with the p-hub median problem (pHMP) and the hub location problem (HLP). In pHMP, p nodes are selected as hubs and the remaining non-hub (spoke) nodes are allocated to them so that the total flow transportation cost via the resulting network is minimized. HLP is similar to pHMP, but p is not exogenously determined and here the objective is to minimize the sum of the total transportation cost and the hub installation fixed cost. If spoke nodes are required to connect to one or more hubs, the allocation pattern is said to be single allocation (SA) or multiple allocations (MA), respectively. Furthermore, if any restriction applies to the least and/or most volumes of flow that can pass through links or hubs, the problem is said to be capacitated (C); otherwise, it is said to be uncapacitated (U).

The uncapacitated single allocation p-hub median problem (USApHMP) was first formulated by O'kelly [1] in 1987 as a quadratic mixed-integer program. O'kelly [2] also presented a quadratic model for the hub location problem. Nonetheless, Campbell [3] proposed the first mixed-integer linear programming (MILP) model for the uncapacitated multiple allocation p-hub median problem (UMApHMP). Using a similar approach, Campbell [4] formulated USApHMP as a MILP. Campbell [4] also formulated single and multiple allocation versions of the uncapacitated hub location problems. Skorin-Kapov et al. [5] applied linear relaxations to the single and multiple versions of the problem proposed by Campbell [3,4] and obtained a better bound. Using arc-based variables, Ernst and Krishnamoorthy [6] proposed a new MILP for the USApHMP requiring fewer variables and constraints. A similar approach was also used by Ernst and Krishnamoorthy [7] for formulating the multiple allocation version of the median problem.

Some studies were restricted the amount of flow passing through links/hubs. Campbell [4] incorporated capacities in the HLP with single and multiple allocations. Ernst and Krishnamoorthy [8] proposed two MILPs for CSAHLP, where the second model was required fewer variables and constrains. Ebery et al. [9] restricted the amount of the incoming flow to the hubs in order to avoid overloading the hubs. Marín [10] proposed a tight MILP for CMAHLP restricting the amount of incoming flow from the spoke nodes to the hub nodes. In [11], an MILP for CMAHLP was proposed, using the multi-commodity flow idea. Capacity constraints were applied to the flow passing through hubs and arcs. They relaxed the fully connected inter-hub network assumption. Contreras et al. [12] considered a CSAHLP, where capacity was applied to the amount of the incoming flow to the hubs. They applied a Lagrangian relaxation to their proposed model. Correia et al. [13] restricted the volume of transitional flow through the hubs. Multiple capacity alternatives with known capacities and installation cost were considered. The objective was to minimizing the sum of the total transportation cost and the hubs establishing cost; furthermore, the appropriate size of the hubs was determined. For the latest survey, see [14].

Considering identical discount factors for the links with different volumes of flow is not realistic, because as the amount of flow passes through a link increases, we are able to install a wider link and enjoy a cheaper transportation. O'kelly and Bryan [15] were first to address the flow-based discount factor. According to [15], flow-independent discount not only miscalculates total network cost, but may also erroneously select optimal hub locations and allocations. They presented a flow-based MILP for UMA*p*HMP, namely FLOWLOC, where the flow-based discount factor on the inter-hub link is formulated as a non-linear cost function that increased with decreasing rate. Piece-wise linear approximation was then exploited to encounter non-linearity. Klincewicz [16] showed that for a fixed set of hubs, FLOWLOC [15] could be solved by using the classical uncapacitated facility location model. In [17] and [18], the unit cost flow was discounted as the volume of flow passing through a link exceeded a specific threshold. Racunica and Wynter [19] used a modified version of the concave cost function used in [15] to formulate the flow-based discount, where the discount on inter-hub links depended on the total flow passing through the link. The cost functions on different links have different structures and each function calibrates independently of the other functions.

According to FLOWLOC [15], increasing the volume of flow passing through any edge of the network would continually decrease the flow unit transportation cost due to the flow agglomeration. The Piece-wise linear approximation of the continuous relation between the volume of flow passing through an edge and flow transportation cost is depicted in Fig. 1.

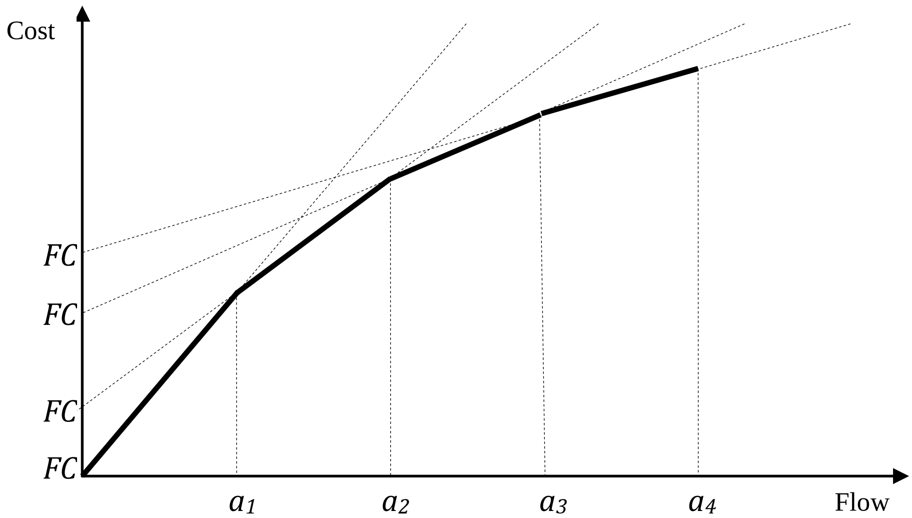


Fig. 1. Total transportation cost on a link (FLOWLOC)

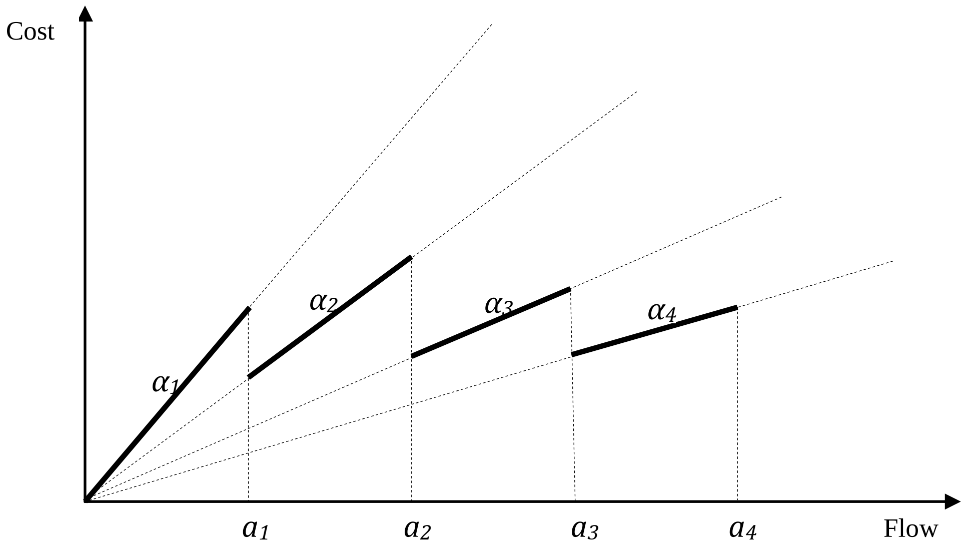


Fig. 2. Total transportation cost on a link (multi-level capacity approach)

Although higher volume of flow allows for using the cheaper wider link, but transportation enjoys much more discount per unit of flow because more volumes of flow allow for the use of wider link types with cheaper transportation costs.

