



## “Technical Note”

**Customer Lifetime Value Models: A literature Survey**

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Markov chain,  
RFM method,  
Survival analysis**ABSTRACT**

*Customer Lifetime Value (CLV) is known as an important concept in marketing and management of organizations to increase the captured profitability. Total value that a customer produces during his/her lifetime is named customer lifetime value. The generated value can be calculated through different methods. Each method considers different parameters. Due to the industry, firm, business or product, the parameters of CLV may vary. Companies use CLV to segment customers, analyze churn probability, allocate resources or formulate strategies related to each segment. In this article we review most presented models of calculating CLV. The aim of this survey is to gather CLV formulations of past 3 decades, which include Net Present Value (NPV), Markov chain model, probability model, RFM, survival analysis and so on.*

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**1. Introduction**

Judith W. Kincaid in the book *Customer Relationship Management: Getting It Right!*, defines customer as: ‘A customer is a human being who can make decisions and use product. A customer is a person who has acquired or is considering the acquisition of one of our (firm’s) products’ (Kincaid, 2003).

After defining Customer, it is good to describe the meaning of Customer Relationship Management (CRM). ‘CRM is a managerial effort to manage business interactions with customers by combining business processes and technologies that seek to understand a company’s customers’ (Kim et al., 2003). CRM is the process of carefully managing detailed information about customers and all customer touch points to maximize customer loyalty (Lewis, 2010).

Customer Lifetime Value is one of the important topics in CRM, which was defined by Kotler about forty

years ago as ‘The present value of the future profit stream expected given a time horizon of transacting with the customer’ (Kotler, 1974). To do so, CLV researchers after recognizing customers, by using different metrics, calculate the customer lifetime value by various approaches. Then use the result of the calculation for formulating strategies related to each segment of customers or other managerial decisions.

CLV can help organizations to identify the profitable users. The definition of profitable customer is: ‘a person, household, or company whose revenues over time exceed, by an acceptable amount, the company costs of attracting, selling, and servicing the customer’ (Kotler and Armstrong, 1996). Limitation of company’s resources makes it necessary to identify profitable and non-profitable customers, So That understanding customers’ profitability and retaining profitable customers in knows as the main part of CRM by Hawkes (Hawkes, 2000). CLV has an important role for studies such as performance measurement (Rust, 2004) or customer valuation, churn analysis, retention management (Nikkhahan et al., 2011), targeting customers (Haenlein et al., 2006), marketing resource allocation (Reinartz et al., 2005; Ming et al., 2008), product offering (Shih and Liu, 2008), pricing

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(Hidalgo et al., 2007), customer segmentation (Kim et al., 2003; Rosset, 2002; Benoit and Van den, 2009) and so on.

We want to have a review on the models of CLV calculation, which can be categorized in the first group of Jain and Singh research in. The foundation of this survey is as follow. In part three we define CLV, and talk about the usage of CLV. In part four, the methodology of this survey is explained. In fifth part we have a review on the papers that presented new models for calculating CLV. In part six, we study other researches which relate to CLV, but do not present new models, but use defined models in some decision makings. In seventh part we have a brief conclusion and in the last part represent the reviewed references.

## 2. What is CLV

Customer Lifetime Value is defined by little differences in researches. In Gupta and Lehmann research in 2003 CLV is defined as present value of all future profits generated from a customer (Gupta and Lehmann, 2003). In Pearson's research in 1996 CLV is defined as the net present value of the stream of contributions to profit that result from customer transactions and contacts with the company (Pearson, 1996). Customer Lifetime Value is the net profit or loss to the firm from a customer over the entire life of transactions of that customer with the firm (Jain and Singh, 2002). CLV appears under different names, such as Lifetime Value (LTV) (Kim et al. 2006), Customer Equity (CE) and customer profitability (Jain and Singh, 2002). CLV is used to answer questions such as: "Which customer is more profitable" and

"how to allocate resources among customers" (Gupta et al., 2006). Traditional marketing tools focus on tangible assets; however it is more effective to focus on tangible assets simultaneously (Sohrabi and Khanlari, 2007). There are some reasons to growing investigations on the concept of CLV (Gupta et al., 2006). First one is the increasing pressure in companies to make marketing accountable. Second reason is the inefficiency of financial metrics. Improving information technology techniques makes it possible for firms to collect enormous amount of customer information, this is the third reason of attempting to CLV. Researchers use different methods to calculate CLV. In this survey we review the presented models. But the basic calculation of CLV is based on NPV which means Net Present Value (Berger and Nasr, 1998). The NPV approach was extended (Gupta and Lehmann, 2003). We will show that other element were included in CLV models step by step.

## 3. Survey Methodology

In 2002 Jain and Singh reviewed CLV researches and categorized them into three parts. The first group is models which calculated CLV for each customer. The second group describes customer base analysis and the third group contains articles which talk about managerial decision makings by CLV. In this paper we want to have a literature review on first type of studies, which are CLV calculation modeling. We studied most papers presenting different models of evaluating CLV from 1985 till 2012. All available journals and conferences were considered; Table 1 contains the name of the reviewed journals.

Tab. 1. List of journal and conference's name

Abbreviation	Name of journal or Conference	Abbreviation	Name of journal or Conference
AJBM	African Journal of Business Management	JDBM	Journal of Database Marketing
AS	American Statistician	JDM	Journal of Direct Marketing
BIT	Behavior and Information Technology	JDU	Journal Of Donghua University
DMKD	Data Mining and Knowledge Discovery	JIE	Journal of Industrial Engineering
DSS	Decision Support Systems	JIM	Journal of Interactive Marketing
E	Emerald	JMI	Journal of Managerial Issues
EJOR	European Journal of Operational Research	JMFM	Journal of Market Focused Management
EMJ	European Management Journal	JM	Journal of Marketing
ESA	Expert Systems with Applications	JMR	Journal of Marketing Research
HBR	Harvard Business Review	JNSH	Journal of Natural Science Heilongjiang University
HBSP	Harvard Business School Press	JR	Journal of Retailing
HC	Heath and Company	JRM	Journal of Relationship Marketing
IAU	IAU, South Tehran Branch	JSR	Journal of Service Research
IMM	Industrial Marketing Management	JSM	Journal of Strategic Marketing
IM	Information and Management	JTM	Journal of Targeting Measurement
ICISE	Information Science and Engineering Conference	JAMS	Journal of the Academy of Marketing Science
ICEB	International Conference on Electronic Business	ManS	Management Science
ICNIT	International Conference on Networking and Information Technology	MC	Marketing Computers
ICSSH	International Conference on Social Science and Humanity	MarS	Marketing Science
IJASE	International Journal of Applied Science and Engineering	Omega	Omega
IJF	International Journal of Forecasting	OMRS	Operations Research and Management Science
IAAR	Iranian Accounting & Auditing Review	PCS	Procedia Computer Science
IJMS	Iranian Journal of Management Studies	QME	Quantitative Marketing and Economics
JBIS	Journal of Business Information Systems	SBP	Social Behavior and Personality
JBR	Journal of Business Research	SI	Scientia Iranica

Related studies in each journal and conference are represented in table 2. This table shows the importance of CLV. Should be noted that the study is mentioned in

each cell and the bold ones which are marked by star (\*), are reviewed articles and books which represent a CLV calculation model.

