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A Novel Clustering Approach for Estimating the Time of Step Changes in Shewhart Control Charts

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ABSTRACT

Although control charts are very common to monitoring process changes, they usually do not indicate the real time of the changes. Identifying the real time of the process changes is known as change-point estimation problem. There are a number of change point models in the literature; however most of the existing approaches are dedicated to normal processes. In this paper we propose a novel approach based on clustering techniques to estimate Shewhart control chart change-point when a sustained shift is occurrs in the process mean. For this purpose we devise a new clustering mechanism, a new similarity measure and a new objective function. The proposed approach is not only capable of detecting process change-points, but also automatically estimates the true values of the out-of-control parameters of the process. We also compare the performance of the proposed approach with existing methods.

1. Introduction

Control charts are widely used to monitor changes in processes by investigating the potential causes of variability. The control chart is a graphical display of the quality characteristic that has been measured or computed from a sample versus the sample number or time. The chart contains three parts: a center line that represents the average value of the quality characteristic corresponding to in-control state, and two other lines, called upper control limit (UCL) and lower control limit (LCL), which are chosen to assure that if the process is in-control, nearly all of the sample points will fall between them (Montgomery [1]). When a control chart produces an out-of-control signal, a search must be initiated to find the assignable cause of the out-of-control state. Knowing the exact time of a change in a process restricts the range of the search for the assignable cause which in turn accelerates the assignable cause identification and appropriate corrective action implementation.

Change-point models focus on finding the point in time where the process has changed in some fashion. Fig. 1 shows a typical shift in the mean of a normal process. The output of this process is modeled by two normal distributions. The process follows the normal distribution $N(\mu_0, \sigma_0^2)$ until the point τ in time, and then follows another normal distribution $N(\mu_1, \sigma_0^2)$. The point τ in which the process shifts to another distribution is called change-point.

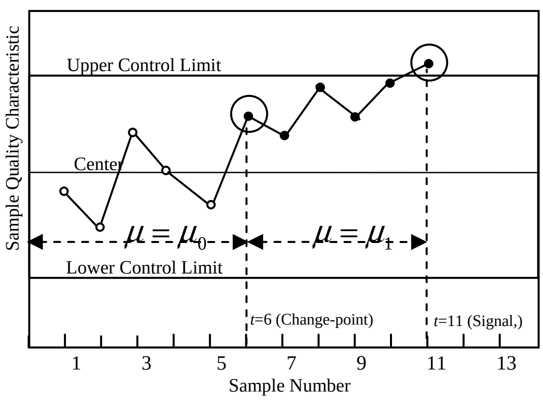

Fig. 1. A $\overline{X}$ control chart with a step change in the mean: the out-of-control signal is illustrated at *t*=11 while the real time of the change is at *t*=6

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Like control charts, the change-point models may be employed for either of the phases I or II. In phase I, a set of process data are gathered and analyzed for detecting any lack of control. Next, assignable causes accounted for out-of-control observations are identified and removed with their related data. By the time, the set of data reflecting expected in-control performance are used to estimate the in-control distribution of observations, including their mean μ_0 and standard deviation σ_0. The major task of change-point models in this phase is to estimate the time at which the change occurrs and parameters μ_0, σ_0.

Unlike the fixed set of data in phase I, phase II data are a never-ending stream which is gathered subsequently. Phase I estimated parameters are required to be plugged into the phase II calculations. In this phase, the quick detection of probable shifts in process parameters is of most importance. Using change-point model in phase II leads to two major tasks; a testing task and an estimation task. The testing task is to decide whether there has indeed been a change, and the estimation task, is to estimate τ, the time at which the change occurrs and perhaps to estimate one or both of the parameters μ_1 and σ_1. This paper focuses on change-point estimation in phase II of normal processes.

The rest of the paper is organized as follows: In section 2, we briefly review the literature of change-point detection. In section 3 we discuss the philosophy of clustering methods application for change point estimation. Section 4 explains the proposed methodology, and Section 5 examines the performance of the proposed method in comparison with some traditional methods. Finally, we present conclusions and further work in Section 6.

