Fuzzy Complexity Analysis with Conflict Resolution for Educational Projects

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KEYWORDS
Conflict resolution, Complexity measure, Fuzzy graph, Fuzzy relation, Project complexity

ABSTRACT
Evaluative and comparative analysis among educational projects remains an issue for administration, program directors, instructors, and educational institutes. This study reports a fuzzy complexity model for educational projects, which has two primary aspects (technical aspects and transparency aspects). These aspects may not be measured precisely due to uncertain situations. Therefore, a fuzzy graph-based model to measure the relative complexity of educational projects is presented that uses an aggregation operator to resolve conflict among experts with respect to a complexity relation. The model maps the fuzzy graph into a scaled Cartesian diagram that depicts the relative degree of complexity among projects. An illustrative example for several educational projects is demonstrated to present the application of the model.

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1. Introduction
The term educational project refers to a comprehensive framework used to inform, educate, and raise awareness among students, practitioners, professionals, and management about specific subjects. Having a set of projects, a comparative analysis from effectiveness point of view becomes important. The effectiveness of an educational project can be analyzed through the complexity factor which may be interpreted by two major aspects: 1- Technical aspect, 2- Transparency aspect. However, due to the uncertainty of many situations, these aspects may not be measured precisely [1]. As a result, project failures are numerous in practice; for example: delivery, target audience, budget and schedule overruns, compromised performance, and missed opportunities [2]. Therefore, adapting educational project management style to the project uncertainty profile, as measured by the dimensions of the project size, project structure, and experience, is required [3]. Also, a project empirical classification method was proposed that used degrees of technical uncertainty and the complexity of the project to map the overall uncertainty [4]. As complexity measures have become an efficient yardstick to manage a group of educational projects, having a quantitative model to analyze the relative complexity under uncertain situations is a must. There are several methods to analyze complexity.

The knowledge base rule uses the knowledge encoded in some form such as rule-based systems and decision tree. Generally, the construction of a complexity model has been carried out by interviewing experts in complexity aspects and painstakingly translating the experts’ opinions into an appropriately structured set of rules (e.g., if-then) [5]. Due to time consuming and complexity of consistency check, a knowledge base approach is not considered. Alternatively, fuzzy Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) approach for complexity analysis is studied. Since the complexity criterion with the highest score has disproportionate impact in the complexity ranking process, the sensitivity analysis cannot be done.
with TOPSIS [6]. Also, Analytic Hierarchy Process (AHP) technique is considered to determine the preferential weight of relative complexity between projects.

This approach works based upon three principles: 1) decomposition, 2) comparative judgments, and 3) synthesis of priorities.

AHP has several shortcomings for complexity analysis, such as man-made inconsistency in pair wise comparisons, and rank reversal when new projects are introduced. Considering the simplicity of and efficiency of the proposed method, this study makes two contributions.

First, by defining a fuzzy relation, a quantitative method for expressing the relative complexity among projects is presented. The method uses an aggregation operator to mitigate experts’ opinions on a complexity relation. Second, a pictorial model mapped in a scaled Cartesian diagram to show relative complexity among educational projects is proposed. Outcomes of this graph can help in an efficient comparative analysis and fair evaluation, budgeting, planning, and allocating soft and hard resources among projects. A hypothetical example for five educational projects is demonstrated to present the application of the model.

### 2. Fuzzy Complexity Analysis

The impact of complexity on outcomes, which are realizable from projects over their life cycle, has become a major concern in today’s educational institute performance. Complexity measures can be derived from technical and transparency aspects of projects. However, quantifying these aspects is often uncertain and vague.

As a result, most of the traditional tools for modeling, reasoning and computing, which are crisp, deterministic, and precise in character, may not be suitable for complexity analysis in educational projects. In this study, a fuzzy relation, which is an element of a fuzzy graph, is proposed to define complexity relations among projects.

A fuzzy complexity graph composed of a set of fuzzy relations can be represented by a fuzzy matrix containing a list of all projects and the degree of membership of the fuzzy relative complexity. In a crisp situation, the relative complexity means in what degree project i is more complex than project j denoted by $P_i \rightarrow P_j$.