For instance, common urban roads could be replaced by speed ways; by using a cheaper road type, each unit of flow enjoys the same unit flow transportation cost and this rate remains fixed for all volumes of flow, unless the road type is changed. Thus, unit flow transportation cost depends on the volume of passing flow, but this dependence is due to improving the quality of link; so, any increase in the volume of flow without changing the link type does not result in a cheaper transportation.

Therefore, due to the limiting link types, increasing the volume of flow passing through a link leads to an increase in the total transportation cost according to a step-wise function instead of a continuous function (Compare Figs. 1 & 2). The discreteness occurs at the link changing point.

Since the different discount factor is due to the different link type, the fixed link installation costs are not the same and should not be ignored. Wider links are preferred, only if the earned economies of scales stems from using cheaper wider links are larger than the additional installation costs imposed by installing wider links. Therefore, the trade-off between fixed link installation cost and flow transportation cost should be considered.

Here, we introduce a new approach, namely the multi-level capacity approach. According to our approach, it is assumed that alternative link types with known capacities, installation costs and discount factors are available, and wider links impose larger fixed installation costs, but provide larger discount factors (cheaper unit flow transportation costs). When any flow passes through an arc, an appropriate type of link should be installed on the arc. The type of link is chosen, so that the sum of total transportation cost and total fixed link installation cost is minimized; furthermore the volume of passing flow should not exceeds the capacity of link.

By installing specific link on an edge, each unit of flow passing this edge enjoys the corresponding discount factor which is fixed and depends on the type of the link. Increasing the amount of flow may lead to a change in the link type or make the extra flow transfer via other routes.

Carello et al. [20] studied CSAHLP, where the capacity restriction was applied to the incoming/outgoing flows to/from the hubs. In this problem, links had modular capacities and the required capacity provided by adding similar links.

The objective was to minimize the sum of the hubs establishing cost and the cost for connecting the spoke nodes to the hub nodes. Yaman [21] studies an uncapacitated version of the study considered by Carello et al. [20]. He also studied the polyhedral characteristics of USAHLP with the modular arc capacity. Yaman and Carello [22] formulated a hub location problem with the single allocation and modular arc capacity as a quadratic mixed-integer program. They also applied restrictions to the amounts of flow passing through the hubs. Yaman [23] proposed two MIPs for the *p*-hub location problem with the single allocation and a star shaped network having modular arc capacities.

In the modular arc problem it is assumed that similar links with the same link installation cost, capacity and economy of scale are available and the required capacity is provided by increasing the number of links installed on any specific edge.

Therefore, if the volume of transmitting flow through a specific edge increases, the passing flow is not enjoying more economies of scale, due to the similarity of links. In the multi-level capacity approach, which has applications in public transportation, the required capacity is provided by installing appropriate link types instead of changing the number of links, so that flow agglomeration allows for the use of cheaper wider links.

In some situations, the amount of flow between two non-hub nodes i and j is large enough to give a justification for the transfer of flow, instead of using the hub and spoke network; specially, when the nodes i and j have a high level of interaction with each other, and their levels of interactivity with other nodes are very low.

Therefore, direct links may be allowed because the additional link installation cost might be smaller than the extra transportation cost. Some studies [17-18] allow direct links between spoke nodes, where there are no discounts applied to the directly transmitted flow.

Allowing direct transmission may lead to unsuitable network structures. Because disregarding the link installation cost and applying no discount to the flow transferred directly, the network cost may not calculated correctly.

In [4], [17] and [18] a flow threshold was considered; where, flow is allowed to passes any link, if the volumes of transitional flow exceed the given threshold. Sohn and Park [24] considered the link installation cost to avoid unnecessary links in USA*p*HMP.

To the best of our knowledge, there is no study to consider multi level capacities and flow-based discounts together. Using the multi level capacity, the flow-based discount is applied to "*all arcs*" of the network regardless of their positions in the network, but according to the types of the arcs; therefore, not only the correct optimum hub location and spoke allocation is determined, but also appropriate link types to be installed on network edges are determined. Table 1 gives a summary of some studies considering flow-based discount, link installation cost and capacity restriction.

Tab. 1. Literature review

Papers	Links							Hubs		
	Direct link	Flow-based discount	Flow threshold	Capacity	Multi-level capacity	Modular arcs	Installation cost	Capacity	Multi-level capacity	Avoiding congestion
[24]							x			
[15][16][19]		x								
[17][18]	x		x							
[4][8][9][10][12]								x		
[11]				X				x		
[13]									x	
[25][26][27][28][29]										x
[20]						x	x	x		
[21][22][23]						x	x			
Our approach	x	x			x		x		x	x

3. Multi-Capacity Multiple Assignment *p*-Hub Median Problem (MCMA*p*HMP)

This section provides the mathematical formulation for MCMA*p*HMP. The objective is to select p hubs and connect spoke nodes to them by installing appropriate link types on network edges, and minimize the sum of total flow transportation cost and links installation cost.

3.1. Parameters

d_{ij} Distance between nodes i and j
c_{ij} Cost of transferring a unit of flow from node i to node j per unit distance
p Number of hubs to be established
q Link type index
Q Number of available link types
a_q Capacity of link type q
b_q Link installation cost per unit distance
α_q Discount factor corresponding to link type q

3.2. Variables

x_{ikmj} Fraction of flow from origin i to destination j via hubs k and m
x_{ij} Binary variable with value 1 if nodes i and j are directly connected, and 0, otherwise
y_k Binary variable with value 1 if a hub is established at node k, and 0, otherwise
t_{ijq} Binary variable with value 1 if arc (i, j) uses link type q, and 0, otherwise
L_{ijq} Total flow passing through arc (i, j) with link type q

3.3. Mathematical Model

Model I: MCMA*p*HMP_1

$$\min\ W = \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{q=1}^{Q}(b_q\, t_{ijq} + \alpha_q c_{ij} L_{ijq}) d_{ij} \qquad (1)$$

s.t.