**Tab. 2. Publications related to the concept of CLV**

	Till 1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	TOTAL
AJBM																(Li and Chang)		(Saiti et al.)	2
AS	(Schmittlein, et al.)																		1
BIT												(Tarokh et al.)							1
DMKD									(Rosset et al.)*										1
DSS							(Verhoef et al.)												1
E																(Singh et al.)	(Chang et al.)		2
EJOR													(Crowder et al.)	(Ming et al.)					2
EMJ								(Bayon et al.)					(Haenlein et al.)*						2
ESA										(Hwang et al.)*		(Kim et al.)		(Shih and Liu)	(Coussemont)	(Benoit)	(Yeh)	(Chan et al.)	7
HBR	(Reichheld et al.)	(Blattberg et al.)						(Reinartz et al.)				(Kumar et al.)							4
HBSP		(Reichheld)																	1
HC	(Barbara)																		1
IAAR													(Sohabi et al.)						1





PCS																(Khajvand) (Khajvand)	2		
QME														(Dieze)				1	
SBP																	(Chang)	1	
SI																	(Cheng)	1	
Book	(Kumar) (Rust) (Pearson) (Chan)					(Blatberg)		(Judith)		(Gupta)* (Fader)* (Hughes)	(Kumar)	(Kumar)			(Rust) (Lemon)			13	
<b>Total</b>	<b>14</b>	<b>5</b>	<b>3</b>	<b>1</b>	<b>4</b>	<b>4</b>	<b>6</b>	<b>9</b>	<b>8</b>	<b>10</b>	<b>12</b>	<b>10</b>	<b>5</b>	<b>4</b>	<b>5</b>	<b>11</b>	<b>9</b>	<b>6</b>	<b>126</b>

**5. Models of Measuring CLV**

We studied related papers to the subject of CLV calculation models from year 1985 to 2012. Here we will have a brief review of each one.

• **F. Robert Dwyer 1997 (Dwyer, 1997)**

Barbara classified industrial buyers into two parts: always-a-share and lost-for-good (Barbara, 1985). A lost-for-good buyer is the buyer who purchases from one vendor for a long time, because switching cost is very high (Dwyer, 1997). Always-a-share buyer is one who can easily experiment with new vendors (Berger and Nasr, 1998). Different groups of models for these two types of buyers were introduced by Dwyer: Retention model and Migration model. Retention model is related to lost-for-good buyers (Pfeifer and Carraway, 2000), and migration model is related to always-a-share buyers (Dwyer, 1997). These two models were used by next researchers.

• **Berger and Nasr 1998 (Berger and Nasr, 1998)**

In 1998 Berger and Nasr introduced a basic CLV model (Kumar and George, 2007) and reviewed a series of models for determination of CLV which were presented before 1995. They said that to calculate CLV, two steps should be taken:

1. Projecting the net cash flows that the firm expects to receive from customer.
2. Calculate the present value of that stream of cash flows (Berger and Nasr 1998).

The concentration of this article is on diagnosing Net contribution margin that can be achieved from each customer, therefore two implications must be considered. Acquisition cost and Fixed cost. Acquisition cost is the cost that a company spends to attract customers until customer’s first purchase occur. Fixed cost is not considered in this article (Berger and Nasr 1998). In addition to Berger and Nasr’s research, Dwyer (Hughes and Wang, 1995), Hughes (Dwyer,

1989) and Wang (Wang and Spiegel, 1994) didn’t consider fixed cost even.

Berger and Nasr’s have a formulation to calculate CLV as follow:

$$CLV = Revenue - (cost\ of\ sales + promotion\ expenses) \quad (1)$$

Cost of sales in equation number 1 is derived from cost of goods sold and cost of order processing, handling and shipping (David, 1995), and promotion expenses in the expense that company spends for customer retention (Berger and Nasr 1998). This method of CLV calculation is an aggregate level approach (Kumar and George, 2007). In aggregate level approach, which is a top-down approach, researchers segment customers then calculate CLV for each segment, and then multiply it to the number of customers (Kumar and George, 2007).

This research gathers some cases which calculate CLV by considering some specific assumptions. Indeed this article is a review on the CLV calculation researches before 1998. These cases have some assumptions in common:

- Specific timing of cash flows
- First sale occurs at the time of determination of CLV
- Revenues and costs of a sale happen at the same time
- Net contribution margin of each customer be constant

Here are the different formulations for calculating CLV before 1998 through five cases:

1. Carpenter in 1995 and Dwyer in 1989, used this formula: (Carpenter, 1995; Dwyer, 1989)

$$CLV = \left\{ GC \times \sum_{i=0}^n \frac{r^i}{(1+d)^i} \right\} - \left\{ M \times \sum_{i=1}^n \frac{r^{i-1}}{(1+d)^{i-0.5}} \right\} \quad (2)$$

The elements of formulation 2 are explained here; these variables are used in next cases too.