2. An Overview of Change-Point Detection Literature

Samuel et al. [2] considered the use of an estimator of the time of the change of a normal process once Shewhart $\overline{X}$ control chart issues a signal. Their estimator was derived based on maximum likelihood estimator (MLE) of the process location change-point. Pignatiello and Simpson [3] investigated a control chart based on likelihood ratio approach that not only provides speedy detection regardless of the magnitude of the process shift, but also supplies useful change-point statistics. Their chart provides point and interval estimates for the time and magnitude of the process shift. Hawkins et al. [4] proposed an unknown parameter change-point formulation for detecting and diagnosing step changes in the process mean when the parameters of the process are unknown. Their approach is competitive with the Shewhart chart for isolated non-sustained special causes. Perry et al. [5] compared the MLE of the process change point designed for linear trends to the MLE of the process change point designed for step changes when a linear trend disturbance is present. Hawkins and Zamba [6] suggested a single control chart using the un-known parameter likelihood ratio test for a change in mean and/or variance in normally distributed data. Their formulation gives a single diagnostic to detect shifts in mean, in variance, or in both. They also demonstrated that their change-point formulation is competitive with the best of traditional formulation for detecting step changes in process parameters.

Samuel et al. [7] proposed a MLE for step changes in the variance of a normal process. Hawkins and Zamba [8] proposed a variance change-point model, based on the likelihood ratio test, for a change in variance with conventional Bartlett correction, adapted for repeated sequential use. Their approach has good performances across the range of possible shifts. Samuel et al. [9] proposed a change-point estimator based on the likelihood function for the Poisson random variable. Perry et al. [10] also proposed a maximum likelihood estimator for the change point of a Poisson rate parameter without requiring prior knowledge regarding the form of the presented effect.

Pignatiello and Samuel [11] suggested an estimator for identifying the time of a step change in the process fraction nonconforming. Their proposed change-point estimator is the maximum likelihood estimator of the time of the step change in the binomial process and can be applied after a p or np chart signals a special cause. Perry et al. [12] proposed a maximum-likelihood estimator for the change point of the process fraction non-conforming without requiring knowledge of the exact change type a priori. They compared their proposed change-point estimator to the maximum-likelihood estimator for the process change point derived under a simple step change assumption. Nedumaran et al. [13] proposed a maximum likelihood estimator for the time of a step change in a multivariate process mean when the observations follow a multivariate normal distribution. They showed their estimator performs effectively and equally well for all process dimensions and shift magnitudes. Zamba and Hawkins [14] suggested an unknown-parameter likelihood ratio test for changes in the mean of p-variate normal data and showed that their approach is able to control the run behavior despite the lack of a large Phase-I sample.

Zou et al. [15] proposed a control chart based on the change-point model that is able to monitor linear profiles whose parameters are unknown but can be estimated from historical data. This chart can detect a shift in the intercept, slope or standard deviation. Mahmoud et al. [16] proposed a change point approach based on the segmented regression technique for testing the constancy of the regression parameters in a linear profile data set. Their change point approach is based on the likelihood ratio test for a change in one or more regression parameters.

3. The Philosophy of Clustering Methods Application for Change-Point Estimation

Change-point models have much in common with clustering methods: 1- There are two possible states in change-point models, namely the in-control state and the out-of-control state, which can be effectively considered as two possible clusters (See Fig. 2).

