



# Multiresponse Surface Optimization Via Harmony Search Algorithm

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## KEYWORDS

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Response surface methodology ;  
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## ABSTRACT

*Response surface methodology is a common tool in optimizing processes. It mainly concerns situations when there is only one response of interest. However, many designed experiments often involve simultaneous optimization of several quality characteristics. This is called a Multiresponse Surface Optimization problem. A common approach in dealing with these problems is to apply desirability function approach combined with an optimization algorithm to determine the best settings of control variables. As the response surfaces are often nonlinear and complex a number of meta-heuristic search techniques have been widely for optimizing the objective function. Amongst these techniques genetic algorithm, simulated annealing, tabu search and hybridization of them have drawn a great deal of attention so far. This study presents the use of harmony search algorithm for Multiresponse surface optimization. It is one of the recently developed meta heuristic algorithms that has been successfully applied to several engineering problems. This music inspired heuristic is conceptualized from the musical process of searching for a perfect state of harmony. The performance of the algorithm is evaluated by an example from the literature. Results indicate the efficiency and outperformance of the method in comparison with some previously used methods.*

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## 1. Introduction

Applications of design of experiment involve optimization of the performance of manufacturing processes to determine the appropriate conditions for process variables in order to meet a desirable quality characteristic. This is usually done through Response Surface Methodology (RSM). This methodology is comprised of a collection of statistical and

mathematical techniques that has been used in developing, improving, and optimizing processes [1]. In most RSM problems, the exact form of relationship between the response variables and the independent variables is not known.

Thus it is required to seek for a suitable approximation for the response variables. Polynomial regression techniques are usually applied in this step. By fitting a low-order polynomial, some curvatures in the system could not be covered. Therefore, a polynomial of higher degrees, mostly second order would be applied in these situations. Afterwards by the method of steepest ascent or steepest descent the most appropriate

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set of inputs for response is determined [2]. RSM studies usually consider situations when there is one response of interest. However, in real world industrial and manufacturing problems it is commonly required to deal with several quality characteristics and optimize them at the same time.

This problem is referred to as a multi response surface optimization problem. For simultaneous consideration of multiple responses, the first step is data collection. It is usually done by design of experiments. The second step is model building. Building appropriate response surface model for each response is of high importance. Then we attempt to find a set of operating conditions that optimize all responses or at least keep them in desired ranges [3]. This is the final step called optimization.

In this study we will assume that the data collection and model building step have been carried out before. In the optimization phase the first question is what to optimize. Amongst different techniques in MRS literature for combining response surfaces into a single value, we will use the desirability function approach. One of the most important advantages of this approach is its flexibility.

Then the one objective function (the overall desirability) should be optimized. Both direct search and gradient based optimization techniques have been applied in the literature for this step. Methods like Hooke-Jeeves and generalized reduced gradient (GRG) have been widely used with desirability approach in MRS. Some insufficient results provoked the authors' interest to use Meta-heuristic algorithms. genetic algorithm, simulated annealing, tabu search and hybridization of them have drawn a great deal of attention so far.

This study presents the use of harmony search (HS) algorithm for Multiresponse surface optimization. HS algorithms one of the recently developed meta heuristic algorithms that has been successfully applied to several engineering problems.

This music inspired heuristic is conceptualized from the musical process of searching for a perfect state of harmony. Section 2 explains the desirability function approach and briefly reviews the literature in MRS. In Section 3 the basic concept of HS is explained. Section 4 presents the proposed method. Section 5 illustrates a numerical example from the literature to show the performance of the method. Finally the last section contains the conclusions.

## 2. Multiresponse Surface Optimization

A very popular approach in multi response surface optimization is to combine the multiple response surfaces into a single value. Then the single model is optimized. Different techniques have been introduced so far to combine surfaces including desirability function approach [4-6], distance functions [7] and loss function approach [8-9]. As desirability function approach is easy to understand and extremely flexible,

it is applied in this study. In this approach an estimated response such as  $\hat{y}_i(x)$  is transformed to a scaled free value that is called desirability. It varies from 0 to 1. The overall desirability (D) also in the [0, 1] interval is obtained by combining all desirabilities ( $d_i$ ) [4]. Derringer and Suich extended the idea and presented a method to construct an overall desirability [5]. Desirability function for the larger-the-better case is as follows:

$$d_i = \begin{cases} 0 & \hat{y}_i(x) \leq y_i^{\min} \\ \left[ \frac{\hat{y}_i(x) - y_i^{\min}}{y_i^{\max} - y_i^{\min}} \right]^t & y_i^{\min} \leq \hat{y}_i(x) \leq y_i^{\max} \\ 1 & \hat{y}_i(x) \geq y_i^{\max} \end{cases} \quad (1)$$

In the above equation, the min and max indexes on the y denote the lower and upper limits accepted for  $\hat{y}_i(x)$  respectively.