$$\sum_{k=1}^{n} y_k = p$$

$$(x_{ij} + \sum_{k=1}^{n}\sum_{m=1}^{n}) w_{ij} = w_{ij}, \qquad \forall\, i, j \neq i \in V \qquad (2)$$

$$\sum_{m=1}^{n} x_{ikmj} \le y_k\,, \qquad \forall\, i, j \neq i, k \in V \qquad (3)$$

$$\sum_{k=1}^{n} x_{ikmj} \le y_m\,, \qquad \forall\, i, j \neq i, m \in V \qquad (4)$$

$$\sum_{q=1}^{Q} L_{ijq} = \sum_{r=1}^{n}\sum_{l=1}^{n}\left(x_{ijrl} w_{il} + x_{rijl} w_{rl} + x_{rlij} w_{rj}\right) + x_{ij} w_{ij}\,, \qquad \forall\, i, j \neq i \in V \qquad (5)$$

$$L_{ijq} \le t_{ijq}\, a_q\,, \qquad \forall\, i, j \neq i \in V, \forall q \qquad (6)$$

$$\sum_{q=1}^{Q} t_{ijq} \le 1\,, \qquad \forall\, i, j \neq i \in V \qquad (7)$$

$$x_{ikmj} \in \{0,1\}\,, \qquad \forall\, i, j, k, m \in V \qquad (8)$$

$$x_{ij} \in \{0,1\}\,, \qquad \forall\, i, j \neq i \in V \qquad (9)$$

$$y_k \in \{0,1\}\,, \qquad \forall\, k \in V \qquad (10)$$

$$t_{ijq} \in \{0,1\}, \qquad \forall\, i, j \neq i \in V, \forall q \tag{11}$$

$$L_{ijq} \geq 0\,; \;\; \forall\, i, j \neq i \in V, \forall q \tag{12}$$

The objective function is to minimize the sum of the link installation costs and the total flow transportation costs. Since b_q and c_{ij} are the link installation cost and unit cost flow "*per unit distance*", they should be multiplied by the link length to compute the fixed link installation and the cost of transferring unit of flow from node i to node j, respectively.

Constraint (1) ensures that exactly p hubs are established. According to Constraint (2), sum of total flow transmitted directly from node i to node j (first term of left hand side) and total flow transmitted indirectly via hubs (second term of left hand side) should be equal to the flow demand from node i to node j.

Note that the cost of self services is set to be zero. According to constraints (3) and (4), the flow from node i to node j is permitted to transfer indirectly via nodes k and m, if the hub facilities are established at nodes k and m.

The left hand side of constraint (5) calculates the total flow transmitted via edge (i,j) that is equal to the sum of flow transferred through all types of links installed on the edge (i,j). Since indirect transmission of flow consists of three part, namely collecting, transferring and distributing, so edge (i,j) might be used as the first, second or third part of some 2-stop paths. First, second and third terms of the right hand side of constraint (5) calculate the total flow transmitted via edge (i,j), when it is the head, inter-hub or tail of some paths, respectively.

Furthermore, the final term in the right hand side of constraint (5) calculates the flow passing the edge (i,j) when flow from node i to node j is transferred directly by using the edge (i,j). So, constraint (5) requires the sum of total flow which is transferred by using the edge (i,j) to be equal to the sum of flow passing through all types of links installed on the edge (i,j).

The constraints (6) restrict the volume of flow passing through edge (i,j) to the capacity of link type q, if link type q is installed on edge (i,j). Therefore, if no link is installed on edge (i,j), then the flow is not permitted to transfer through the edge.

According to constraints (7) at most one type of link can be installed on each edge. The constraints (8)-(12) define the type of variables used in model I. Even if a direct link is available between nodes i and j, the flow from node i to node j may transfer indirectly, due to the capacity restriction.

Replacing x_{ij} by $\sum_{q=1}^{Q} t_{ijq}$ in constraint (2) and (5), the w_{ij} is required to be transferred directly, if such a link is available. Using this approach, we obtain the following model, namely MCMA*p*HMP_2.

Model II: MCMA*p*HMP_2

$$\min\; W = \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{q=1}^{Q}\left(b_q t_{ijq} + \alpha_q c_{ij} L_{ijq}\right) d_{ij}$$

s.t.

Constraints (1), (3), (4), (6), (7), (8), (10), (11), (12) and:

$$\Big(\sum_{q=1}^{Q} t_{ijq} + \sum_{k=1}^{n}\sum_{m=1}^{n} x_{ikmj}\Big) w_{ij} = w_{ij}\,, \qquad \forall\, i, j \neq i \in V \tag{13}$$

$$\sum_{q=1}^{Q} L_{ijq} = \sum_{r=1}^{n}\sum_{l=1}^{n}\left(x_{ijrl} w_{il} + x_{rijl} w_{rl} + x_{rlij} w_{rj}\right) + \sum_{q=1}^{Q} t_{ijq}\, w_{ij}\,, \qquad \forall\, i, j \neq i \in V. \tag{14}$$

The MCMA*p*HMP_2 is not only requires fewer variables, but also provides a better LP relaxation (see Section 5).

4. Multi-Capacity Multiple Assignment Hub Location Problem

Hubs have three main tasks, namely collecting, transferring and distributing the flows from origins to destinations. When the amount of flow passing through a hub increases, some amounts of flow would require more time at the hub nodes to be transferred. In these cases, hubs are said to be congested. Some studies have addressed congestion at hub nodes. Marianov and Serra [25] formulated a hub network as an M/D/C queue. Rodríguez et al. [26] used a similar approach to prevent congestion at the hubs, modeling the hubs as an M/M/1 queue. They considered minimizing the delivery time being the sum of the transportation time and time spent at the hubs. Costa et al. [27] considered minimizing the amount of processing flow at the hubs as a second objective besides the traditional objective in USAHLP. They proposed two bi-objectives MILP for this problem. Indeed, they wanted to find the appropriate trade-offs between the hub size and the transportation cost.

Elhedhli and Hu [28] incorporated the congestion effect as a non-linear convex function in the objective function approximating the maximum of a set of piece-wise linear and tangent hyper planes to avoid congestion at the hubs. Camargo et al. [29] incorporated a congestion function similar to the one proposed in [28] in the objective function. They decomposed the problem into two sub-problems using general Bender's decomposition. A master problem was to determine the hub locations and a sub-problem was considered for the flow balance and congestion at the hubs. Correia et al. [13] considered the volume of the flow passing through the hubs to have multiple level capacities.

Similar to the multiple capacities approach defined for links, one can classify the hub facilities; therefore, two separate costs regarding hub facilities would be considered, namely establishing cost and operational cost. Hub establishing cost depends on the hub type and is not directly affected by the volume of flow; thus, depending on the hub type, a specific fixed cost is considered for establishing hub facilities. Hub operational cost is directly dependent on the volume of flow; to avoid congestion, the hub operational cost can be set to be larger for the larger hubs (See Fig. 3).

The hub establishing cost is intended to keep the number of hubs as low as possible. But, decreasing the number of hubs might lead to hub congestion; considering both costs together takes the trade-off between the number of the hubs and the congestion effect into account.