- **GC**: expected yearly gross contribution margin for each customer
- **M**: relevant promotion costs for each customer in one year
- **n**: length of the period of cash flows
- **r**: retention rate in one year
- **d**: discount rate in one year

2. In this case the time period is constant, but is not one year such as case 1.

2.1. If the sales happen more than once a year the formulations is as follow:

$$CLV = \{GC' \times \sum_{i=0}^{pm} \frac{(r')^i}{(1+d)^{\frac{i}{p}}}\} - \{M' \times \sum_{i=1}^{pm} \frac{(r')^{i-1}}{(1+d)^{\frac{i-0.5}{p}}}\} \quad (3)$$

2.2. If the sales happen less than once a year:

$$CLV = \{GC' \times \sum_{i=0}^{\frac{n}{q}} \frac{(r')^i}{(1+d)^{iq}}\} - \{M' \times \sum_{i=1}^{\frac{n}{q}} \frac{(r')^{\frac{i-1}{q}}}{(1+d)^{i-0.5}}\} \quad (4)$$

Reichheld in 1996 and Sasser in 1990 used the model below. In this model M and GC are not constant (Reichheld, 1996; Reichheld and Sasser, 1990).

3. Reichheld in 1996 and Sasser in 1990 used the model below. In this model M and GC are not constant (Reichheld, 1996; Reichheld and Sasser, 1990).

$$CLV = \sum_{i=0}^g \{ [ht^2 + v] \times [\frac{r^t}{(1+d)^t}] \} + \sum_{t=q+1}^n \{ [ [hg^2 + v] + [N(1 - e^{-t+g})] ] \times [\frac{r^t}{(1+d)^t}] \} \quad (5)$$

Here cash flows are assumed to be discrete (Weston and Brigham, 1993).

$$CLV = \pi(0) + \int_0^n \pi(t) \times r^t \times e^{-tLn(1+d)} dt = \pi(0) + \int_0^n \pi(t) \times [\frac{r}{(1+d)^t}] dt \quad (6)$$

4. For previous four models, if a customer leave the transactions with the company, and return after a period, he/she will be treated as a new customer. Migration model helps not to consider such customers as new ones (Dwyer, 1997). Migration model is used in this case.

$$C_i = \sum_{j=1}^i [C_{i-j} \times P_{t-j} \times \prod_{K=1}^j (1 - P_{t-j+k})] \quad (7)$$

$(P_t = 0)$

After reviewing these five models, Berger and Nasr, explain relationship marketing, which is a process in which a company tries to make a relationship with a customer, and maintains it and makes profit from it (Kotler and Armstrong, 1996). Companies need to quantify this relationship to calculate the profits obtained (Berger and Nasr 1998). Carpenter in 1995 presented a model to quantify the relationship between a company and its users (Carpenter, 1995).

Berger and Nasr, believe that there is a difference between customer equity and customer lifetime value. The difference is that, customer equity takes acquisition into consideration (Berger and Nasr 1998).

#### • *Blattberg and Deighton 1996 (Blattberg and Deighton, 1996)*

In 1996, Blattberg and Deighton introduced a procedure to identify acquisition and retention cost on the base of maximizing customer lifetime value (Blattberg and Deighton, 1996).

$$Customer\ Equity = am - A + a \times (m - \frac{R}{r}) \times (\frac{r^n}{1 - r^n}) \quad (8)$$

$$(\frac{r^n}{1 + d})$$

The elements of above equation are defined as follow:

- a**: acquisition rate
- m**: the margin on a transaction
- A**: the acquisition cost per customer
- R**: the retention cost per customer
- r**: yearly retention rate
- d**: yearly discount rate

There are other models to calculate dynamic interaction of acquisition and retention cost that was presented 1994 (Wang and Spiegel, 1994).

#### • *Pfeifer and Carraway 2000 (Pfeifer and Carraway, 2000)*

Carraway and Pfeifer used Markov model to calculate CLV in 2000. The advantages of Markov model is as follow:

- This model is flexible. This flexibility is needed to model different kinds of customer relationship situations.
- Markov model, can be used both for customer retention and customer migration models.
- Markov model is probabilistic, which is useful in uncertain situations.
- Markov is a model that can be used in decision-making (Puterman, 1994).

In Markov model possible states of relationships with a customer in counted. The probability of moving from one state to another in a single period is called transition probability. If the number of possible states is n, an n×n transition probability matrix (P) represents the transitions between states. We can have a t-step matrix to show the probabilities of moving from one step to others in t periods. In Markov model, we can arrive to a V<sup>T</sup> vector. V<sup>T</sup> is an n×1 vector of expected present value over t periods:

$$V^T = \sum_{t=0}^T [(1+d)^{-1} P]^t R \tag{9}$$

If the time period is infinite, V can be measured by equation number 10, where I is identify matrix (Pfeifer and Carrawa, 2000).

$$V = \lim_{T \rightarrow \infty} V^T = \{ I - (1+d)^{-1} P \}^{-1} R \tag{10}$$

Companies can use Markov chain model to evaluate proposed customer relationships and manage and improve them. Markov chain model is also helpful in retention and termination decisions (Pfeifer and Carrawa, 2000).

• **Rust 2000 (Rust et al., 2000)**

Rust developed an approach to determine CLV that incorporates customer-specific brand switching metrics (Rust et al., 2004). The Markov model is used in this study to model customer's probability of switching from one brand to another by transition matrix (Kumar and George, 2007). This model is an aggregate level approach.

$$CLV_i = \sum_{t=0}^{T_{ij}} \frac{1}{(1+d_j)^t} V_{ijt} \times \pi_{ijt} \times B_{ijt} \tag{11}$$

$$CLV = m \left( \frac{r}{1+i-r} \right) \tag{13: when the average margin is constant}$$

$$CLV = m \left( \frac{r}{1+i-r(1+g)} \right) \tag{14: if the margin grow at a constant rate g per period (Gupta et al., 2004)}$$

- m:**Contant average margin
- i:** Discount rate
- r:** Discount retention rate

• **Hwang, Jung, Suh 2004 (Hwang et al., 2004)**

This research proposes a model to measure LTV. The authors consider three factors which were not mentioned in previous studies of CLV. The factors are: Past profit contribution, Potential benefit and defection probabilities of customer. They also represent a framework to analyze customer value and segment them based on their values.