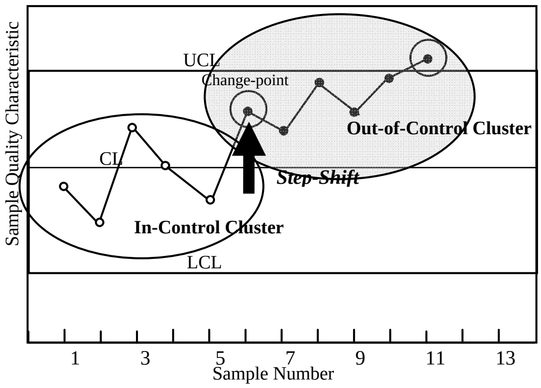

Fig. 2. A simultaneous illustrations of in-control and out-of-control states and related clusters

2- Similar to clustering methods, change point models are used to classify patterns: almost all in-control observations can be considered as a random pattern, and all out-of-control observations can be characterized as a non-random pattern. 3- Like clustering methods, in the change point models, an observation is characterized as an out-of-control observation based on its proximity to the out-of-control state parameters and its distance from the in-control state parameters 4- There is a close relationship between the control of the mean of the process using change-point model and the use of between variation in clustering on the one hand, and the control of the variance of the process using change-point model and the use of within variation in clustering on the other hand.

4. The Proposed Clustering Methodology

Using traditional change point models and clustering algorithms, in this paper we develop a new clustering approach to effectively estimate process change-point. The performance of the proposed approach directly depends on: 1- the structure, 2-the similarity measure and 3- the objective function of the algorithm.

4-1. The Structure

The proposed approach has a simple structure: after receiving an out of control signal from the control chart, it calculates the objective function for a specific number of in- and out-of-control clusters' combinations. Next, the algorithm finds the most probable in- and out-of-control clusters respect to the fitness function, and introduces the point in the out-of-control cluster with the lowest order as the process change point. In this regard, suppose that the control chart produces an out-of-control signal at time t and the objective is to find the point τ in time that the mean of the process shifts form μ_0 to $\mu_1 \neq \mu_0$. According to what we mentioned in section 3, two possible in-control and out-of-control clusters are considered on such process observations. Also, all observations before τ belong to in-control cluster and all observations beyond τ belong to the out-of control cluster. As a result, to estimate most probable τ, all possible positions of τ are examined sequentially based on the proposed objective function, and the point by which the proposed objective function is minimized is introduced as the change-point. The structure of the proposed approach is shown in Fig. 3.

4-2. The Proposed Similarity Measure

We have conducted extensive simulation studies to assess the performance of different similarity measures in the estimation of the process change point. In this regard, two representatives of distance measures, namely Euclidian (one and two dimensional) and Mahalanobis distances, and the probability of membership to each cluster have been compared to each other. In the conducted simulation studies samples of size n, ($n = 1, 5$, and 10) are randomly generated from a normal distribution with $\mu = 100, \sigma = 5$ for subgroups 1 to 100. Then, starting with subgroup 101, observations are randomly generated from a normal distribution with the mean $100 + \delta$ and standard deviation 5. This will continue until the control chart produces a signal. Then, in each simulation run, the comparing similarity measures are used to estimate the change-point of the process. For this purpose, the representatives of distance measures are employed by Fuzzy C-Mean (FCM) algorithm (Bezdek [17]) and probability of membership measure is used with the proposed objective function which will be discussed in the next section. The procedure is repeated 1,000 times for each of the values of $\delta = 0.5\sigma, \sigma, 1.5\sigma, 2\sigma$ and 3σ.

Then the average of estimates is computed to compare the performance of the measures. Table 1 illustrates the results of the comparison. Regarding the simulation results the probability measure outperforms the distance measures. It seems that as in SPC and consequently the change point models we are working with random variables and they inherit probability characteristics, the probability measure performs better than other measures. It should also be noted that one may use other factors to achieve better results. Hence, in this study we make use of the probability measure. In this regard, for each assumed in- or out-of-control clusters we calculate its covered observations' probability of membership as the similarity measure. For example, in Fig. 2, we calculate observations 6 to 12 probability of membership to the out-of-control cluster and observations 1 to 5 probability of membership to the in-control cluster as their similarity measures to related clusters.