A wide variety of optimization methods have been used to optimize the overall desirability (D). Table 1 summarizes the prominent works.

**Tab. 1. Literature comparison**

| Authors              | Year | stages in MRS    |                              |
|----------------------|------|------------------|------------------------------|
|                      |      | Combining method | Optimization                 |
| Derringer & Suich    | 1980 | Desirability     | Hook & Jeeves                |
| Del Castillo et al.  | 1996 | Desirability     | Generalized reduced gradient |
| Li et al.            | 2003 | Desirability     | Genetic Algorithm            |
| Hsu                  | 2004 | Desirability     | Tabu Search                  |
| Pasandideh & Niaki   | 2006 | Desirability     | Genetic Algorithm            |
| Noorossana & Rabbani | 2007 | Desirability     | Memetic Algorithm            |
| Chang                | 2008 | Desirability     | Simulated Annealing          |
| Noorossana et al.    | 2009 | Desirability     | Genetic Algorithm            |

To read more about the mentioned papers in the table see [10-14]. Generally the authors' interest to use meta-heuristics in the optimization phase has aroused especially in recent years. However, In the authors' understanding, recently developed meta-heuristic algorithms like Harmony search algorithm was not still applied for multi response optimization problems.

### 3. Harmony Search Algorithm

Harmony search, a music inspired evolutionary algorithm, was presented by Geem et al [15,16]. It was originally introduced for discrete optimization but was expanded later on to continuous optimization [17]. The algorithm was conceptualized from the natural phenomena of music players' behavior when they play their musical instruments together to find a perfect state of harmony [18]. In this algorithm each music player is analogous to a decision variable. Collection of notes in the music players' memories is analogous to values of the decision variables. The goal is to find a pleasant harmony using the notes stored in their memories. The pleasant harmony that the group is trying to find it, is found through three musical rules [19]:

- I. Harmony is achieved by playing a note taken from harmony memory (HM)
- II. Harmony is achieved by playing a note that is close to a harmony stored in the HM.
- III. Harmony is achieved by playing an arbitrary note from the whole note range.

A pleasing harmony can be achieved from a combination of the stated three rules [19]. HS has few parameters and this is considered as one of its advantages [17]. The parameters are Harmony memory size (HMS), the harmony memory considering rate (HMCR), the pitch adjusting rate (PAR), the band width or step size for variable perturbation during pitch adjustment (bw), and finally the number of improvisations (NI) [20]. In the harmony memory a set of solution vectors for decision variables are kept [20]. HMCR and PAR are applied to improve the vector of solutions stored in HM and to increase the diversity of searching [15,16,20]. A new harmony (a new solution vector) is produced by these parameters and the following procedures [20]:

- i. Memory consideration
- ii. Pitch adjustment
- iii. Random selection

Main steps of the harmony search algorithm are as follows [21]:

1. Initialize the optimization problem and algorithm parameters
2. Initialize the harmony memory with the random solution vectors
3. Improvise a new harmony vector from the harmony memory
4. Update the harmony memory
5. Repeat steps 3 and 4 until the stopping criteria is satisfied

The global optimization problem can be summarized as follows:

$$\text{Min } f(x) \text{ st: } x(j) \in [LB(j), UB(j)], j = 1, 2, \dots, n$$

$f(x)$  is the objective function,  $x = (x(1), x(2), \dots, x(n))$  is the set of control variables,  $n$  is the number of control variables, and  $LB(j)$  and  $UB(j)$  show respectively the lower and upper bounds for the control variable  $x(j)$  [00]. The algorithm parameters as were described before are HMS, HMCR, PAR, BW and NI.

The harmony memory initialization is the next step. HM consists of a certain number of randomly generated solutions for the optimization problem.

$$HM = \begin{bmatrix} x_1^1 & \dots & x_2^1 \\ \vdots & \ddots & \vdots \\ x_1^{HMS} & \dots & x_N^{HMS} \end{bmatrix} \quad (2)$$

A new harmony vector is generated and evaluated against the worst harmony in the memory [21]. As was explained before, this procedure is done through memory consideration, pitch adjustment and random selection. In the memory consideration, values of the new harmony vector are selected randomly from existing vectors in the HM with a probability of HMCR. In the randomization, control variables are randomly chosen considering the feasible range with a probability of (1-HMCR) [16]. The operators are illustrated in the following equation [21]:

$$\begin{aligned} x_{new}(j) &\leftarrow \{x_{new}(j) \in (x_1(j), x_2(j), \dots, x_{HMS}(j)) \text{ with probability } HMCR\} \\ x_{new}(j) &\in [LB(j), UB(j)] \text{ with probability } 1 - HMCR \end{aligned} \quad (3)$$

Pitches are adjusted with a probability of  $HMCR \times PAR$  in the pitch adjusting operation that follows:

$$x_{new} = x_{old} + bw \times \varepsilon \quad (4)$$

$x_{old}$  represents the existing pitch, chosen from HM,  $x_{new}$  is the new pitch after the pitch adjusting operation, bw is the bandwidth distance for continuous problems, and  $\varepsilon$  shows a random number in the range from -1 to 1 [21].