When the flow from node i to node j (w_{ij}) is transferred indirectly, w_{ij} visits at most two hubs; so, each hub might be used as the first and/or the second hub in some 2-stop paths, and thus the total volume of flow transmitted via each hub (say k) is equal to the sum of total flow transmitted via this hub, when it is the first or the second hub in all the 2-stop paths. Therefore, the total volume of flow transmitted via hub k, when it is the first or the second hub of some paths, is calculated by:

$$F_k = \sum_{i}^{n} \sum_{\substack{j \neq i \\ j \neq k}}^{n} \sum_{m \neq i}^{n} x_{ikmj} w_{ij} + \sum_{i \neq k}^{n} \sum_{\substack{j \neq i \\ j \neq m}}^{n} \sum_{m \neq k}^{n} x_{imkj} w_{ij}, \qquad \forall\, k \in V. \tag{15}$$

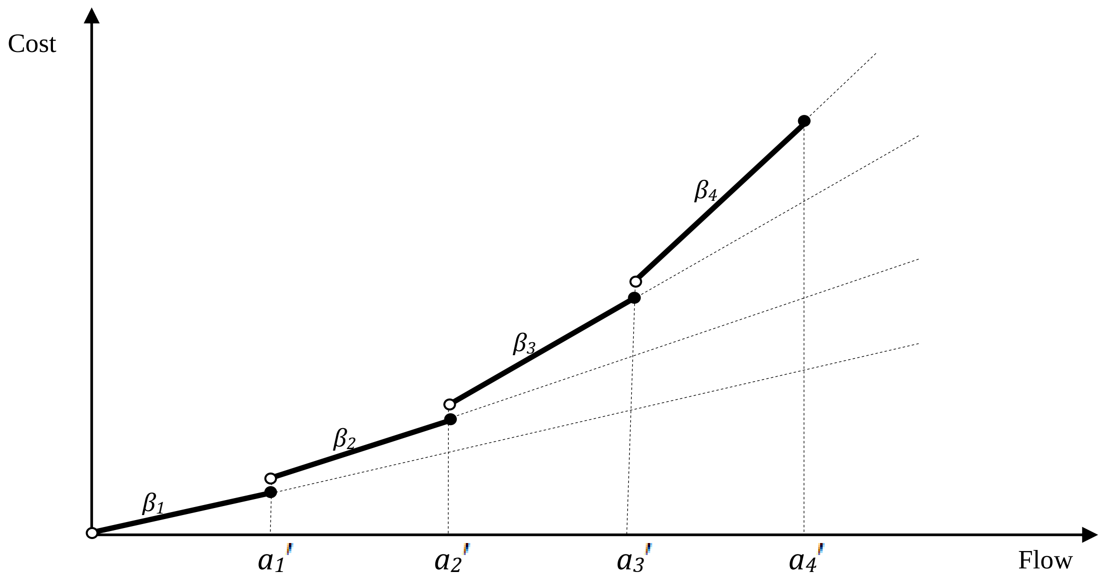


Fig. 3. Total transferring cost on each hub type

Since multiple alternatives are available for establishing the hubs, the F_k could be replaced by $\sum_{q'}^{Q'} F_{kq'}$, where $F_{kq'}$ denotes the amount of flow processed by hub k of type q'.

4.1. Parameters

- q' Hub class index
- Q' Number of available hub types
- $a_{q'}$ Capacity of hub class q'
- $b_{q'}$ Hub class q' establishing cost
- β_q Diseconomy of scale factor corresponding to hub type q'

4.2. Variables

- $t'_{kq'}$ Binary variable with value 1 if class q' is established at node k, and 0, otherwise
- $F_{kq'}$ Total flow processed by class q' hub at node k

4.3. Mathematical Model

Model III: MCMAHLP_1

$$\min W = \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{q=1}^{Q}(b_q t_{ijq} + \alpha_q c_{ij} L_{ijq}) d_{ij} + \sum_{k}^{n}\sum_{q'}^{Q'}(b'_{q'} t'_{kq'} + \beta_{q'} F_{kq'})$$

s.t.

Constraints (2), (5), (7), (8), (9), (11), (12), and

$$\sum_{q'}^{Q'} F_{kq'} = \sum_{i}^{n}\sum_{\substack{j\neq i\\ j\neq k}}^{n}\sum_{m\neq i}^{n} x_{ikmj} w_{ij} + \sum_{i\neq k}^{n}\sum_{\substack{j\neq i\\ j\neq m}}^{n}\sum_{m\neq k}^{n} x_{imkj} w_{ij} \tag{16}$$

$$F_{kq'} \leq t'_{kq'} a'_{q'}, \qquad \forall\, k \in V, \forall q' \tag{17}$$

$$\sum_{q'=1}^{Q'} t'_{kq'} \leq 1, \qquad \forall\, k \in V \tag{18}$$

$$t'_{kq'} \in \{0,1\}, \qquad \forall\, k \in V, \forall q' \tag{19}$$

$$F_{kq'} \geq 0, \qquad \forall\, k \in V, \forall q'. \tag{20}$$

According to constraint (16), sum of total flow passing through all types of hubs installed on node *k* ($\sum_{q'}^{Q'} F_{kq'}$) is required to be equal to the volume of flow transmitted indirectly via hub *k*, when node *k* is the first or the second hub in some 2-stop paths. constraint (17) ensures that total flow passes through hub *k* should not exceed the capacity of hub type *q'*, if type *q'* of hubs is established on node *k*; therefore, if no hub is installed on node *k* ($\sum_{q'=1}^{Q'} t'_{kq'} = 0$), then the flow is not permitted to be transferred indirectly via hub *k*. According to constraint (18), at most one type of hubs should be established on each network node.

Replacing constraints (2) and (5) by (13) and (14), respectively, we can require a flow between nodes *i* and *j* to be transfered directly if such a link is available.

Model IV: MCMAHLP_2

$$\min W = \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{q=1}^{Q}(b_q t_{ijq} + \alpha_q c_{ij} L_{ijq}) d_{ij} + \sum_{k}^{n}\sum_{q'}^{Q'}(b'_{q'} t'_{kq'} + \beta_{q'} F_{kq'})$$

s.t.

Constraints (6) - (8), (11) - (14) and (16) - (20).

5. Computational Results

To test the effectiveness of the proposed models, GAMS 22.9 with the CPLEX 11.0 solver was used on a Laptop (2.80 GHz CPU, 4GB RAM). Four link types and also four hub types were defined. Tables 2 and 3 summarize the specification of links and hubs, respectively. The CAB data set [1] customization is given in Appendix I. The unit flow transportation cost per unit distance is set to be 1.

Tab. 2. Link type specification

Link type	Capacity	Installation cost (per mile)	Discount factor
Type I	20000	24000	1
Type II	50000	55000	0.9
Type III	150000	140000	0.75
Type IV	400000	400000	0.55

Tab. 3. Hub type specification

Hub type	Capacity	Establishing cost	Variable cost
Type I	40000	75000000	300
Type II	100000	80000000	450
Type III	300000	88000000	700
Type IV	800000	100000000	1000

