



Sensitivity Analysis of Single Flexible Link Based on Sobol's Method

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ABSTRACT

The goal of this paper is to use Sensitivity Analysis (SA) method to improve the performance of the system and to make the best decision of choosing the appropriate dimensions of the flexible link to optimize the system. In this paper the SA of the geometric parameters such as: length, thickness and width of a single link flexible manipulator on maximum deflection (MD) of the end effector and vibration energy (VE) of that point by using Sobol's method are conducted. The full dynamic model of the system is developed based on Gibbs-Appel (G-A) formulation. In this paper, Timoshenko beam theory (TBT) and Euler-Bernoulli beam theory (EBBT) are used to find the end-point MD and VE of the end effector. So, at first the assumption of TBT has been performed to consider the effects of shear and rotational inertia. Then, EBBT is used. In continue Sobol's sensitivity analysis method is applied to determine how VE and end-point MD is influenced by those geometric parameters. Finally, results of two mentioned theories are compared.

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1. Introduction

Using of robotic manipulators is widespread in industries to help dangerous and tedious jobs. Most of the existing manipulators are built in a rigid body to minimize the vibration of the end-effector to achieve the acceptable accuracy by using the heavy materials and massive design. Hence, it is shown that, the rigid manipulators have some disadvantages like the high power consumption and low speed. So building the robotic manipulators in a flexible form is very desirable to reduce the weight of the arms to increase their speed of operation. Due to the importance and usefulness of these topics, understanding and analyzing of flexible manipulations has concerned researchers worldwide for many years [1-6], and proper modeling can be used to the understanding of the process.

SA is fundamental tool in the building, use and understanding of mathematical models of all forms [7]. Specifying the model resemblance with the factors that mostly take apart in the output variability, SA may be used [8-9]. While this method has been used extensively in other sciences [10-15], but this type of analysis has not been used widely to our knowledge for the analysis of the flexible manipulators. SA provides information regarding the behavior of the simulation model being evaluated. The results of SA will be very useful to adjust the dimensions of the flexible link to choose the appropriate link, to achieve optimum design.

It is essential to use SA as a reliable tool to determine the effect of each parameter while the others also changing. Local SA and global SA are two main kinds of SA that is recognized so far [16]. The global SA is used to study the effect of random input variables on the response variability of a computer code [17]. In most of studies, global SA techniques are used instead of local SA [18-20]. A commonly used method in

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global SA is the method of Sobol [21-23]. The Sobol indices are commonly used to distinguish the contribution of each input variables in the response variance decomposition.

As a problem definition in this paper, dynamics of a single link flexible manipulator is solved and sensitivity analysis of all geometric parameters of a dynamic model during the manipulation have been developed to achieve how each parameter is influenced on vibrations and deflections of the end-effector. In this case dynamic model of system is developed based on G-A formulation and AMM.

Then VE and MD of the end-effector of the flexible link are achieved by using both TBT and EBBT theories. After that, Sobol's SA method is used to evaluate how VE and end-point MD are influenced by geometric parameters. Finally, the results of two mentioned theories are compared.

2. Kinematics of the Single-Link Elastic Robotic Manipulator

The single-link elastic manipulator system is shown in Fig. 1, where XYZ and xyz represent the stationary and moving coordinate's frames, respectively.

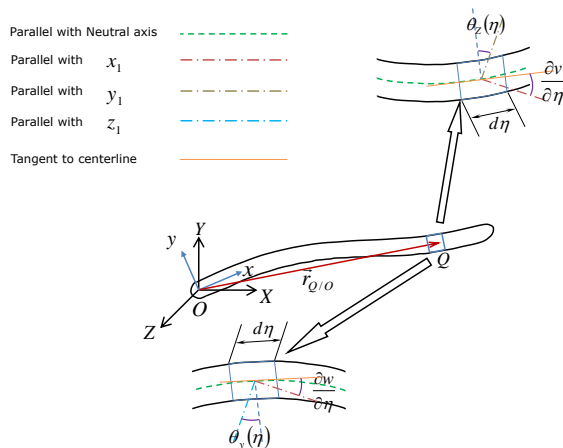


Fig. 1. Single link flexible robotic manipulator

The position vector of differential element Q with respect to the base reference system xyz is shown by $\vec{r}_{Q/o}$. To incorporate the deflection of the link, the approach of modal analysis is used. $\vec{r}_{Q/o}$ can be expressed as,