• **Blattberg 2001 (Blattberg et al., 2001)**

In this study customer lifetime value is the sum of three components, which are: return on acquisition, return on retention and return on add-on selling (Blattberg et al., 2001). This model is an aggregate model (Kumar and George, 2007). The formulation is as follow:

$$CLV(t) = \sum_{i=0}^I [N_{i,t} \alpha_{i,t} (S_{i,t} - c_{i,t}) - N_{i,t} B_{i,t} + \sum_{k=1}^{\infty} N_{i,t} \alpha_{i,t} \left( \prod_{j=1}^k \rho_{j,t+k} \right) \times S_{i,t+k} - c_{i,t+k} - B_{i,t+k} - B_{i,AO,t+k} \left( \frac{1}{1+d} \right)^k] \tag{12}$$

- N<sub>i,t</sub>:** Number of potential customers at time t for segment i
- α<sub>i,t</sub>:** Acquisition probability at time t for segment i
- ρ<sub>i,t</sub>:** Retention probability at time t for a customer in segment i
- B<sub>i,a,t</sub>:** Marketing cost per prospect (N) for acquiring customers at time t for segment i
- B<sub>i,r,t</sub>:** Marketing costs in time period t for retained customers for segment i
- B<sub>i,AO,t</sub>:** Marketing costs in time period t for add-on selling for segment i
- d:** Discount rate
- S<sub>i,t</sub>:** Sales of the product/services offered by the firm at time t for segment i
- c<sub>i,t</sub>:** Cost of goofs at time t for segment i
- I:** The number of segments
- i:** The segment of designation
- t<sub>0</sub>:** The initial time period (Kumar and George, 2007)

• **Gupta and Lehmann 2003 (Gupta and Lehmann, 2003)**

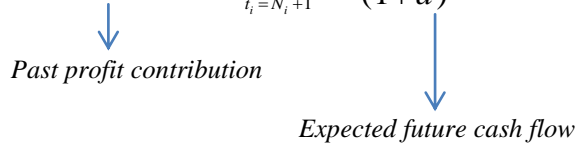
Gupta and Lehmann had some assumptions to calculate CLV: Constant retention rate and infinite projection period. They showed that if margins and retention rates are constant over time, time can be infinite (Gupta et al., 2006). Based on these assumptions the formulation of CLV is defined as follow: (Gupta and Lehmann, 2003)

The value of customer is divided to three groups and the segmentation of customers is related to them. Three groups are: current value, potential value and customer loyalty.

In this model customer defection and cross-selling opportunities in business in attended to. Cross-selling, up-selling and customer retention are necessary works to increase customer value (Kim, 2000).



$$LTV_i = \sum_{t_i=0}^{N_i} \pi_p(t_i)(1+d)^{N_i-t_i} + \sum_{t_i=N_i+1}^{N_i+E(i)+1} \frac{\pi_f(t_i)+B(t_i)}{(1+d)^{t_i-N_i}} \tag{15}$$



The preferences of this model is considering future financial contribution and potential profit generation of a customer in addition to past profit contribution. There

is a framework that helps to understand the proposed model:

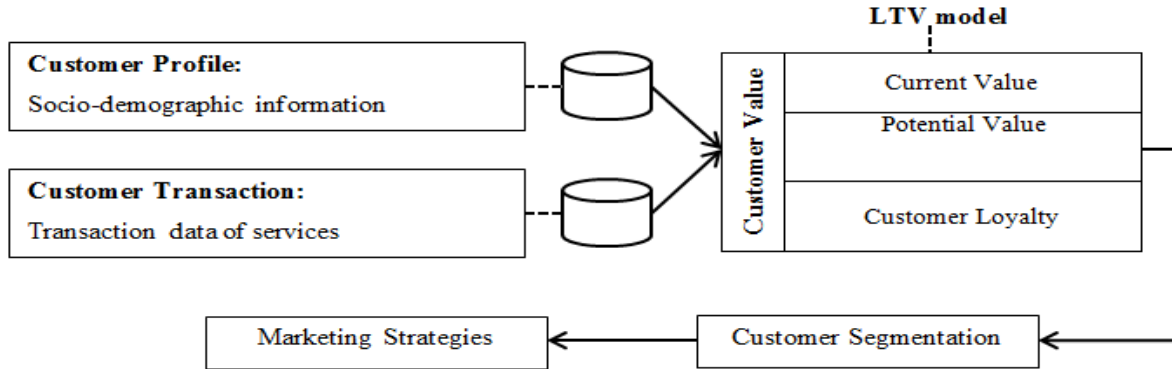


Fig. 1. Conceptual framework (Hwang et al., 2004)

Current value in this research is defined as follow: “a profit contributed by a customer during a certain period to time” (which was 6 month in the case of wireless telecommunication company).

Potential value is the expected profit that can be obtained from a certain customer. This element considers the opportunity of cross-selling. Potential value can be calculated by this formulation:

$$Potential\ value_i = \sum_{j=1}^n (prob_{ij} \times prof_{ij}) \tag{16}$$

**Prob:** is a factor that shows a customer will use an optional service presented by the company, or not. To calculate Prob, data mining methods such as R<sup>2</sup> method is used. Data should be classified based on defined variables. The related techniques introduced in this

research are decision tree, artificial neural network and logistic regression.

**Prof:** is the expected profit when a company provides a certain potential profit for a customer.

Customer loyalty measurement is very important. Even if the volume of a customer purchase is high, but the loyalty is down, the customer may not be considered as a high value customer for the company. Customer loyalty is a metric that shows the customer retention.

$$Customer\ Loyalty = 1 - Churn\ rate \tag{17}$$

To calculate churn rate of a customer models such as decision trees, neural networks and logistic regressions are used. In this study some variables effective to churn rate are introduced as follow:

Tab. 3. Effecting variables on churn rate (Hwang et al., 2004)

Variable name	Description
SEX	Gender of a customer
SMS_IN0	Whether a customer uses SMS or not
CHG_NAME	The number of times the name in charge was transferred
CHG_PYMT	The number of times the way of payments was changed
CLASS_S	The level of a customer according to the cyber-point
DEL_STAT	Weather there is any delinquent charge
DEPOMETH	Payment method of deposits
D_DEGREE	The degree of amounts in arrears in the datum month
FEE_METH	Payment method of the registration fee
MDL_NBR1	Month passed after a phone launched
MUST_MON	The obligatory period of use
OCCU_GP	Occupation of a customer
PRICE_1	The type of monthly charge
VAS_CNT	The number of optional services used
INB_MD	The number of inquiry on the phone

The proposed model to calculate CLV in this study is presented here:

$$CustomerValue = \frac{Average\ amount\ asked\ to\ pay - Cumulative\ amount\ in\ arrears}{total\ service\ period} \tag{18}$$

$$LTV_i = \sum_{t_i=0}^{N_i} \pi_p(t_i)(1+d)^{N_i-t_i} + \sum_{t_i=N_i+1}^{N_i+\lfloor \frac{1}{P_{cum}(i)} \rfloor + 1} \frac{\pi_f(t_i) + B(t_i)}{(1+d)^{t_i-N_i}} \tag{19}$$

In some article after calculating CLV, results are used to segment customers and formulate appropriate strategies for each segment.