Tab. 4. A structural comparison of MCMA*p*HMP_1 and MCMA*p*HMP_2

Model	No. of nodes	No. of hubs	CPU time	No. of iterations	Variable	Binary variable	Constraint	Objective function	LP relaxation	Relative gap (%)
MCMA*p*HMP_1	5	1	0.319	879	510	425	306	4.1318E+08	3.1794E+08	23.05
MCMA*p*HMP_2	5	1	0.263	576	490	405	306	4.1318E+08	3.6881E+08	10.74
MCMA*p*HMP_1	5	2	0.852	3351	510	425	306	3.9259E+08	2.2850E+08	41.80
MCMA*p*HMP_2	5	2	1.024	1614	490	405	306	3.9259E+08	2.8143E+08	28.31
MCMA*p*HMP_1	5	3	0.429	1176	510	425	306	3.8932E+08	2.2850E+08	41.31
MCMA*p*HMP_2	5	3	0.403	1141	490	405	306	3.8932E+08	2.7746E+08	28.73
MCMA*p*HMP_1	5	4	0.659	1255	510	425	306	3.8932E+08	2.2850E+08	41.31
MCMA*p*HMP_2	5	4	0.167	793	490	405	306	3.8932E+08	2.7746E+08	28.73
MCMA*p*HMP_1	5	5	0.444	792	510	425	306	3.8932E+08	2.6527E+08	31.86
MCMA*p*HMP_2	5	5	0.316	614	490	405	306	3.8932E+08	3.0609E+08	21.38
MCMA*p*HMP_1	10	1	115.8	102571	8120	7750	2261	1.6339E+09	8.9678E+08	45.11
MCMA*p*HMP_2	10	1	92.34	114514	8030	7660	2261	1.6339E+09	1.2813E+09	21.58
MCMA*p*HMP_1	10	2	30300	19524715	8120	7750	2261	1.4963E+09	8.9678E+08	40.07
MCMA*p*HMP_2	10	2	29032	32846723	8030	7660	2261	1.5026E+09	1.1072E+09	26.32
MCMA*p*HMP_1	10	3	37590	19063655	8120	7750	2261	1.4351E+09	8.9678E+08	37.51
MCMA*p*HMP_2	10	3	14435	11941858	8030	7660	2261	1.4351E+09	1.0384E+09	27.64
MCMA*p*HMP_1	10	4	203762	138034467	8030	7660	2261	1.4042E+09	8.9678E+08	36.14
MCMA*p*HMP_2	10	4	56192	39130005	8030	7660	2261	1.4042E+09	9.9042E+08	29.47
MCMA*p*HMP_1	10	5	30722	22675500	8120	7750	2261	1.3782E+09	8.9678E+08	34.93
MCMA*p*HMP_2	10	5	41668	27702437	8030	7660	2261	1.3891E+09	9.5923E+08	30.95
MCMA*p*HMP_1	10	10	7275.0	7321464	8120	7750	2261	1.3531E+09	8.9678E+08	33.72
MCMA*p*HMP_2	10	10	5516.3	10145920	8030	7660	2261	1.3607E+09	8.9678E+08	34.09

According to Table 4, for specific number of nodes and hubs, both MCMA*p*HMP_1 and MCMA*p*HMP_2 require equal constraints, but the latter requires fewer binary variables. Since MCMA*p*HMP_1 has higher degrees of freedom, corresponding optimal solution value is smaller than or equal to one for MCMA*p*HMP_2; nevertheless, MCMA*p*HMP_2 provides a better LP relaxation, and the relative gap between the optimal solution and the LP relaxation for MCMA*p*HMP_2 is almost smaller than as compared to MCMA*p*HMP_1. Fewer variables and tighter LP bounds turn MCMA*p*HMP_2 to be more efficient than MCMA*p*HMP_1 (see Table 4, column 4). Even if hub facilities are established on all nodes, a fully connected network was not formed and some pairs use indirect transmission. Similarly, MCMAHLP_2 requires less iteration and CPU time than MCMAHLP_1, as seen in Table 5.

Tab. 5. A structural comparison of MCMAHLP_1 and MCMAHLP_2

Model	No. of nodes	CPU time	No. of iteration	Variable	Binary variable	Constraint	Objective function	LP relaxation	Relative gap (%)
MCMAHLP_1	5	0.631	510	540	440	170	5.1637E+08	2.2850E+08	55.75
MCMAHLP_2	5	0.428	796	520	420	170	5.1637E+08	3.8516E+08	25.41
MCMAHLP_1	10	161.71	127568	8180	7880	690	1.8178E+09	8.9678E+08	50.67
MCMAHLP_2	10	82.233	75998	8090	7790	690	1.8178E+09	1.4870E+09	18.19
MCMAHLP_1	15	20769	7478491	43170	42270	1560	5.8008E+09	3.1700E+09	45.35
MCMAHLP-2	15	3330	908034	42960	42060	1560	5.8073E+09	4.7759E+09	17.76

Tab. 6. Comparison results for MCMA*p*HMP_1 and MCMA*p*HMP_2

Model	No. of nodes	Hubs	Objective function	Transferring Percentage		Achieved discount factor	Link utilization (%)	Average congestion	Max. congestion	C.V. congestion
				Direct	Indirect					
MCMA*p*HMP_1	5	5	4.1318E+08	35.7	64.3	0.8439	69.1	165922	165922	-
MCMA*p*HMP_2	5	5	4.1318E+08	35.7	64.3	0.8439	69.1	165922	165922	-
MCMA*p*HMP_1	5	2,5	3.9259E+08	46.3	53.7	0.9118	77.8	69269	70270	0.02
MCMA*p*HMP_2	5	2,5	3.9259E+08	46.3	53.7	0.9118	77.8	69269	70270	0.02
MCMA*p*HMP_1	5	2,4,5	3.8932E+08	68.9	31.1	0.9274	83.1	31813	68268	0.99
MCMA*p*HMP_2	5	2,4,5	3.8932E+08	68.9	31.1	0.9274	83.1	31813	68268	0.99
MCMA*p*HMP_1	5	2,3,4,5	3.8932E+08	68.9	31.1	0.9274	83.1	23859	68268	1.27
MCMA*p*HMP_2	5	2,3,4,5	3.8932E+08	68.9	31.1	0.9274	83.1	23860	68268	1.27
MCMA*p*HMP_1	5	All	3.8932E+08	19.9	80.1	0.9274	83.1	74952	119158	0.53
MCMA*p*HMP_2	5	All	3.8932E+08	68.9	31.1	0.9274	83.1	25580	74224	1.18
MCMA*p*HMP_1	10	9	1.6339E+09	65.7	34.3	0.8886	73.2	342570	342570	-
MCMA*p*HMP_2	10	9	1.6339E+09	65.7	34.3	0.8886	73.2	342570	342570	-
MCMA*p*HMP_1	10	4,9	1.4963E+09	65.1	34.9	0.8706	80.2	211830	255053	0.29
MCMA*p*HMP_2	10	4,6	1.5026E+09	62.3	37.7	0.8476	75.7	228427	282340	0.33
MCMA*p*HMP_1	10	4,7,9	1.4351E+09	63.0	37.0	0.8676	87.7	146005	288462	0.90
MCMA*p*HMP_2	10	4,7,9	1.4351E+09	63.0	37.0	0.8676	87.7	146005	288462	0.90
MCMA*p*HMP_1	10	4,5,7,9	1.4042E+09	59.5	40.5	0.8688	88.0	118635	265920	0.66
MCMA*p*HMP_2	10	4,5,7,9	1.4042E+09	59.5	40.5	0.8688	88.0	122506	265920	0.82
MCMA*p*HMP_1	10	4,5,6,6,9	1.3782E+09	50.0	50.0	0.8670	87.1	117567	192700	0.56
MCMA*p*HMP_2	10	4,5,6,7,9	1.3891E+09	63.2	36.8	0.8798	86.0	108912	195163	0.55
MCMA*p*HMP_1	10	All	1.3531E+09	59.3	40.7	0.8918	91.0	104764	180466	0.46
MCMA*p*HMP_2	10	All	1.3607E+09	60.7	39.3	0.8708	89.4	72534	202860	0.94