For updating the harmony memory, first for the new harmony the value of the objective function is calculated. If the value is better than the worst one in the HM, then the new harmony will be included in the HM and the worst harmony is left out [21].

For a detailed look at the HS steps and its application one can refer to [15-22].

### 4. The Proposed Method

As it is depicted in the figure, the proposed method starts with define the objective function. The objective function is obtained from the desirability approach,

combining multiresponse surfaces into a single function called overall desirability function. Then the algorithm parameters are specified. Next Harmony memory is initialized. New Harmony generation is the following step if the stopping criteria are not still met.

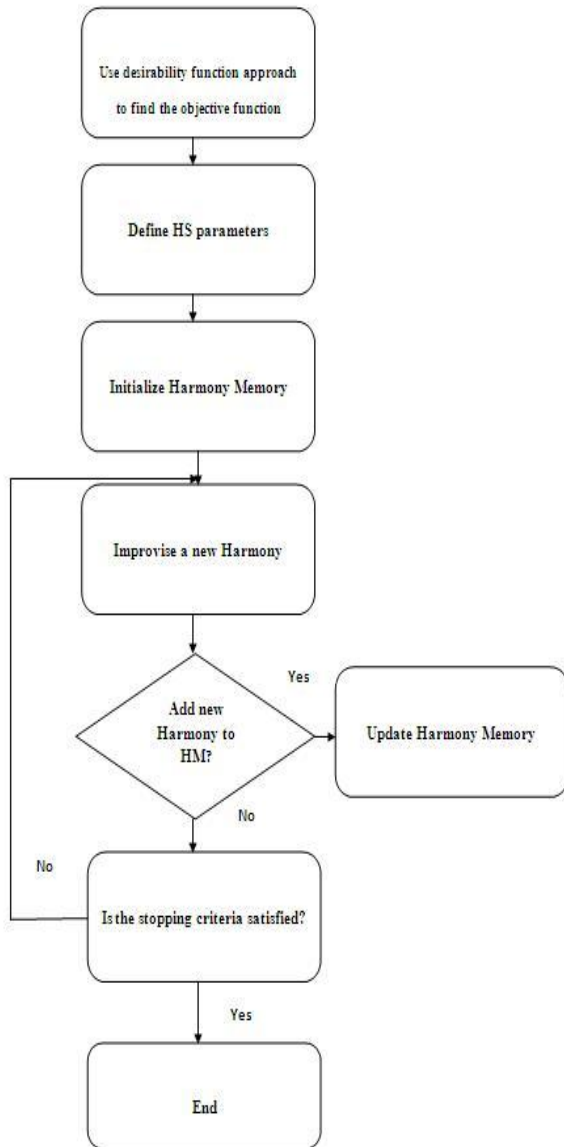


Fig. 1. Flowchart of the proposed algorithm

5. A Numerical Example

In this section the proposed algorithm is illustrated with an example from the literature. Del Castillo et al. presented a wire bonding process optimization problem from the semiconductor industry[6]. There are three control factors that exert influence on the temperature at the wire bond. N2 flow rate ( $x_1$ ), N2 temperature ( $x_2$ ), and the heater block temperature ( $x_3$ ). Six response variables are:

$Y_1$ : maximum temperature at position A

$Y_2$ : beginning bond temperature at position A

$Y_3$ : finish bond temperature at position A

$Y_4$ : maximum temperature at position B

$Y_5$ : beginning bond temperature at position B

$Y_6$ : finish bond temperature at position B

A Box-Behnken design was used that is available in Table 2.