$$\vec{r}_{Q/o} = \vec{\eta} + \{u \quad v \quad w\}^T = \vec{\eta} + \sum_{i=1}^m \delta_i(t) \vec{r}_i(\eta) \quad (1)$$

where $\vec{\eta} = \{\eta \quad 0 \quad 0\}^T$ is the position vector of differential element Q with respect to o (when the flexible link is undeformed); u, v and w are small displacements along the ox, oy and oz axes, respectively; $\vec{r}_i = \{x_i \quad y_i \quad z_i\}^T$ is the eigen function

vector whose components x_i , y_i and z_i are i-th longitudinal and transverse mode shapes of the link; δ_i is the i-th time dependent modal generalized coordinate of the link; and m is the number of modes used to express the deflection of the link. The center line's total transverse displacement of differential element Q is due to bending and shear. The total slopes of the deflected centerline about oy and oz axes due to the bending and shear deformation can be represented as,

$$-\frac{\partial w}{\partial \eta} = \varphi_y + \theta_y \quad (2)$$

$$\frac{\partial v}{\partial \eta} = \varphi_z + \theta_z \quad (3)$$

Where φ_y and φ_z are the slope of the deflected centerline due to shear and θ_y , θ_z are the slope of the deflected centerline due to bending. Since the shear has not any effects on rotating the differential element Q so, this differential element undertakes rotations only due to bending and torsion. Hence the rotation of this element around the ox, oy and oz axes can be considered as θ_x , θ_y and θ_z , respectively.

3. The systems Gibbs Function and its Derivatives

The G-A method makes use of a scalar function in terms of accelerations to derive the equations of motion, analogous to the concept of using kinetic energy in Lagrange's equations. In this section the acceleration energy of the system and its derivatives with respect to quasi-accelerations are developed to construct the G-A formulation. Considering the assumption of TBT, at first the acceleration energy of a differential element Q should be presented. Then, integration of this differential acceleration energy over the link gives the link's total contribution.

$$S = \int_0^l \left(\frac{1}{2} \mu(\eta) (\ddot{\vec{r}}_Q^T \cdot \ddot{\vec{r}}_Q) + \frac{1}{2} \ddot{\theta}^T \cdot J(\eta) \ddot{\theta} \right) d\eta \quad (4)$$

It should be noted that with the assumption of EBBT only the first term of Eq. (4) should be preserved. Also $\mu(\eta)$ and $J(\eta)$ are mass per unit length and mass moment of inertia per unit length, respectively. $\ddot{\vec{r}}_Q$ and $\ddot{\theta}$ are linear and angular acceleration of differential element Q that can be stated as,

$$\ddot{\vec{r}}_Q = \ddot{\vec{r}}_{Q/o} + 2\vec{\omega} \times \dot{\vec{r}}_{Q/o} + \dot{\vec{\omega}} \times \vec{r}_{Q/o} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{Q/o}) \quad (5)$$

$$\ddot{\theta} = \sum_{j=1}^m \ddot{\delta}_j(t) \bar{\theta}_j(\eta) \tag{6}$$

Note that, in above expressions, $\dot{\omega}$ and $\ddot{\omega}$ are angular velocity and angular acceleration of the link, respectively. Also the velocity and the acceleration of differential element Q with respect to the origin of the local reference system are shown by $\dot{r}_{Q/O}$ and $\ddot{r}_{Q/O}$, respectively. One part of dynamic equations of the system using G-A formulation will be obtained by differentiating of Gibbs' function with respect to quasi-accelerations. These two terms can be represented as,

$$\frac{\partial S}{\partial \dot{q}} = \frac{\partial \dot{\omega}^T}{\partial \dot{q}} \cdot (\bar{B}_1 + 2B_2 \dot{\omega} + B_3 \ddot{\omega} + \dot{\omega} \bar{B}_3 \dot{\omega}) \tag{7}$$

$$\frac{\partial S}{\partial \ddot{q}} = \sum_{k=1}^m \ddot{\delta}_k (C_{1jk} + C_{5jk}) - 2\dot{\omega}^T \cdot \sum_{k=1}^m \dot{\delta}_k \bar{C}_{2jk} - \dot{\omega}^T \cdot \beta_j \dot{\omega} + \dot{\omega}^T \cdot \bar{\alpha}_j \tag{8}$$