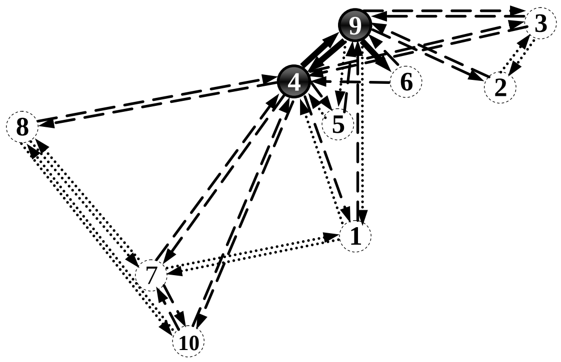
According to tables 6 and 7, total percentage of directly transferred flows is larger for MCMA*p*HMP_2 and MCMAHLP_2 is compared to MCMA*p*HMP_1 and MCMAHLP_1, respectively; Indeed, the formers require connecting nodes to transfer corresponding flows directly.

Since the link installation cost is being taken into account, all the presented models were intended to transfer further flows via installed links to avoid extra installation costs. According to tables 6 and 7, for an instance with 5 nodes, the MCMAHLP models select no hub and all the flows are transferred directly; therefore, links are not utilized properly. On the other hand, since MCMA*p*HMP model establishes *p* hub, flow agglomeration occurs and the average link utilization is always bigger than 68% of the capacity. The last three columns in tables 6 and 7 give some information about congestion at the hubs. As expected, the MCMAHLP models offer less congested hubs as compared to MCMA*p*HMP models.

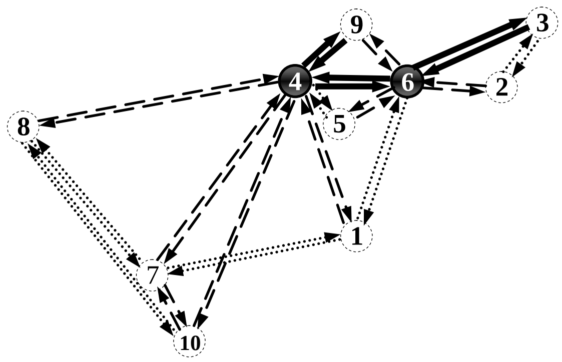
Fig. 4 shows the resulting network using MCMA*p*HMP_1 and MCMA*p*HMP_2 (10 nodes; 2 hubs) as well as MCMAHLP-1 and MCMAHLP_2 (10 nodes).

Tab. 7. Comparison results for MCMAHLP_1 and MCMAHLP_2

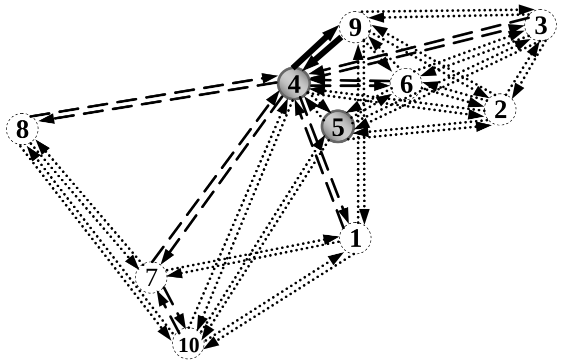
Model	No. of nodes	Hubs	Objective function	Transferring Percentage		Achieved discount factor	Link utilization (%)	Average congestion	Max. congestion	C.V. congestion
				Direct	Indirect					
MCMAHLP_1	5	-	5.1637E+08	100.0	0.0	0.9572	48.0	-	-	-
MCMAHLP_2	5	-	5.1637E+08	100.0	0.0	0.9572	48.0	-	-	-
MCMAHLP_1	10	4,5	1.8178E+09	81.4	18.6	0.9234	68.4	97400	98864	0.02
MCMAHLP_2	10	4,5	1.8178E+09	81.4	18.6	0.9234	68.4	97400	98864	0.02
MCMAHLP_1	15	1,4,7	5.8008E+09	69.3	30.7	0.8583	74.2	246149	551772	1.08
MCMAHLP_2	15	1,4,7	5.8073E+09	69.9	30.1	0.8626	74.7	241330	532423	1.04



A - MCMA*p*HMP-1 (10 nodes; 2 hubs)



B - MCMA*p*HMP-2 (10 nodes; 2 hubs)



C - MCMAHLP-1 & MCMAHLP-2 (10 nodes)

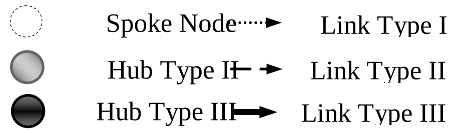


Fig. 4. Hub and spoke configuration

The MCMA*p*HMP models appear to use fewer and wider links as compared to the MCMAHLP models. Using wide links leads to congestion at the hub nodes, due to transmission the high amount of flow to the hubs. The MCMA*p*HMP models use type III hubs, but the hub location models use type II hubs. Besides avoiding congestion, the MCMAHLP models appear to balance the flow among the hubs tacitly.

It is easily inferred that spoke nodes located close to the center of configuration would transfer their input/output flows indirectly; however, spoke nodes located far from the hubs (e.g., 7, 8 and 10) use non-stop transmission.

6. Conclusion

Here, some basic assumptions of traditional *p*-hub median and hub location problems were relaxed. Direct links between spoke nodes were allowed to permit major spokes to enjoy non-stop transportations. To avoid a fully noneconomic connected structure or even noneconomic multiple allocations, a link installation cost was considered. The alternative link types with specific capacities, installation costs and discount factors were considered, where wider links (i.e., higher capacity) provide cheaper transportations, but require larger installation costs. Using a multi-level capacity approach to restrict the amount of flow passing through the links resulted in applying flow based discount to the flow transportation cost; therefore, hub and spoke structure costs were calculated correctly and optimal hubs location and spoke allocations were determined properly.

Furthermore, the link types to be installed on network edges were chosen from alternative link types. Two MILPs were presented for MCMA*p*HMP, where a second formulation required connecting nodes to transfer corresponding flows directly, if such a link was available, even if it was not the shortest path. By using a multi-level capacity and allowing for direct transmission of the flow, besides the inclusion of link installation cost, model I and model II do not require the network nodes to be connected by a pre-specified structure (say hub and spoke structure) to enjoy the flow agglomeration.

The network edges are allowed to select the best combination of link types minimizing the total network costs while considering capacity restrictions of the links. This realistically simulates the real life decision making process for public transportation, when some alternative types of roads/railways/airlines, and etc., with known capacities, unit flow transportation costs and installation costs are available for connecting the cities and optimal selection of them is intended.

A multi level capacity approach was also applied to the hubs in hub location problem. Two types of costs were considered for the hubs, establishing cost and operational cost. Two MILP formulations were proposed for the MCMAHLP, where congestion at the hubs was taken into account, while aiming to balance the transitional flow among the hubs.