In uncertain situations, experts may express the fuzzy relative complexity between project i and project j by values in a range $[1-9]$ where the spectrum of the linguistic variables and corresponding values in responding to the question of if $P_i$ is more complex than $P_j$ can be expressed by linguistic variables associated with Triangular Fuzzy Number (TFN) in Table 1.

#### Tab. 1. Complexity term and TFN

<table>
<thead>
<tr>
<th>Fuzzy Number</th>
<th>Linguistic Variable</th>
<th>TFN</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Equally Complex</td>
<td>1,1</td>
</tr>
<tr>
<td>3</td>
<td>Weakly Complex</td>
<td>2,3</td>
</tr>
<tr>
<td>5</td>
<td>Essentially Complex</td>
<td>4,5</td>
</tr>
<tr>
<td>7</td>
<td>Very Strongly Complex</td>
<td>6,7</td>
</tr>
<tr>
<td>9</td>
<td>Absolutely Complex</td>
<td>8,9</td>
</tr>
</tbody>
</table>

By using a membership function, the degree of membership can be calculated for the fuzzy relative complexity obtained from experts. Since experts do not often agree on the relative complexity between projects, an aggregation operator is used to mitigate conflict of experts’ opinions. As a result, the fuzzy matrix is composed of the aggregated degrees of membership. By using the first and the second projects of the expected value of the fuzzy matrix, the fuzzy complexity graph can be mapped into a scaled Cartesian diagram. This diagram and the membership function are the elements for computing the coefficient factor used for comparing the educational projects.

### 3. Complexity Relations Under Fuzzy Situations

Consider the fuzzy relation $\tilde{R}$ that represents the relative complexity between the projects, Eq.(1). The crisp relative complexity $P_i \rightarrow P_j$ determines if project i is more complex than project j by a crisp number (e.g., No=0 and Yes=1 in a non-fuzzy situation). In a fuzzy situation, the relative complexity can be defined by a fuzzy number in Table 1.

Thus, the fuzzy relations are fuzzy subsets of $P_i \times P_j$ that is mapping from $P_i \rightarrow P_j$. Let $P_i, P_j \subseteq \tilde{R}$ be universal project sets, then $\tilde{R}$ is called a fuzzy relation on $P \times P$ (Figure 1).

$$\mu_{\tilde{R}}(P_i, P_j)$$

Fig. 1. Degree of complexity relation between $P_i$ and $P_j$.

$$\tilde{R} = \{ \mu_{\tilde{R}}(P_i, P_j) \mid P_i \rightarrow P_j \subseteq P \times P \}$$

(1)

To calculate the degree of membership of the relative complexity defined in Eq.(1), experts are required to express their opinions in what degree project i is more complex than project j by a fuzzy value in a range $[1-9]$ as depicted in Table 1. To resolve the conflict, the relative complexity between projects i and j can be calculated by

$$RC_j = \frac{(I' + A'I' + I')}{6}$$

(2)
Where $f,m,r$ are average values of TFNs associated with fuzzy numbers that are obtained from $k$ experts with respect to project $i$ and $j$. There are many functions for assigning the degree of membership to a fuzzy number (i.e., relative complexity). These functions must be convex and assign the degree of membership in a range $[0,1]$ [7]. Here, the membership function (Eq.3) is used for simplicity (Figure 2).

$$\mu_R(P_i,P_j) = \frac{1}{10}(RC_{ij})$$

Fig. 2. Membership function of complexity relation $P_i \rightarrow P_j$.

4. Complexity Model
A graph is made of up a crisp set of nodes and a set of edges. Sometimes a pair of nodes is connected by multiple edges yielding a multi-graph. When a node is connected to itself by an edge, it is called a loop, yielding a pseudo-graph as shown in Figure 3. Finally, edges can also be given a direction yielding a directed graph (or digraph).