Tab. 2. Experimental runs

| $x_1$ | $x_2$ | $x_3$ | $Y_1$ | $Y_2$ | $Y_3$ | $Y_4$ | $Y_5$ | $Y_6$ |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 40    | 200   | 250   | 139   | 103   | 110   | 110   | 113   | 126   |
| 120   | 200   | 250   | 140   | 125   | 126   | 117   | 114   | 131   |
| 40    | 450   | 250   | 184   | 151   | 133   | 147   | 140   | 147   |
| 120   | 450   | 250   | 210   | 176   | 169   | 199   | 169   | 171   |
| 40    | 325   | 150   | 182   | 130   | 122   | 134   | 118   | 115   |
| 120   | 325   | 150   | 170   | 130   | 122   | 134   | 118   | 115   |
| 40    | 325   | 350   | 175   | 151   | 153   | 143   | 146   | 164   |
| 120   | 325   | 350   | 180   | 152   | 154   | 152   | 150   | 171   |
| 80    | 200   | 150   | 132   | 108   | 103   | 111   | 101   | 101   |
| 80    | 450   | 150   | 206   | 143   | 138   | 176   | 141   | 135   |
| 80    | 200   | 350   | 183   | 141   | 157   | 131   | 139   | 160   |
| 80    | 450   | 350   | 181   | 180   | 184   | 192   | 175   | 190   |
| 80    | 325   | 250   | 172   | 135   | 133   | 155   | 138   | 145   |
| 80    | 325   | 250   | 190   | 149   | 145   | 161   | 141   | 149   |
| 80    | 325   | 250   | 180   | 141   | 139   | 158   | 140   | 148   |

In the table the factors are not coded. They have changed to coded variables with low level of -1 and high level of +1.

To build the regression models ordinary least squares (OLS) estimation techniques have been used. The models are as follows:

$$\hat{y}_1 = 174.93 + 23.38x_2 + 3.62x_3 - 19x_2x_3$$

$$\hat{y}_2 = 141 + 6x_1 + 21.02x_2 + 14.12x_3$$

$$\hat{y}_3 = 139.53 + 7.25x_1 + 16x_2 + 19.75x_3$$

$$\hat{y}_4 = 154.90 + 10.10x_1 + 30.60x_2 + 6.30x_3 - 11.20x_1^2 - 11.303x_1x_2$$

$$\hat{y}_5 = 139.29 + 4.63x_1 + 19.75x_2 + 16.13x_3 - 5.41x_1^2 + 7x_1x_2$$

$$\hat{y}_6 = 146.86 + 4.87x_1 + 15.62x_2 + 27x_3 - 3.98x_1^2 + 4.75x_1x_2$$

In order to use the desirability function approach, the process engineer determines the lower, upper and

target values for the individual desirabilities ( $d_i$ ). These are shown in Table 3.

**Tab. 3. lower, upper, and target values for individual desirabilities**

| Response | $Y_{\min j}$ | $T_j$ | $Y_{\max j}$ | $d_{Y_{\min}}$ | $d_T$ | $d_{Y_{\max}}$ |
|----------|--------------|-------|--------------|----------------|-------|----------------|
| $Y_1$    | 200          | 250   | 139          | 103            | 110   | 110            |
| $Y_2$    | 200          | 250   | 140          | 125            | 126   | 117            |
| $Y_3$    | 450          | 250   | 184          | 151            | 133   | 147            |
| $Y_4$    | 450          | 250   | 210          | 176            | 169   | 199            |
| $Y_5$    | 325          | 150   | 182          | 130            | 122   | 134            |
| $Y_6$    | 325          | 150   | 170          | 130            | 122   | 134            |

In this example the overall desirability, is the geometric mean of the six individual desirabilities:

$$D(x) = [d_1(Y_1(x))d_2(Y_2(x))\dots d_6(Y_6(x))]^{\frac{1}{6}} \quad (5)$$

D is optimized by the Harmony search algorithm. The algorithm is coded in Matlab 2009[22]. The results are shown in Table 4 .

**Tab. 4. Results**

| Optimization method | Optimal solution(x*) | D*   |
|---------------------|----------------------|------|
| GRG                 | (0.1,1,0.7)          | 0.30 |
| Hooke-Jeeves        | (0.1,1,0.8)          | 0.30 |
| SA-GA               | (0.02,0.4,0.2)       | 0.31 |
| HS (proposed)       | (1,0,0.84,0.7)       | 0.36 |

The larger D\* that is achieved from the HS algorithm demonstrates the outperformance of method in comparison with existing methods.

## 6. Conclusions

This study presents the use of harmony search algorithm for Multiresponse surface optimization. It is one of the recently developed meta-heuristic algorithms that have been successfully applied to several engineering problems. As this algorithm has few parameters, changing the ranges to see how it affects the result is easy and it can lead to finding better set of control variables that optimize the objective function. Such new meta-heuristic algorithms outperforms exact optimization methods and show efficiency in comparison with some of the well known meta-heuristic techniques.

Future research areas are consideration of improved versions of Harmony search algorithm for this problem. In addition, other techniques can be carried out in MRS data collection and model building stages.

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