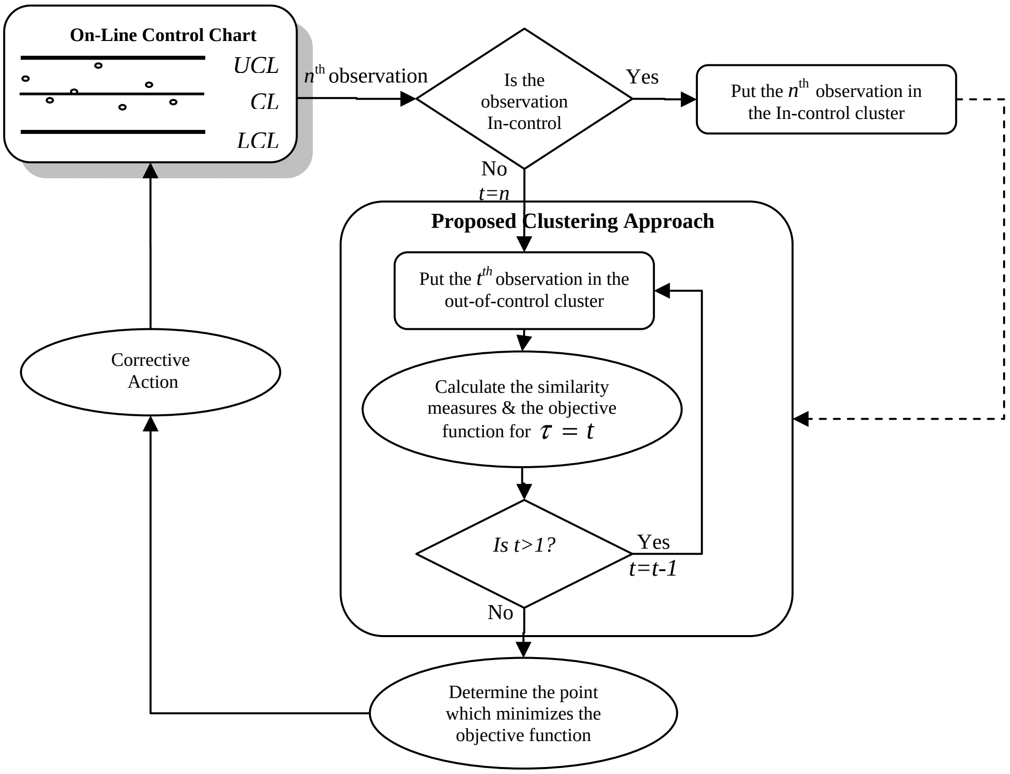

Fig. 3. The structure of the proposed approach

Tab. 1. The Results of simulation studies for different measures comparison

Method	n	$\bar{\hat{\tau}}$ / $\sigma_{\hat{\tau}}$	0.5	1	1.5	2	3
One Dimensional Euclidian Distance	1	$\bar{\hat{\tau}}$	238.4	113.0	82.9	75.2	73.9
		$\sigma_{\hat{\tau}}$	159.7	50.3	25.1	26.4	28.2
	5	$\bar{\hat{\tau}}$	102.0	74.9	74.9	73.1	71.7
		$\sigma_{\hat{\tau}}$	37.6	26.1	27.7	28.5	28.4
	10	$\bar{\hat{\tau}}$	81.7	74.5	73.6	72.9	65.4
		$\sigma_{\hat{\tau}}$	23.1	27.7	28.3	27.5	28.8
Two Dimensional Euclidian Distance	1	$\bar{\hat{\tau}}$	99.4	66.9	75.6	77.3	77.8
		$\sigma_{\hat{\tau}}$	83.0	16.1	4.1	1.7	1.3
	5	$\bar{\hat{\tau}}$	68.4	79.4	79.4	79.3	79.2
		$\sigma_{\hat{\tau}}$	13.6	1.1	0.7	0.8	0.7
	10	$\bar{\hat{\tau}}$	77.5	79.7	79.5	79.4	79.4
		$\sigma_{\hat{\tau}}$	5.1	0.6	0.6	0.6	0.6
Mahalanobis Distance	1	$\bar{\hat{\tau}}$	219.3	117.0	84.3	82.4	89.1
		$\sigma_{\hat{\tau}}$	165.1	69.5	43.8	33.6	25.1
	5	$\bar{\hat{\tau}}$	100.7	82.1	91.7	93.4	96.3
		$\sigma_{\hat{\tau}}$	56.3	32.9	21.6	18.4	4.6
	10	$\bar{\hat{\tau}}$	2.1	89.5	24.9	95.7	95.3
		$\sigma_{\hat{\tau}}$	41.9	24.9	24.9	9.6	4.6
Probability Measure	1	$\bar{\hat{\tau}}$	103.2	100.0	99.7	99.6	99.4
		$\sigma_{\hat{\tau}}$	23.2	8.0	5.4	5.0	5.3
	5	$\bar{\hat{\tau}}$	100.1	99.5	99.5	99.7	100.0
		$\sigma_{\hat{\tau}}$	6.5	5.2	4.9	3.6	1.4
	10	$\bar{\hat{\tau}}$	99.6	99.4	99.8	100.0	100.0
		$\sigma_{\hat{\tau}}$	5.5	5.1	3.4	1.4	0.5