where

$$\begin{aligned} \bar{B}_1 &= \sum_{j=1}^m \ddot{\delta}_j \bar{\alpha}_j & B_2 &= \sum_{j=1}^m \dot{\delta}_j \beta_j \\ B_3 &= C_3 + \sum_{j=1}^m \delta_j (C_{4j} + \beta_j) & \bar{\alpha}_j &= \bar{C}_{6j} + \sum_{k=1}^m \delta_k \bar{C}_{2kj} \\ \beta_j &= C_{4j} + \sum_{k=1}^m \delta_k C_{7kj} & C_{1jk} &= \int_0^l \mu \bar{r}_j^T \cdot \bar{r}_k d\eta \\ \bar{C}_{2jk} &= \int_0^l \mu \bar{r}_j \bar{r}_k d\eta & C_3 &= \int_0^l \mu \bar{\eta}^T \bar{\eta} d\eta \\ C_{4j} &= \int_0^l \mu \bar{\eta}^T \bar{r}_j d\eta & C_{5jk} &= \int_0^l \bar{\theta}_j^T \cdot J \bar{\theta}_k d\eta \\ \bar{C}_{6j} &= \int_0^l \mu \bar{\eta} \bar{r}_j d\eta & C_{7jk} &= \int_0^l \mu \bar{r}_j^T \bar{r}_k d\eta \end{aligned} \tag{9-20}$$

4. Dynamic Equations of Flexible Link Manipulator

Motion equation of elastic robotic manipulator will be completed by considering the effect of gravity and the generalized forces which are caused by the remaining internal and external force terms. The effect of gravity on manipulator can be considered simply by inserting $\ddot{r}_O = \bar{g}$, where \bar{g} is the acceleration of gravity. To represent the strain potential energy stored in flexible link as internal forces, two theories are existed; TBT and EBBT. For the first assumption the strain potential energy will be represented in terms of deflections and rotations as,

$$\begin{aligned} V_e &= \frac{1}{2} \int_0^l \left[kAG(\varphi_y^2 + \varphi_z^2) + EI_y \left(\frac{\partial \theta_y}{\partial \eta} \right)^2 + EI_z \left(\frac{\partial \theta_z}{\partial \eta} \right)^2 \right. \\ &\quad \left. + EA \left(\frac{\partial u}{\partial \eta} \right)^2 + GI_x \left(\frac{\partial \theta_x}{\partial \eta} \right)^2 \right] d\eta \end{aligned} \tag{21}$$

But for the second assumption, the first term of the above integral will be eliminated. In Eq. (21), E and G are elasticity and shear modulus, respectively; I_x is the polar area moment of inertia about ox axis; I_y and I_z are the area moment of inertia about oy and oz axes, respectively; A is the cross section area of the link and k is shear correction factor. To derive the motion equation of the elastic robotic manipulators, the partial derivatives of strain potential energy with respect to generalized coordinates are needed. Finally, the generalized forces which are caused by the remaining external force terms should be considered. There is no external load on the link of the considered robotic manipulator. So, the generalized forces in the deflection equations will be zero. The generalized force in the joint equation is the torque τ that applies to the joint. With this assumption, the dynamic equations of motion in the G-A formulation will be completed as follows,

- The joint equations of motion

$$\frac{\partial S}{\partial \dot{q}} + \frac{\partial V_e}{\partial q} = \tau \tag{22}$$

- The deflection equations of motion

$$\frac{\partial S}{\partial \ddot{\delta}_j} + \frac{\partial V_e}{\partial \delta_j} = 0 \quad j = 1, 2, \dots, m \tag{23}$$

5. Sobol's Sensitivity Analysis Method

Sobol's sensitivity analysis is one of the well-known statistical methods that is used successfully to non-linear mathematical models, so it is reasonable to use this method to make the best decision to optimize and also to improve the performance of the system by analyzing the behavior of the system. It is shown that, it can be used efficiently for model based analysis of real-world rough-terrain robotic systems [24]. At first the region of input factors should be defined to explain Sobol's method as follows,

$$\Omega^k = (X | 0 \leq x_i \leq 1; i = 1, 2, \dots, k) \tag{24}$$

where x_1 is input factors vector, that are perpendicular to each other's. The main idea behind the Sobol's method is that, the function $f(x)$ is derived from the sum of the following functions;