• **Liu, Zhao, Zhang and Lu 2004 (Liu et al., 2004)** “Companies can acquire customers through costly but fact-acting marketing investments, or through slower but cheaper word-of-mouth processes” (Villanueva et al., 2007).

Liu, Zhang and Lu believe that some factors are missed in previous calculations of CLV. One concept that must be added is ‘Word-of-mouth’ marketing value (Reinartz and Kumar, 2002). The aspects that should be considered in CLV calculation due to this study are: Past profit contribution, Potential value and word-of-mouth marketing value. Word-of-mouth marketing value is needed to calculate aspects such as current value, potential value, customer loyalty and communicative value (Liu et al., 2004).

$$LTV_i = \sum_{t_i=0}^{N_i} \pi_p(t_i)(1+r)^{N_i-t_i} + \sum_{t_i=0}^{N_i} S_p(t_i)(1+r)^{N_i-t_i} + \sum_{t_i=N_i+1}^{N_i+E(i)+1} \frac{\pi_f(t_i) + B(t_i)}{(1+r)^{t_i-N_i}} + \sum_{t_i=N_i+1}^{N_i+E(i)+1} \frac{S_f(t_i)}{(1+r)^{t_i-N_i}} \tag{20}$$

**S<sub>p</sub>(t<sub>i</sub>):** past word-of-mouth contribution  
**S<sub>f</sub>(t<sub>i</sub>):** future word-of-mouth contribution

Word-of-mouth in this study is divided into two parts: Direct Word-of-mouth (S<sub>0</sub>) and indirect word-of-mouth (S<sub>1</sub>).

$$S_p(t_i) = S_{p0}(t_i) + S_{p1}(t_i) \tag{21}$$

$$S_f(t_i) = S_{f0}(t_i) + S_{f1}(t_i) \tag{22}$$

$$S_1(t_i) = \{ (E(X_{t_i}) - E(X_{t_i-1})) \times p(t_i) - m(t_i) \} \times c \times \lambda \tag{23}$$

**m(t<sub>i</sub>):** The number of customers who were directly attracted by customer I at period t<sub>i</sub>.  
**E(X<sub>ti</sub>):** Expected number of customers from period 0 to period t<sub>i</sub>.

**c:** The cost that the corporation attracts a new customer by itself.

**λ:** Profit sharing ratio of indirect word-of-mouth marketing.

**p(t<sub>i</sub>):** Ratio that the new customers attracted by customers in the whole new customer at period t<sub>i</sub>.

E(X<sub>ti</sub>) can be arrived from this equation:

$$E(X_{t_i}) = \frac{N}{1 + (N - 1)e^{-t_i \times k \times N}} \tag{24}$$

• **Venkatesan and Kumar 2004 (Venkatesan and Kumar, 2004)**

Customer lifetime value is defined as a metric for selecting customers and allocating marketing resources in this study. Berger and Colleagues (2002) believe that resource allocation is necessary to maximize CLV (Berger et al., 2002).

Different models of CRM are as follow:

- Customer lifetime value
- Customer equity
- Data Base marketing
- CLV antecedents
- CLV-based resource allocations.

This study had accomplished a comparison on these models. Table 4 is imitated from the article.

Until 2004 two studies had attended to resource allocation in their CLV researches, the first was Berger and Bechwati’s in 2001, and the next was Venkatesan and Kumar’s study in 2004. The privilege of Kumar’s study is below items that were not considered in Bechwati’s research. The items are Return allocation for each customer, Resource allocation across channels, comparison of customer-based metrics and statistical details.

This formula is extracted in this research to compute CLV. Purchase frequency, contribution margin and marketing cost are considered item in CLV calculation.

$$CLV_i = \sum_{t=1}^n \frac{Future\ contribution\ margin_{it} - Future\ cost_{it}}{(1+r)^t} \tag{25}$$

**i:** Customer  
**t:** Time  
**n:** Forecast horizon  
**r:** Discount rate

Tab. 4. Comparing the proposed CLV frameworks with existing models (Venkatesan and Kumar, 2004)

Model	Representative Research	Return on investment modeled and calculated	CLV calculation	How marketing communication affect CLV	CLV-based resource allocation	Resource allocation for each customer	Resource allocation across channels	Comparison of customer – based metrics	Statistical details
CLV	Berger, Nasr 1998	NO	YES	YES	NO	NO	NO	NO	YES
	Berger et al. 2002	YES	YES	YES	YES	NO	YES	NO	NO
Customer Equity	Blattberg, Deighton 1996	YES	YES	NO	YES	NO	NO	NO	YES
Database	Libai et al.	YES	YES	YES	NO	NO	NO	NO	NO
Marketing CLV	Reinartz, Kumar 2000	YES	YES	NO	NO	NO	NO	NO	YES
antecedents	Bolton et al. 2004	YES	YES	YES	NO	NO	NO	NO	YES
CLV- based resource allocation	Reinartz, Kumar 2003	YES	YES	YES	NO	NO	NO	YES	YES
	Rust et al. 2004	YES	YES	YES	NO	NO	NO	YES	YES
	Berger, Bechwati 2001	YES	YES	YES	YES	NO	NO	NO	NO
	Venkatesan, Kumar 2004	YES	YES	YES	YES	YES	YES	YES	YES

In most of the studies of CLV, two points are mentioned, first is that the factors effecting CLV are industry-based. And the second point is that the time considered in CLV calculations can be finite or infinite. For contractive customers the time can be finite, and for non-contractive relationships the time may be unlimited.