Since the proposed approach is developed to monitor the mean of normal processes, the membership probability of each observation to the in-control cluster is equal to the probability that the observation follows $N(\mu_0, \sigma_0^2)$ distribution:

$$P(x_i \in In-Control Cluster) = P(x_i \in N(\mu_0, \sigma_0^2))$$
$$P(x_i \in N(\mu_0, \sigma_0^2)) = \begin{cases} 2\times P(X \le x_i \mid X \sim N(\mu_0, \sigma_0^2)) & X \le \mu_0 \\ 2\times P(X \le -x_i \mid X \sim N(\mu_0, \sigma_0^2)) & X > \mu_0 \end{cases} \quad (1)$$

The above probability is similar to the operation characteristics function of the distribution parameters (Freund [18]). The membership probability of each observation to the out-of-control cluster is equal to the probability that the observation follows $N(\mu_1, \sigma_0^2), \mu_1 \neq \mu_0$ distribution:

$$P(x_i \in Out-of-Control Cluster) = P(x_i \in N(\mu_1, \sigma_0^2))$$
$$P(x_i \in N(\mu_1, \sigma_0^2)) = \begin{cases} 2\times P(X \le x_i \mid X \sim N(\mu_1, \sigma_0^2)) & X \le \mu_1 \\ 2\times P(X \le -x_i \mid X \sim N(\mu_1, \sigma_0^2)) & X > \mu_1 \end{cases} \quad (2)$$

In the above relation μ_1 is estimated by averaging the observations in the out-of-control cluster. This probability is similar to the power function of the distribution parameters (Freund [18]).

4-3. The Proposed Objective Function

Different types of objective functions including: entropy function, likelihood function and the logarithm of likelihood function have been studied and examined for the proposed approach.

The final structure of the proposed objective function which must be minimized by the most probable τ is as follows:

$$F = -\sum_{x_i=1}^{\tau-1} \ln\left(P\left(x_i \in In-Control \quad Cluster\right)\right) - \sum_{x_i=\tau}^{n} \ln\left(P\left(x_i \in Out-of-Control \quad Cluster\right)\right) \quad (3)$$

Originally the proposed objective function is a multiplication of the operation characteristics and power functions of observations, which is moderated by a logarithm operator. From another point of view the above objective function is likelihood function of observation membership to in- or out-of-control cluster.