$$\begin{aligned} f(x_1, x_2, \dots, x_k) &= f_0 + \sum_{i=1}^k f_i(x_i) + \sum_{1 \leq i < j \leq k} f_{ij}(x_i, x_j) \\ &\quad + \dots + f_{1,2,\dots,k}(x_1, x_2, \dots, x_k) \end{aligned} \tag{25}$$

where f_0 is constant and it is determined as,

$$f_0 = \int_{\Omega^k} f(x) dx \tag{26}$$

Sobol showed that, the decomposition of Eq. (25) is unique. Also, all terms of the mentioned equation can be computed via multi dimensional integrals,

$$f_i(x_i) = -f_0 + \int_0^1 \dots \int_0^1 f(x) dx \tag{27}$$

$$f_{ij}(x_i, x_j) = -f_0 - \sum f_i(x_i) + \int_0^1 \dots \int_0^1 f(x) dx_{-ij} \tag{28}$$

where dx_{-i}, dx_{-ij} represent integration over all variables except x_i and x_j , respectively. Hence, for higher-order terms, continuous formula can be obtained. In the sensitivity indices which is based on variance, total variance of $f(x)$ "D" is expressed to be,

$$D = \int f^2(x) dx - (f_0)^2 \tag{29}$$

Partial variances are computed as follow,

$$D_{i_1, \dots, i_s} = \int f_{i_1, \dots, i_s}^2(x_{i_1}, \dots, x_{i_s}) dx - (f_0)^2 \tag{30}$$

After squaring and integrating Eq. (25) over all variables, expression "D" is simplified as follow,

$$D = \sum_{i=1}^k D_i + \sum_{1 \leq i < j \leq k} D_{ij} + D_{1,2, \dots, k} \tag{31}$$

So the sensitivity measures $S_{1,2, \dots, k}$, are given by,

$$S_{1,2, \dots, k} = \frac{D_{1,2, \dots, k}}{D} \quad 1 \leq i_1 \leq \dots \leq i_s \leq k \tag{32}$$

The total sensitivity analysis index is obtained by adding all the sensitivity indices involving the factor in Eq.(32). In the proposed method, Sobol's sensitivity analysis is applied to evaluate the optimal value of dimensions of the flexible link with respect to the VE and end-effector's MD minimization.

5-1. Sensitivity Analysis of MD using TBT and EBBT

In this section both TBT and EBBT assumptions are used to achieve MD of the single link flexible manipulator for each application. To do this, the variation intervals of each parameter should be extracted, firstly. Table.1, presents those intervals. Then, Sobol's sampling method is applied to generate 1152 uniform random numbers on intervals presented in Table1. Using the approach discussed in the previous section the end-point MD of the flexible link is obtained with respect to each extracted random number. The results of the SA of the flexible link with

respect to TBT and EBBT are shown in Table.2. Also, the Pie chart diagrams of the SA of the MD of the elastic link using TBT and EBBT are illustrated in Figs. 2 and 3, respectively.

Tab. 1. Properties of the link.

Parameter	Specifications	Unit
Density(ρ)	2700	kg/m ³
Young module (E)	70	Gpa
Length	(20 , 140)	Cm
Thickness	(0.1 , 0.2)	Cm
Width	(4 , 7)	Cm

Tab.2. The SA results of MD using TBT and EBBT.

Sensitivity Indices	Values(TBT)	Values(EBBT)
S_L	0.3602	0.3599
S_T	0.2193	0.2188
S_W	0	0
S_{LT}	1.0175	1.0173
S_{LW}	0.7234	0.7234
S_{TW}	0.7287	0.7288