Allowing direct links between the spoke nodes lead to less hub congestion because of elimination of the direct transferring flow from the hub and spoke network. The proposed models were aimed to transfer more flows on the installed links and implicitly were intended to enhance the link utilization. According to [15], some pairs may interact via their non-least-cost path in the optimal solution.

MCMA*p*HMP and MCMAHLP models could be seen from client perspective, where the system optimality is the main concern. One can incorporate service levels to improve user equilibrium. Since the installation costs are paid for longer periods as opposed to the transportation cost, or usually flow data assumes to be for the total planning horizon, one can formulate a situation where the flow and unit cost flow were available for multiple periods; however, installation costs only incur once, at the start of the planning horizon.

References

[1] O'Kelly, M.E., *A Quadratic Integer Program for the Location of Interacting Hub Facilities*, European Journal of Operational Research, Vol. 32, No. 3, 1987, pp. 393-404.

[2] O'Kelly, M.E., *Hub Facility Location with Fixed Costs*, Papers in Regional Science, Vol. 71, No. 3, 1992, pp. 293-306.

[3] Campbell, J.F., *Location and Allocation for Distribution Systems with Transshipments and Transportation Economies of Scale*, Annals of Operations Research, Vol. 40, No. 1, 1992, pp. 77-99.

[4] Campbell, J.F., *Integer Programming Formulations of Discrete Hub Location Problems*, European Journal of Operational Research, Vol. 72, No. 2, 1994, pp. 387-405.

[5] Skorin-Kapov, D., Skorin-Kapov, J., O'Kelly, M.E., *Tight Linear Programming Relaxations of Uncapacitated p-Hub Median Problems*, European Journal of Operational Research, Vol. 94, No. 3, 1996, pp. 582-593.

[6] Ernst, A.T., Krishnamoorthy, M., *Efficient Algorithms for the Uncapacitated Single Allocation p-Hub Median Problem*, Location Science, Vol. 4, No. 3, 1996, pp. 139-154.

[7] Ernst, A.T., Krishnamoorthy, M., *Exact and Heuristic Algorithms for the Uncapacitated Multiple Allocation p-Hub Median Problem,* European Journal of Operational Research, Vol. 104, No. 1, 1998, pp. 100-112.

[8] Ernst, A.T., Krishnamoorthy, M., *Solution Algorithms for the Capacitated Single Allocation Hub Location Problem*, Annals of Operations Research, Vol. 86, 1999, pp. 141-159.

[9] Ebery, J., Krishnamoorthy, M., Ernst, A., Boland, N., *The Capacitated Multiple Allocation Hub Location Problem: Formulations and Algorithms*, European Journal of Operational Research, Vol. 120, No. 3, 2000, pp. 614-631.

[10] Marín, A., *Formulating and Solving Splittable Capacitated Multi-Allocation Hub Location Problems*, Computers & Operations Research, Vol. 32, No. 12, 2005, pp. 3093-3109.

[11] Rodríguez-Martín, I., Salazar-González, J., *Solving a Capacitated Hub Location Problem,* European Journal of Operational Research, Vol. 184, No. 2, 2008, pp. 468-479.

[12] Contreras, I., Díaz, J.A., Fernández, E., *Lagrangean Relaxation for the Capacitated Hub Location Problem with Single Assignment*, OR Spectrum, Vol. 31, No. 3, 2009, pp. 483-505.

[13] Correia, I., Nickel, S., Gama, F.S., *Single-Assignment Hub Location Problems with Multiple Capacity Levels*, Transportation Research - Part B, Vol. 44, Nos. 8-9, 2010, pp. 1047-1066.

[14] Alumur, S., Kara, B.Y., *Network Hub Location Problems: The State of the Art*, European Journal of Operational Research, Vol. 190, No. 1, 2008, pp. 1-21.

[15] O'Kelly, M.E., Bryan, D.L., *Hub Location with Flow Economies of Scale*, Transportation Research - Part B, Vol. 32, No. 8, 1998, pp. 605-616.

[16] Klincewicz, J.G., *Enumeration and Search Procedures for a Hub Location Problem with Economies of Scale*, Annals of Operations Research, Vol. 110, Nos. 1-4, 2002, pp. 107-22.

[17] Skorin-Kapov, D., *On Cost Allocation in Hub-Like Networks*, Annals of Operations Research, Vol. 106, Nos. 1-4, 2001, pp. 63-78.

[18] Skorin-Kapov, D., Skorin-Kapov, J., *Threshold Based Discounting Networks: The Cost Allocation Provided by the Nucleolus*, European Journal of Operational Research, Vol. 166, No. 1, 2005, pp. 154-159.

[19] Racunica, I., Wynter, L., *Optimal Location of Intermodal Freight Hubs*, Transportation Research - Part B, Vol. 39, No. 5, 2005, pp. 453-477.

[20] Carello, G., Della Croce, F., Ghirardi, M., Tadei, R., *Solving the Hub Location Problem in Telecommunication Network Design: A Local Search approach*, Networks, Vol. 44, No. 2, 2005, pp. 94-105.

[21] Yaman, H., *Polyhedral Analysis for the Uncapacitated Hub Location Problem with Modular Arc Capacities*, SIAM Journal on Discrete Mathematics, Vol. 19, No. 2, 2005, pp. 501-522.

[22] Yaman, H., Carello, G., *Solving the Hub Location Problem with Modular Link Capacities*, Computers & Operations Research, Vol. 32, No. 12, 2005, pp. 3227-3245.

[23] Yaman H., Star p-hub median problem with modular arc capacities, Computers & Operations Research, Vol. 35, No. 9, 2008, pp. 3009-3019.

[24] Sohn, J., Park, S., *Efficient Solution Procedure and Reduced Size Formulations for p-Hub Location Problems*, European Journal of Operational Research, Vol. 108, No. 1, 1998, pp. 118-126.

[25] Marianov, V., Serra, D., *Location Models for Airline Hubs Behaving as M/D/C Queues*, Computers & Operations Research, Vol. 30, No. 7, 2003, pp. 983-1003.

[26] Rodríguez, V., Alvarez, M.J., Barcos, L., *Hub Location Under Capacity Constraints*, Transportation Research - Part E, Vol. 43, No. 5, 2007, pp. 495-505.

[27] Costa, M.G., Captivo, M.E., Clímaco, J., *Capacitated Single Allocation Hub Location Problem-A bi-Criteria Approach*, Computers & Operations Research, Vol. 35, No. 11, 2008, pp. 3671-3695.

[28] Elhedhli, S., Hu, F.X., *Hub-and-Spoke Network Design with Congestion*, Computers & Operations Research, Vol. 32, No. 6, 2005, pp. 1615-1632.

[29] Camargo, R.S., Miranda, Jr. G., Ferreira, R.P.M., Luna, H.P., *Multiple Allocation Hub-and-Spoke Network Design Under Hub Congestion*, Computers & Operations Research, Vol. 36, No. 12, 2009, pp. 3097-3106.

Appendix I

CAB Data Set Customization

This appendix presents CAB Data Set Customization for examples presented in this paper.