In the article of Werner, Reinartz and Kumar in 2000, a variable named A(Live) is defined. A(Live) shows the probability that a customer be alive in future, which can be specified by previous purchase behavior of the customer (Reinartz and Kumar, 2000). In the study of Dwyer et al. by considering last purchase of a customer, frequency of customer’s future purchase will be predicted (Dwyer et al., 1987). Based on this prediction, the CLV function can be measured as follow:

$$CLV_i = \sum_{y=1}^{T_i} \frac{CM_{i,y}}{(1+r)^{frequency_y}} - \sum_{l=1}^n \frac{\sum_{m=1}^m c_{i,m,l} \times x_{i,m,l}}{(1+r)^{l-1}} \quad (26)$$

where:

- CLV<sub>i</sub>: Lifetime value of customer i
- CM<sub>i,y</sub>: predicted contribution margin from customer i in purchase y
- r: discount rate
- C<sub>i,m,l</sub>: unit marketing cost for customer I in channel m in year l
- X<sub>i,m</sub>: number of contacts to customer i in channel m in year l
- Frequency: predicted purchase frequency for customer i
- n: number of years to forecast
- T<sub>i</sub>: predicted number of purchases made by customer I until the end of planning period

The results of verification testing of this model show that, the defined metrics are more profitable than other

three metrics which are Customer Lifetime Duration (CLD), Previous Period Customer Revenue (PCR) and Past Customer Value (PCV).

To design CLV-based resource allocation strategies, the researchers of this study, estimated customer’s responsiveness through variable β<sub>s</sub>. Then they defined the level of covariance for each customer that would maximize CLV (Venkatesan and Kumar, 2004). Major conclusions are as follow: (Venkatesan and Kumar, 2004)

- Marketing communication across various channels affects CLV nonlinearly
- CLV performs better than other commonly used customer-based metrics for customer selection
- Managers can improve profits by designing marketing communications that maximize CLV.

• **Junxiang Lu and Overland Park (Junxiang)**

Junxiang et al. presented a model for CLV using survival analysis. The factors that they presented in telecommunication industry are customer’s monthly margin and customer’s survival curve. They believe that the factors may differ in each industry. Where the competition is severe among companies in one industry, customer retention becomes more important, therefore it is needed to calculate customer survival curve. To estimate survival curve, survival analysis is used in this article (Junxiang). Logistic regressions and decision trees also are important methods to predict churn and survival rate.

In telecommunication industry, formulation number 27 is released to assess CLV.

$$LTV = MM \times \sum_{i=1}^T \frac{p^i}{(1 + \frac{r}{12})^{i-1}} \quad (27)$$

MM: monthly margin (for last three month in this case)

**T:** the number of months

**r:** discount rate

**P<sup>i</sup>:** series of customer survival probabilities. It is estimated by customer survival model (Junxiang)

In this study CLV is based on Single Product value, but it can be extended to cross-sell probabilities to estimate CLV in Multi Product scenario (Junxiang).

• **Fader, Hardie, Lee 2005 (Fader et al., 2005)**

RFM model is a well-known model which is used in marketing for decades. In 1999 Colombo and Jiang used stochastic RFM to choose customers to target them with an offer in marketing (Colombo et al., 1999). Before RFM model, demographic data of customers were used for such purpose (Gupta et al., 2006). In 2005, Fader et al. presented a new model that links RFM with CLV (Fader et al., 2004). RFM model creates groups of customers based on three factors which are Recency, Frequency and Monetary (Gupta et al., 2006).

Recency refers the duration time between last customer purchasing and present time, Frequency refers the total number of customer purchasing during life time and Monetary refers to the average money spending during past customer purchases (Tabaei and Fathian, 2011; Jonker et al., 2002).

In Fader’s study, based on three factors of RFM, customers are grouped into 125 cells, because each factor can have five values. Therefore the number of cells is 5×5×5=125 (Fader et al., 2004). The main assumption of RFM is that future behavior of customer is based on past and present behavior pattern.

• **Liu and Shih 2005 (Liu and Shih, 2005)**

Three factors of RFM model may have different impacts on various industries. A novel methodology was presented in this research to determine relative weights of RFM variables to evaluate customer’s lifetime value which is named W-RFM (Weighted-RFM). To determine the relative weights of RFM variables, analytic hierarchy process (AHP) was used. To judge the weightings of W-RFM in this study three administrative managers, two business managers in sales, one marketing constant and five customers were invited. If  $C_I^j$  is integrated rating of cluster J, the formulation to find its amount is as follow:

$$C_I^j = w_R C_R^j + w_F C_F^j + w_M C_M^j \tag{28}$$

• **Gupta 2006 (Gupta et al., 2006)**

As a related literature review we can mention to Gupta et.al. research in 2006. This article reviews CLV models and divides them into six groups. We will expand the mentioned researches and add other articles which were published after 2006 and missed researches. But as a summary, we summarize Gupta’s study.

1. The first group of CLV models is RFM model. In this group Hughes’s research in 2005 (Fader et al., 2005) was considered as an example.

2. The second group contains probability models. Probability model “is a representation in which observed behavior is viewed as the realization of an under lying stochastic process governed by latest behavioral characteristics, which in turn vary across individuals” (Gupta et al., 2006). In order to measure CLV of a customer, it is important to predict whether the customer is active in next transaction or not (Schmittlein et al., 1987). One of the related articles is based on Pareto/NBD model by Schmittlein et al. in 1987.

3. Economic models are categorized in third group. This models share the underlying philosophy of probability models in Gupta’s opinion (Gupta et al., 2006). This group of articles generally models customer acquisition, customer retention and customer expansion and the combination of them to estimate CLV (Gupta et al., 2006). As an example to model relationship duration, Gamma calculation was used (Venkatesan and Kumar, 2004).

4. Persistence model focus on modeling the behavior of acquisition, retention and cross-selling. “The major contribution of persistence modeling is that it projects the long-run or equilibrium behavior of a variable or a group of variables of interest” (Gupta et al., 2006).

5. Computer science models are based on theory. These models include neural network models (Huang et al., 2004), decision tree models, spline-based models and so on.