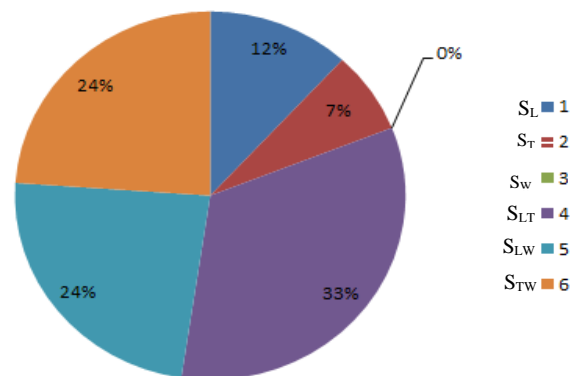


Fig. 2. The results of SA based on TBT

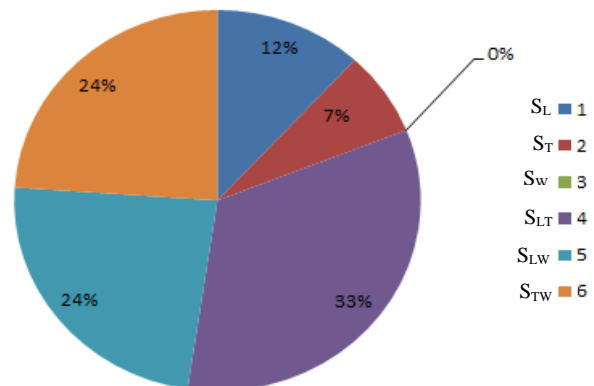


Fig. 3. The results of SA based on EBBT

Although, according to the Figs.2 and 3, it is understood that the SA results of both TBT and EBBT assumptions are the same, but in fact, they are different as noted in Table.2. As shown in Figs.2 and 3, among the first sensitive indices, the most percentage of the sensitivity corresponds to the length which is shown by S_L . To clarify the relation of MD corresponds to the link's dimensions, simulations are done by applying

both TBT and EBBT assumptions and finally, their results are presented in Figs.4, 5 and 6. Moreover, to see the results simultaneously and compare them with each other, both results are shown in single plot.

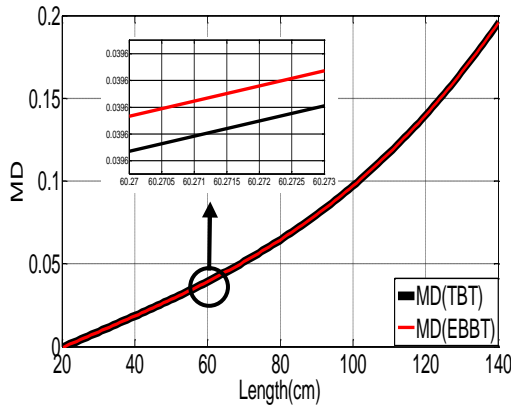


Fig.4. MD versus the link's length using TBT and EBBT

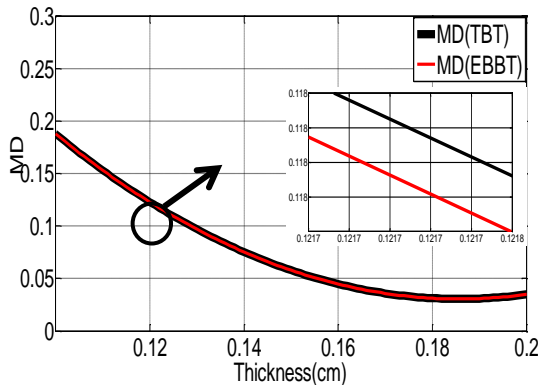


Fig.5. MD versus the link's thickness using TBT and EBBT

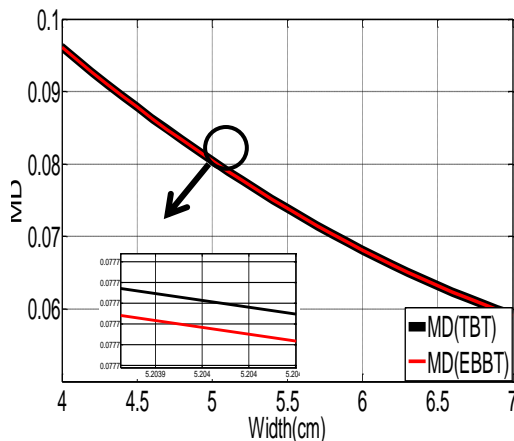


Fig.6. MD versus the link's width using TBT and EBBT

The results presented in Figs.4-6, show that, to decrease the MD, the flexible link with short length and high width and thickness should be used. As it is seen in Figs.4, 5 and 6, the minimum values of MD and corresponding dimensions can be obtained from Sobol's sensitivity analysis method which is lead to

appropriate determination of geometric parameters of the flexible link manipulator system. To achieve the best decision of choosing the optimum dimensions due to MD minimization, the factor of l/wt is defined. Fig.7 shows the relation between MD and l/wt , by using TBT and EBBT assumptions. According to the Fig.7, the best value of l/wt is about 26(1/Cm). Hence the dimensions of the flexible link are selected so that the factor of l/wt is equaled to 26.