5. Simulation Studies

Monte Carlo simulations are conducted to study and compare the performance of the proposed approach with two most common change-point models in the literature, namely Hawkins et al. [4] and Samuel et al. [2] methods. Like section 4.2 in the conducted simulation studies samples of size n, $(n=1,5,\text{ and }10)$ are randomly generated from a normal distribution with $\mu=100, \sigma=5$ for subgroups 1 to 100. Then, starting with subgroup 101, observations are randomly generated from a normal distribution with a mean of $100+\delta$ and the standard deviation of 5. This will continue until the control chart produces a signal. Then, in each simulation run, the comparing approaches estimate the change-point of the process. The procedure is repeated 10,000 times for each of the values of $\delta = 0.5\sigma, \sigma, 1.5\sigma, 2\sigma$, and 3σ. The average of estimates from 10,000 simulation runs is computed to compare the performance of the proposed approach and other methods. In this regard, Table 2 has tabulated the average estimate and the standard errors of ($\bar{\hat{\tau}}$) for the comparing methods for different sample sizes and different shift sizes.

Tab. 2. Average change-point estimates and associated standard errors for change-point $\tau = 100$

Row	Method	Sample Size	Change –point estimate / Standard Deviation	Shift Size 0.5	1	1.5	2	3
1	Hawkins et al. [4]	1	Change - point estimate ($\bar{\hat{\tau}}$)	119.27	100.80	99.88	99.64	99.38
			Standard Deviation ($\sigma(\bar{\hat{t}})$)	56.15	7.75	5.26	4.65	5.76
		5	Change - point estimate ($\bar{\hat{\tau}}$)	100.28	99.59	99.50	99.74	99.97
			Standard Deviation ($\sigma(\bar{\hat{t}})$)	6.75	4.98	4.96	3.67	1.14
		10	Change - point estimate ($\bar{\hat{\tau}}$)	99.93	99.45	99.81	99.98	100.00
			Standard Deviation ($\sigma(\bar{\hat{t}})$)	4.69	5.14	2.92	0.43	0.06
2	Samuel et al. [2]	1	Change - point estimate ($\bar{\hat{\tau}}$)	104.45	100.39	99.94	99.70	99.61
			Standard Deviation ($\sigma(\bar{\hat{t}})$)	23.07	7.15	3.93	3.71	3.49
		5	Change - point estimate ($\bar{\hat{\tau}}$)	100.23	99.65	99.65	99.79	99.99
			Standard Deviation ($\sigma(\bar{\hat{t}})$)	5.80	4.30	3.62	2.88	0.18
		10	Change - point estimate ($\bar{\hat{\tau}}$)	99.87	99.62	99.86	99.98	100.00
			Standard Deviation ($\sigma(\bar{\hat{t}})$)	4.09	3.83	2.04	0.42	0.05
3	The proposed approach	1	Change - point estimate ($\bar{\hat{\tau}}$)	103.17	100.01	99.70	99.56	99.42
			Standard Deviation ($\sigma(\bar{\hat{t}})$)	23.24	7.95	5.36	4.98	5.27
		5	Change - point estimate ($\bar{\hat{\tau}}$)	100.09	99.48	99.46	99.70	99.96
			Standard Deviation ($\sigma(\bar{\hat{t}})$)	6.48	5.22	4.85	3.60	1.37
		10	Change - point estimate ($\bar{\hat{\tau}}$)	99.60	99.38	99.75	99.96	99.99
			Standard Deviation ($\sigma(\bar{\hat{t}})$)	5.52	5.14	3.38	1.41	0.51

As the actual change-point for simulations occurs at time 100, the average estimated time of process change, $\bar{\hat{\tau}}$, should be close to 100. From Table 2, it can be seen that for a process step change of standardized magnitude $\delta = .5$ and sample size $n=1$, Hawkins et al. [4] estimates the change-point at time 119.27 on average. In this case, the average estimated time of the process change is 104.45 by Samuel et al. [2], and 103.17 by the proposed approach. The same results are illustrated for $\delta = 1, 1.5, 2, 3$ in Table 1. The results of Table 2 show that the number of subgroups required to detect a change in a process parameter by the Hawkins et al. [4] method are biased specially for small shifts and small sample sizes. For example, if a step changes of $\delta = .5$ occurs in the process mean, then using individual observations, their method estimates the change-point about 19 subgroups after the process actual change. On the other hand Samuel et al. [2] and the proposed approach work faily close to the real change-point. However, the proposed approach works slightly better than the Samuel et al. [2] for small sample size and small changes, while Samuel et al. has a slightly better performance than the proposed approach for large sample size and large changes.