6. The final group is Diffusion/Growth model. CLV is the long-run profitability of an individual customer (Gupta et al., 2006). Forecasting the acquisition of future customers is typically achieved in two ways. First approach is disaggregated and second approach is aggregated method (Kumar and George, 2007). In aggregate data approach, diffusion or growth models are used to predict the number of customers. For example this model was presented to forecast the new number of customers at time t:

$$n_t = \frac{\alpha \gamma \exp(-\beta - \gamma t)}{[1 + \exp(-\beta - \gamma t)]^2} \tag{29}$$

$\alpha$ ,  $\beta$  and  $\gamma$  are the parameters of the customers  
By such forecasting the CLV calculation was as follow:

$$CLV = \int_{k=0}^{\infty} \int_{t=k}^{\infty} n_k m_{t-k} e^{-ik} e^{-\frac{1+i-r}{r}(t-k)} dt dk - \int_{k=0}^{\infty} n_k c_k e^{-ik} dk \tag{30}$$

• **Hwang, Jung, Suh, Kim 2006 (Kim et al., 2006)**

In this article a framework to analyze customer’s value and segment them based on their value is presented. After the segmentation, strategies related to each

segment are proposed. Similar to Hwang et al. study in 2003, customer defection and cross-selling opportunities is considered here. Three approaches to segment customers are presented as follow:

- Segmentation by using only LTV values
- Segmentation by using LTV components
- Segmentation by considering both LTV values and other information.

$$CLV = \{ZU_1(T_1) - C_1(T_1)\} I(T_1 \leq r_1) + \{ZU_1(r_1) - C_1(r_1)\} I(T_1 > r_1) + \{ZU_2(T_2) - C_2(T_2)\} I(T_1 > r_1) I\{ZU_1(r_1) > v_1\}$$
(31)

**Z:** is a random effect that represents a personal value factor.

• **Haenlein, Kaplan and Beeser 2007 (Haenlein et al., 2007)**

A model to determine customer lifetime value is represented in this article. This determination is based on Markov chain model and classification and regression tree. To validate the model 6.2 billion dataset was used.

In the first step customers were classified into subgroups based on their profit contribution by means of regression trees. Then transition matrix was produced by Markov chain to transit between different states of customers. In third step CLV was computed by transition matrix. Used data in this research were categorized into four groups: age, demographic, product ownership and activity level. For each group, some metrics were proposed. For example here is the metrics of product ownership group:

**Tab. 5. Operationalization of Potential Profitability Drivers: Type and Intensity of Product Ownership (Haenlein et al., 2007)**

Type of product ownership	
TP-01	Client owns transaction account
TP-02	Client owns custody account
TP-03	Client owns savings deposits
TP-04	Client owns savings plan
TP-05	Client owns home saving agreement
TP-06	Client owns own home financing
TP-07	Client owns complex home financing
TP-08	Client owns personal loan
TP-09	Client owns arranged credit product
TP-10	Client owns life insurance
TP-11	Client owns other insurance products
Intensity of product ownership	
IP-01	Positive balance transaction account
IP-02	Negative balance transaction account
IP-03	Positive balance custody account
IP-04	Negative balance custody account
IP-05	Market value custody account
IP-06	Turnover custody account during last 12 months
IP-07	Balance saving deposits
IP-08	Balance of saving plans
IP-09	Balance home savings agreement
IP-10	Balance own home financing
IP-11	Balance complex home financing
IP-12	Balance personal loan
IP-13	Cash payments and withdrawals during last 3 months
IP-14	Form-based fund transfers during last 3 months
IP-15	Transactions custody account during last 12 months

• **Crowder, Hand and Krzanowski 2007 (Crowder et al., 2007)**

Two stochastic aspects are included in CLV model calculation by these authors. First one is the expected income of a customer during his/her lifetime. Second one is customer-specific effect which means that some customers may make more income than other customers. The research tries to optimize these two aspects.

• **Razmi and Ghanbarie 2008**

In 2008 Razmi and Ghanbarie presented a new model to determine CLV. The model was a combination of RFM and ROI. The probability of futures customer's contract and his/her loyalty was embedded in the model. This probability was combined to the amount of customer's profitability; this metric is used to prioritize customers. The proposed model considers two dimensions: customer's aliveness in next period and customer churn.

Two metrics are defined in this study to handle the specified dimensions. One is contract indicator ( $\alpha$ ) which shows the probability of customer's purchase in next period. Second is loyalty indicator ( $\beta$ ) which represents customer churn's probability.  $\alpha$  and  $\beta$  are calculated as bellow:

$$\alpha = \frac{T_1}{T_2}$$
(32)

$$\beta = \frac{n}{N}$$
(33)

**T<sub>1</sub>:** Time spent between customer acquisition and last purchase

**T<sub>2</sub>:** Time spent between customer acquisition and probability estimation period

**n:** Number of periods that customer did purchase

**N:** Number of all periods

Range of  $\alpha$  varies between 0 and 1. Authors believe that these two factors ( $\alpha$  and  $\beta$ ) should be included in CLV computation.

$$CLV_i = \frac{P_i}{n} (\alpha + \beta)$$
(34)

$P_i$  is present profit value achieved from customer, and can be obtained as mentioned in equation .....

$$P_i = \sum_{j=1}^N \frac{R_{ij} - C_{ij}}{(1+r)^L}$$
(35)

If the cost of each relation through channel  $m$ , is  $C_m$ , and the number of channel usage is  $N_m$ , the marketing cost can be calculated by equation number 34.

$$CLV_i = \frac{P_i}{n} (\alpha + \beta) \left[ \frac{P_i}{n} - \sum_m (C_m \times N_m) \right] \quad (36)$$

To predict CLV in the next  $k$  periods, equation number 37 is presented.

$$CLV_{ik} = \sum_{j=1}^k \left( \prod_{j=1}^k \alpha_{ij} + \prod_{j=1}^k \beta_{ij} \right) \left[ \frac{P_i}{n_i} \left( \sum_{m=1}^{\infty} C_m \times N_m \right) \right] \frac{1}{(1+r)^{j-1}} \quad (37)$$

• **I-Cheng Yeh, King-Jang Yang, Tao-Ming Ting 2009 (Yeh et al., 2009)**

This study expands RFM model by including Time since first purchase and Probability (Yeh et al., 2009). This model is named RFMTC. For this model the researches have some assumptions:

1. The probability that a customer respond to a marketing campaign is  $P$  which is called response probability
2. A customer having response to a marketing campaign means the customer is still active.
3. If no response to a marketing campaign means the probability that a customer is still active is  $1-Q$ , where  $Q$  is churn probability (Yeh et al., 2009)

By considering these assumptions expected value of times that a customer will respond in the next  $L$  marketing campaigns can be calculated by this formulation: (Yeh et al., 2009)

$$E(X_L) = (1-Q)^n P \cdot \left[ \sum_{k=0}^{L-1} k (1-Q)^k Q + L(1-Q)^L \right] \quad (38)$$

• **Cheng, Chiu and Wu 2011 (Chenga et al., 2012)**

CLV is sum of future value and current value. Three groups of techniques are used in this essay: Logistic regressions and decision tree models to predict customer churn's probability. Regression analysis to determine customer behavior's indicators and Markov chains to model transaction probabilities of changes in customer's behavior are in the second group. And the third group is neural networks to predict revenues gained by a customer.