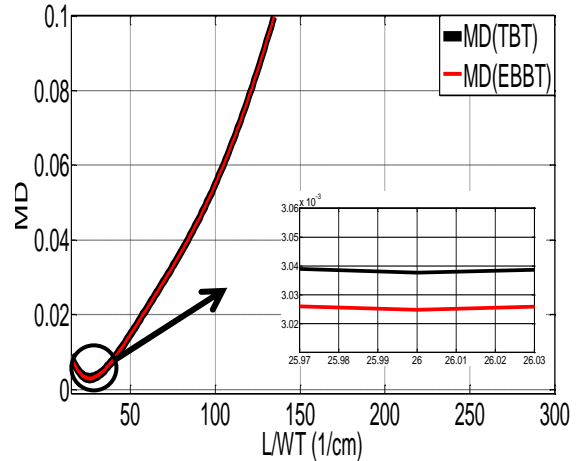


Fig.7. MD versus (l/wt) using TBT and EBBT

5-2. Sensitivity Analysis of VE of the End-Effector Using TBT and EBBT

Like the previous sub-section, the assumptions of TBT and EBBT are used here. VE of the end-effector corresponding to those 1152 random numbers are computed. Using the Sobol's method, the SA results of TBT and EBBT assumptions are obtained and they are presented in Table.3. Also, the pie chart diagrams of the SA are illustrated in Figs.8 and 9.

Tab. 3. The SA results of VE using TBT and EBBT.

Sensitivity Indices	Values(TBT)	Values(EBBT)
S_L	0.1195	0.119211
S_T	0.1004	0.100412
S_w	0.0074	0.00741
S_{LT}	0.8359	0.836077
S_{LW}	0.7515	0.7508
S_{TW}	0.9379	0.9369

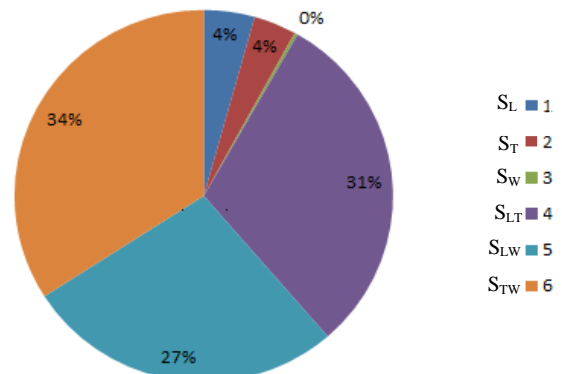


Fig.8. The results of SA based on TBT

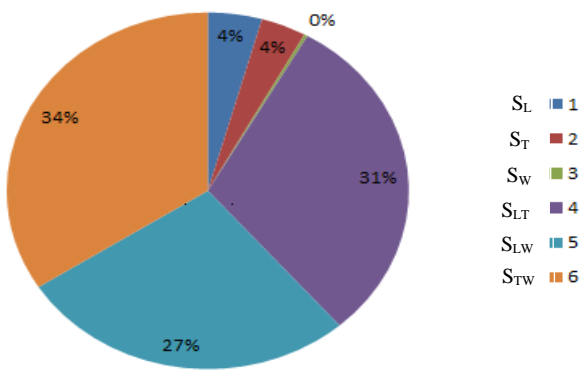


Fig.9. The results of SA based on EBBT

Due to Figs.8 and 9, it is understood that, the SA results of the VE of the end-effector of the elastic link by using the both TBT and EBBT assumptions are the same. But like the previous sub-section, by studying the numerical results of the SA, concluded that the results are different. As shown in Figs.8 and 9, the most effective parameter between the first sensitivity indices, is S_L , which shows the sensitivity amount of length on the VE of the end-effector. To discover how each parameter influenced on the VE of the end-effector, simulations are done and the results are presented in Figs.10, 11 and 12. Each figure shows the results corresponding to the both TBT and EBBT simultaneously.