As shown in Table 2, the run length measure of the approaches under study has a large standard deviation, which makes the comparisons hard.

Hence, like Samuel et al [2], along with studying the run length of different approaches, we calculate the observed frequency with which each compared estimator of the time of the change is within m subgroups of the actual time of the change, for $m = 0, 1, 2, \ldots, 10$, 15 and sample size 1, 5, 10. Since the results for various sample sizes are almost similar, here we only bring the results for $n = 1$ (see Table 2 to 5). These tables provide an indication of the precision of the estimators. Typically, the higher the proportion of the 10,000 runs in which the estimated time of change is within $\pm m$ of the actual change, the more the precision of the estimator.

In Table 3 it can be seen that out of the 10,000 simulations conducted for $\delta = .5$ and sample size 1, the Hawkins et al. [4] method identified the change-point exactly, in 7 % of the times. This means that in 7 % of the times the approach gives the exact time of the change in the process mean without any error. It can be also seen that in 15.79% of the trials, the change-point was estimated to be within ± 1 of the actual time of the process change. Moreover, in 22.68% of the trials, the estimates are within ± 2 subgroups; in 22.68 % of the trials it is within ± 3 subgroups and so on. Such interpretation is true for the other magnitudes of the shifts. Besides, Table 4 calculates 15 confidence intervals of Samuel et al. [2] method based on 10,000 simulations. As it can be seen, the Samuel et al. [2] method estimates a change of $\delta = 1$ without any error in 25.7% of the times. It can be also seen that in 47.13% of the times, the change-point was correctly estimated to be within ± 1.

Besides, in 60.31% of the times, the estimates are within ± 2 subgroups and so on. Finally within subgroups of size ± 9 and more the probabilities pass 90% and tend to one. Such interpretation can be extended for other magnitudes of the shifts.

Tab. 3. Confidence Intervals for sample size 1 and different magnitudes of changes for Hawkins et al. [4] method

Shift / Probability	0.5	1	1.5	2	3
P(T=t)	0.070	0.257	0.443	0.607	0.818
P(|T-t|<=1)	0.158	0.462	0.689	0.835	0.940
P(|T-t|<=2)	0.227	0.589	0.806	0.916	0.965
P(|T-t|<=3)	0.276	0.671	0.871	0.950	0.976
P(|T-t|<=4)	0.320	0.734	0.908	0.967	0.982
P(|T-t|<=5)	0.360	0.782	0.933	0.977	0.984
P(|T-t|<=6)	0.398	0.815	0.950	0.982	0.986
P(|T-t|<=7)	0.427	0.843	0.963	0.985	0.987
P(|T-t|<=8)	0.458	0.865	0.971	0.988	0.988
P(|T-t|<=9)	0.485	0.883	0.977	0.989	0.989
P(|T-t|<=10)	0.506	0.901	0.982	0.990	0.990
P(|T-t|<=11)	0.548	0.927	0.986	0.992	0.990
P(|T-t|<=13)	0.566	0.936	0.988	0.993	0.991
P(|T-t|<=14)	0.584	0.943	0.990	0.993	0.991
P(|T-t|<=15)	0.600	0.951	0.991	0.993	0.992

Tab. 4. Confidence Intervals for sample size 1 and different magnitudes of changes in for Samuel et al. [2] method