1. CAB Data Set Customization for 5 Nodes

1.1. Flow Data

Node	1	2	3	4	5
1	0	6469	7629	20036	4690
2	6469	0	12999	13692	3322
3	7629	12999	0	35135	5956
4	20036	13692	35135	0	19094
5	4690	3322	5956	19094	0

1.2. Distance Data

Node	1	2	3	4	5
1	0	576.9631	946.4954	597.5972	373.8127
2	576.9631	0	369.5327	613.0386	429.1079
3	946.4954	369.5327	0	858.3308	749.6018
4	597.5972	613.0386	858.3308	0	255.0303
5	373.8127	429.1079	749.6018	255.0303	0

2. CAB Data Set Customization for 10 nodes

2.1. Flow Data

Node	1	2	3	4	5	6	7	8	9	10
1	0	6469	7629	20036	4690	6194	11688	2243	8857	7248
2	6469	0	12999	13692	3322	5576	3878	3202	6699	4198
3	7629	12999	0	35135	5956	14121	5951	5768	16578	4242
4	20036	13692	35135	0	19094	35119	21423	27342	51341	15826
5	4690	3322	5956	19094	0	7284	3102	1562	7180	1917
6	6194	5576	14121	35119	7284	0	5023	3512	10419	3543
7	11688	3878	5951	21423	3102	5023	0	11557	6479	34261
8	2243	3202	5768	27342	1562	3512	11557	0	5615	7095
9	8857	6699	16578	51341	7180	10419	6479	5615	0	4448
10	7248	4198	4242	15826	1917	3543	34261	7095	4448	0

2.2. Distance Data

Node	1	2	3	4	5	6	7	8	9	10
1	0	576.9631	946.4954	597.5972	373.8127	559.7673	709.0215	1208.328	603.6477	695.208
2	576.9631	0	369.5327	613.0386	429.1079	312.8831	1196.489	1502.14	405.8975	1241.961
3	946.4954	369.5327	0	858.3308	749.6018	556.0706	1541.273	1764.791	621.3306	1603.165
4	597.5972	613.0386	858.3308	0	255.0303	311.3071	790.1213	907.4331	237.0703	932.2173
5	373.8127	429.1079	749.6018	255.0303	0	225.8954	794.1726	1080.374	238.944	879.5647
6	559.7673	312.8831	556.0706	311.3071	225.8954	0	1009.689	1216.868	94.2588	1104.574
7	709.0215	1196.489	1541.273	790.1213	794.1726	1009.689	0	663.8762	982.7378	221.422
8	1208.328	1502.14	1764.791	907.4331	1080.374	1216.868	663.8762	0	1143.791	874.5181
9	603.6477	405.8975	621.3306	237.0703	238.944	94.2588	982.7378	1143.791	0	1094.906
10	695.208	1241.961	1603.165	932.2173	879.5647	1104.574	221.422	874.5181	1094.906	0

3. CAB Data Set Customization for 15 nodes

3.1. Flow Data

Node	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0	6469	7629	20036	4690	6194	11688	2243	8857	7248	3559	9221	10099	22866	3388
2	6469	0	12999	13692	3322	5576	3878	3202	6699	4198	2454	7975	1186	7443	1162
3	7629	12999	0	35135	5956	14121	5951	5768	16578	4242	3365	22254	1841	23665	6517
4	20036	13692	35135	0	19094	35119	21423	27342	51341	15826	28537	65387	12980	44097	51525
5	4690	3322	5956	19094	0	7284	3102	1562	7180	1917	2253	5951	1890	7097	2009
6	6194	5576	14121	35119	7284	0	5023	3512	10419	3543	2752	14412	2043	15642	5014
7	11688	3878	5951	21423	3102	5023	0	11557	6479	34261	10134	27350	6929	7961	4678
8	2243	3202	5768	27342	1562	3512	11557	0	5615	7095	10753	30362	1783	3437	8897
9	8857	6699	16578	51341	7180	10419	6479	5615	0	4448	5076	22463	4783	24609	9969
10	7248	4198	4242	15826	1917	3543	34261	7095	4448	0	4370	17267	3929	8602	2753
11	3559	2454	3365	28537	2253	2752	10134	10753	5076	4370	0	15287	3083	4092	7701
12	9221	7975	22254	65387	5951	14412	27350	30362	22463	17267	15287	0	5454	15011	17714
13	10099	1186	1841	12980	1890	2043	6929	1783	4783	3929	3083	5454	0	3251	1126
14	22866	7443	23665	44097	7097	15642	7961	3437	24609	8602	4092	15011	3251	0	5550
15	3388	1162	6517	51525	2009	5014	4678	8897	9969	2753	7701	17714	1126	5550	0

3.2. Distances Data

Node	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0	576.9631	946.4954	597.5972	373.8127	559.7673	709.0215	1208.328	603.6477	695.208	680.709	1936.572	332.4644	592.5679	908.7715
2	576.9631	0	369.5327	613.0386	429.1079	312.8831	1196.489	1502.14	405.8975	1241.961	960.3459	2318.076	786.5959	949.5669	938.7461
3	946.4954	369.5327	0	858.3308	749.6018	556.0706	1541.273	1764.791	621.3306	1603.165	1250.962	2600.078	1137.335	1266.851	1124.778
4	597.5972	613.0386	858.3308	0	255.0303	311.3071	790.1213	907.4331	237.0703	932.2173	406.3386	1741.873	485.5564	1186.858	345.8738
5	373.8127	429.1079	749.6018	255.0303	0	225.8954	794.1726	1080.374	238.944	879.5647	533.156	1889.528	402.3291	947.3188	598.541
6	559.7673	312.8831	556.0706	311.3071	225.8954	0	1009.689	1216.868	94.2588	1104.574	694.9153	2047.122	627.115	1084.5	626.1548
7	709.0215	1196.489	1541.273	790.1213	794.1726	1009.689	0	663.8762	982.7378	221.422	447.8044	1249.763	411.1133	1097.608	851.8228
8	1208.328	1502.14	1764.791	907.4331	1080.374	1216.868	663.8762	0	1143.791	874.5181	551.6299	841.624	880.0728	1714.651	694.0088
9	603.6477	405.8975	621.3306	237.0703	238.944	94.2588	982.7378	1143.791	0	1094.906	636.9045	1978.943	620.488	1151.868	535.0244
10	695.208	1241.961	1603.165	932.2173	879.5647	1104.574	221.422	874.5181	1094.906	0	642.2092	1375.635	477.459	963.7202	1046.119
11	680.709	960.3459	1250.962	406.3386	533.156	694.9153	447.8044	551.6299	636.9045	642.2092	0	1358.213	378.5906	1236.192	405.0906
12	1936.572	2318.076	2600.078	1741.873	1889.528	2047.122	1249.763	841.624	1978.943	1375.635	1358.213	0	1608.082	2335.816	1530.57
13	332.4644	786.5959	1137.335	485.5564	402.3291	627.115	411.1133	880.0728	620.488	477.459	378.5906	1608.082	0	858.251	700.8213
14	592.5679	949.5669	1266.851	1186.858	947.3188	1084.5	1097.608	1714.651	1151.868	963.7202	1236.192	2335.816	858.251	0	1500.774
15	908.7715	938.7461	1124.778	345.8738	598.541	626.1548	851.8228	694.0088	535.0244	1046.119	405.0906	1530.57	700.8213	1500.774	0