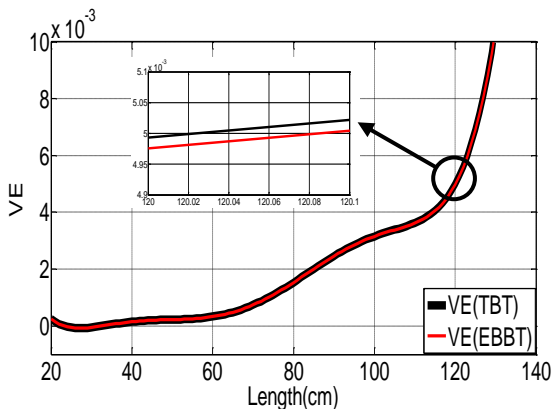


Fig.10. VE versus the link's length using TBT and EBBT

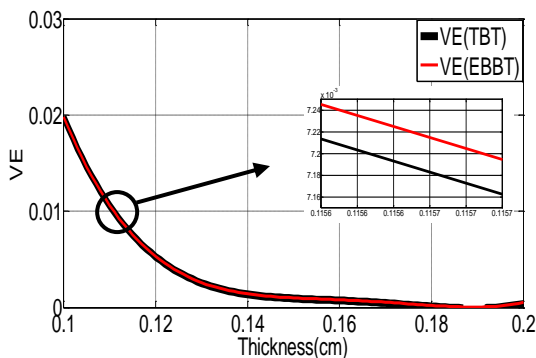


Fig.11. VE versus the link's thickness using TBT and EBBT

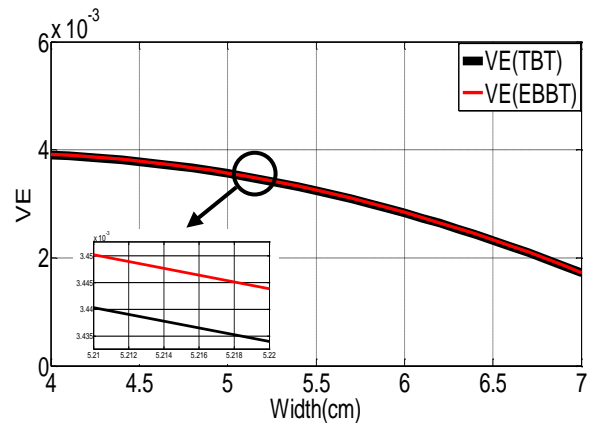


Fig.12. VE versus the link's width using TBT and EBBT

Due to Figs.10, 11 and 12, the VE of the end-effector is increased by the growth of length. But increasing the amount of thickness and also width are led to decrease the VE. So, like the previous sub-section, to find the optimal values of dimensions to achieve the minimum VE, the factor of l/wt is also defined here. Fig.13, shows the relation between VE and l/wt , by using TBT and EBBT assumptions. According to the Fig.13, the best values of l/wt is 81(1/Cm). As noted, the dimensions of the flexible link should be selected so that the value of l/wt must be equal to 81(1/Cm).

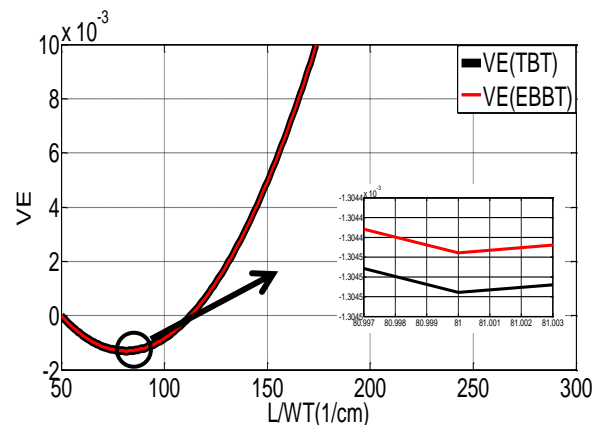


Fig.13. VE versus (l/wt) using TBT and EBBT

6. Conclusion

In this paper, dynamic modeling of the single link flexible manipulator is developed based on G-A formulations. VE and MD of the end-effector are selected to study their behavior for obtaining appropriate criteria for mechanical design of the system. For this reason, both TBT and EBBT are applied to achieve the VE and MD of the end-effector. Understanding the effects of each geometric parameter on VE and MD, SA is done by using Sobol's method. The effects of each geometric parameter are studied. Moreover, the relation between those parameters and VE and MD are presented. The results show that, for

decreasing the VE and MD, the flexible link with low length and high thickness and width should be selected. It is shown that, the most sensitive parameter corresponds to length, either TBT or EBBT. Moreover, the most percentage of sensitivity among all the other sensitivity indices is corresponded to S_{LT} , which expresses the effects of length and thickness simultaneously. Based on the results the optimum values of VE and MD occur at 81 (1/cm) and 26 (1/cm) for l/wt , respectively.

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