Shifts / Probability	0.5	1	1.5	2	3
P(T=t)	0.079	0.257	0.625	0.615	0.824
P(|T-t|<=1)	0.178	0.471	0.841	0.840	0.948
P(|T-t|<=2)	0.259	0.603	0.924	0.919	0.971
P(|T-t|<=3)	0.321	0.687	0.956	0.955	0.979
P(|T-t|<=4)	0.373	0.750	0.972	0.970	0.984
P(|T-t|<=5)	0.419	0.794	0.981	0.979	0.987
P(|T-t|<=6)	0.457	0.832	0.986	0.984	0.989
P(|T-t|<=7)	0.494	0.859	0.989	0.987	0.991
P(|T-t|<=8)	0.528	0.884	0.990	0.989	0.992
P(|T-t|<=9)	0.557	0.903	0.992	0.990	0.993
P(|T-t|<=10)	0.584	0.916	0.993	0.991	0.993
P(|T-t|<=11)	0.630	0.941	0.994	0.993	0.994
P(|T-t|<=13)	0.649	0.948	0.995	0.994	0.994
P(|T-t|<=14)	0.667	0.954	0.995	0.994	0.995
P(|T-t|<=15)	0.685	0.961	0.995	0.995	0.995

Table 5 shows the probabilities of correct change-point identification within different intervals for the proposed approach. For example, it can be seen that out of 10,000 simulations conducted for $n = 1$ and $\delta = 2$, the proposed approach identify the change-point exactly 59.87% of the trials. It can also be seen that in 82.61% of the trials, the change-point was estimated to be within ± 1 of the actual time of the process change. In addition, in 90.87% of the trials, the estimates are within ± 2 subgroups; in 94.29% of the trials, it is within ± 4 subgroups and so on. Again it can be seen that the probabilities are approaching one with an increase in the value of $\pm m$. The explanations can be extended to other magnitudes of the shifts.

Tab. 5. Confidence Intervals for sample size 1 and different magnitudes of changes for the proposed approach

Shifts / Probability	0.5	1	1.5	2	3
P(T=t)	0.082	0.246	0.439	0.598	0.820
P(|T-t|<=1)	0.182	0.453	0.686	0.826	0.939
P(|T-t|<=2)	0.266	0.581	0.802	0.909	0.965
P(|T-t|<=3)	0.323	0.667	0.869	0.943	0.974
P(|T-t|<=4)	0.374	0.730	0.907	0.963	0.978
P(|T-t|<=5)	0.414	0.779	0.933	0.971	0.982
P(|T-t|<=6)	0.460	0.820	0.950	0.977	0.984
P(|T-t|<=7)	0.491	0.850	0.962	0.981	0.985
P(|T-t|<=8)	0.524	0.874	0.971	0.985	0.987
P(|T-t|<=9)	0.559	0.898	0.976	0.986	0.988
P(|T-t|<=10)	0.581	0.914	0.980	0.987	0.988
P(|T-t|<=11)	0.631	0.938	0.984	0.989	0.990
P(|T-t|<=13)	0.654	0.947	0.985	0.990	0.990
P(|T-t|<=14)	0.674	0.955	0.986	0.990	0.991
P(|T-t|<=15)	0.691	0.960	0.988	0.991	0.991

In summary, the results of Tables 2, 3, 4 and 5 show that there is not any significant difference between the proposed approach and the Samuel et al. [2] method. Besides, they work better than the Hawkins et al. [4] method. In addition, the proposed method acts superior to both the Hawkins et al. [4] and the Samuel et al. [2] methods for small shifts and small sample sizes, however Samuel et al. [2] performs better than the proposed approach for large shifts and large sample sizes.

6. Conclusions

Despite numerous applications in process monitoring, control charts are not effective tools for detecting process real time of change. In this paper, based upon clustering techniques, we proposed a new approach to detect step-changes in the mean of a normal process in phase II.

We examined the performance of the proposed approach and showed that it performs as effectively as the popular approaches like the maximum likelihood estimator. The proposed approach automatically estimates the true values of in and out-of-control parameters of the process effectively. Moreover, the proposed approach can be effectively used for other distributions of the process observations. Finally, the approach can be generalized for estimating the time step changes in non-normal processes which is planned for future work.

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