Depend on the loyalty of a customer, lifetime of customers may differ. To predict lifetime of a customer, this research, uses demographic data and historical transaction records to measure loyalty (Chenga et al., 2012). To model customer's possible purchasing frequency, Markov chain model is used, and to estimate the amount of customer's profit, neural networks are utilized.

Here transition matrix is used to model customer's transit from one certain purchasing behavior to another.

The states considered in this essay involve only one variable, which is the number of visits by customers in one year. More variables should be involved in the study. Handling more variables may be difficult by Markov chain model; therefore authors motioned other techniques such as simulation techniques (Chenga et al., 2012).

• **Ahmadi, Taherdoost, Fakhravar and Jalaliyoon 2011 (Ahmadi et al., 2011)**

The authors believe that CLV models should include three elements. The elements are market risk affecting customer cash flow, flexibility of firm reacting to changes and cost of customer attraction and cost of customer retention. By considering these factors, the research presents CLV model for four kinds of buyer seller relationships. (Reinartz and Kumar, 2000; Cannon et al., 2001)

- **Model 1:** When environmental risk is low and suppliers are not flexible. Simple NPV analysis is used.

$$CLV = \sum_{t=0}^n \left( \frac{m \times q}{(1+i)^t} - A_t \right) \quad (39)$$

- **Model 2:** When environmental risk is low and suppliers are flexible. Simple NPV analysis is used here.

- **Model 3:** When environmental risk is high and suppliers are not flexible. Extended NPV is used (Gupta et al., 2006).

$$CLV = \sum_{t=0}^n \left( \frac{m \times q_t}{(1+i)^t} - A_t \right) \quad (40)$$

$$CLV = m \times q_0 - A_0 + \frac{(p \times u + (1-p) \times d) \times m \times q_0}{1+i} - A_1 \quad (41)$$

- **Model 4:** When environmental risk is high and suppliers are flexible. Real options analysis is used for this case.

$$CLV = \sum_{t=0}^n \frac{\max(m \times q_t(S) - S, m \times q_t)}{(1+i)^t} - A_t \quad (42)$$

This research shows that real option analysis determines future cash flow of a customer and calculates CLV more accurate than NPV-based models (Ahmadi et al., 2011).

## 5. CLV Related Studies

As mentioned before the result of calculating CLV can be used in different fields, such as customer segmentation, churn analysis, retention management, targeting customers, marketing resource allocation, product offering, pricing and so on. We do not aim to cover all related studies to CLV, but will mention some of them to show the usability of CLV calculation.

- Hwang et al. in 2004 used LTV models to segment customers in wireless telecommunication industry. To have effective customer relationship management it is important to have customer value information (Hwang et al., 2004).
- Kumar et al. use CLV calculations to select customers and allocate resource strategies for them to have effective use of scarce resources (Venkatesan and Kumar, 2004).
- Duen-Ren Liu et al. in 2005 used CLV calculations to recommend products to customers. Product recommendation is a business activity that is vital to attract customers (Liu and Shih, 2005).
- Su-Yeon Kim et al. in 2006 proposed a framework for analyzing customer value and segmenting customers based on their value. After segmenting customers, they developed strategies related to each segment (Kim et al., 2006).
- Ya-Yueh Shih et al. in 2008 introduced product recommendation approaches based on CLV. Recommender systems help companies to have one-to-one marketing strategies to have better relationship with customers. Collaborative filtering as mentioned in this study used the results of WRFM (Shih and Liu, 2008).
- Sublaban et al. in 2009 talks about the importance of customer equity to help business monitor in order to make marketing investment decisions (Satico et al., 2009).

- Tarokh et al. in 2011 (Nikkhahan et al., 2011) used CLV models in RFM technique to segment customers of an online toy store, and formulated business strategies to each segment.
- Khajvand et al. in 2011 used two approaches, RFM and Extended RFM, to segment customers of a health and beauty company. Without computation techniques it would be hard to have correct analysis on customers (Khajvand et al., 2011).
- Khajvand and Tarokh in 2011 analyzed customer segmentation based on customer value components. This research tries to provide a methodology for segmenting customers with respect to the customer lifetime value (Khajvand and Tarokh, 2011).

## 6. Conclusion

Resources of companies are limited, therefore it is important to know customers, and define specific strategies for customers to better allocate resources between them. In order to segment customers, formulate strategies, allocate resources and so on, it is a necessary to measure customer lifetime value. There are a lot of formulations to calculate CLV. In this survey most of them are reviewed, and their main points and elements are represented. Knowing these models is important to define a good and more effective method which may consider more factors. Table 6 represents a brief review of researches which introduced a new model to calculate CLV, or considered new important factors in CLV equations.

**Tab. 6. CLV models and necessary element of CLV calculations**

Authors	Year of publish	Considered elements / defined model
Berger and Nasr	1998	diagnosing Net contribution margin
Carpenter	1995	quantify the relationship between a company and its users
Blattberg and Deighton	1996	identify acquisition and retention cost
Wang, P. and Spiegel, T.	1994	dynamic interaction of acquisition and retention cost
Pfeifer and Carraway	2000	Markov chain model
Rust	2000	Markov chain model to incorporate brand switching metrics
Blattberg	2001	return on acquisition, return on retention and return on add-on selling
Gupta and Lehmann	2003	Infinite projection period
Hwang, Jung, Suh	2003	current value, potential value and customer loyalty
Liu, Zhao, Zhang and Lu	2004	Word-of-mouth
Venkatesan and Kumar	2004	Consider resource allocation
Junxiang Lu and Overland Park		survival analysis
Fader, Hardie, Lee	2005	RFM
Colombo and Jiang	1999	stochastic RFM
Liu and Shih	2005	WRFM
Hughes	2005	RFM
Hwang, Jung, Suh, Kim	2006	customer defection and cross-selling opportunities
Crowder, Hand and Krzanowski	2007	expected income, customer-specific effect
Haenlein, Kaplan and Beeser	2007	Markov chain model
I-Cheng Yeh, King-Jang Yang, Tao-Ming Ting	2009	RFMTC model
Cheng, Chiu and Wu	2011	Markov chain model
Ahmadi, Taherdoost, Fakhravar and Jalaliyoon	2011	Market risk, flexibility of firm, cost of customer attraction/retention

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