Fig. 3. Typical graph (left), Loop (right)

The fuzzy complexity graph is a directed graph made up of a crisp set of nodes and a fuzzy set of relations. Generally, let $\mathcal{G}(V,\mathcal{R})$ be a complexity graph where $V = \{P_1, P_2, \ldots, P_n\}$ is a set of nodes representing projects and $\mathcal{R} = \left\{ \mu_{ij} : P_i \rightarrow P_j \subseteq P \times P \right\}$ is a fuzzy set of complexity relations between projects. This complexity graph can be presented by a square matrix $\mathcal{S}$ where its elements are $\mu_{ij}$ for all projects.

The expected value of the matrix $\mathcal{S}$ denoted by $\mathcal{S}'$ is required to find direct and indirect complexity relations among projects (Use Eq.(4)).

$$\mathcal{S}' = \lim_{n\to\infty} \mathcal{S}^n$$

Where $S^n_k = \max\{\min(s_{ik}, s_{kj}) \mid k = 1,2,\ldots,n\} \forall i, j$

The expected value of matrix $\mathcal{S}$ is equal to $\mathcal{S}'$ where $\mathcal{S}^n = \mathcal{S}^{n+1}$ (i.e., $s^n_k = s^{n+1}_k \forall i, j$) or ranking orders of the projects based on $\bar{\alpha}$ and $\bar{\beta}$ in $\mathcal{S}'$ and $\mathcal{S}^n$ are similar. Using $\mathcal{S}'$, the degree of membership of relative complexity of projects denoted by $\bar{\alpha}$ can be derived by the first projection, Eq.(5). Also, the second projection depicts the relaxation $\bar{\beta}$ of a project, Eq.(6).

$$\bar{\alpha} = \left\{ (P_i \rightarrow P_j, \max(m_i)) \mid P_i \rightarrow P_j \subseteq P \times P \ orall j \right\}$$

$$\bar{\beta} = \left\{ (P_i \rightarrow P_j, \max(m_i)) \mid P_i \rightarrow P_j \subseteq P \times P \ orall i \right\}$$

Thus, mapping the complexity graph in $\mu_{ij} = \mu_{ij}$ Cartesian diagram presents a scaled degree of complexity and of relaxation memberships. Considering the required budget, and soft and hard resources for a base project, this scaled graph can be used to assess the complexity, and estimate budget, soft and hard resources for other projects [8, 9, 10]. In this model, it is assumed a $\Delta$ difference in relative complexities of two projects translates to $\Delta\%$ difference in their budgets. The model can be implemented in the following steps:

- Step 1. List all projects
- Step 2. Obtain fuzzy numbers from $k$ experts describing the complexity relations among projects.
- Step 3. Aggregate the fuzzy numbers to a single TFN using Table 1 and calculating the average values for $l^i, m^i$, and $r^i$.
- Step 4. Using Eq.(2) and conflict resolution approach, calculate a fuzzy number representing relative complexity.
- Step 5. Construct fuzzy complexity graph.
- Step 6. Generate matrix $\mathcal{S}$ representing the fuzzy complexity graph.
- Step 7. Compute expected value of matrix $\mathcal{S}$ called $\mathcal{S}'$.
- Step 8. Find the first projection $\bar{\alpha}$ and the second projection $\bar{\beta}$ for all projects.
- Step 9. Calculate intensity and relation of projects.

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Step 10. Map the graph in scaled Cartesian diagram. 
Step 11. Use relative complexity measures for estimating required budget, soft and hard resources for projects based on a base project.

5. An Illustrative Example

To illustrate the model, a hypothetical example is presented in this section. Consider an educational institute with five workshop projects for a communication company. These projects require a budget and resources that can be estimated by using the relative complexity of the projects to the base project. On the other hand, the resource allocation must be performed based on the effectiveness of the workshops. Figure 4 shows the complexity graph for these workshops that has five nodes representing the workshops and fuzzy relations representing relative complexities.

Fig. 4. Complexity graph.

To determine the degrees of membership of the relative complexities, relative complexities among the workshops are obtained from three experts. Each expert is asked to determine in what degree workshop i is more complex than workshop j by a fuzzy value in the range [1-9] from Table 1. In this study, the experts not only do not agree on ranking order of the workshops based on complexity measures but also assign different values for the relative complexities among projects. Executing steps 1 to 5, Table 2 represent the matrix S. Using Eq.(4), the expected value of matrix S must be calculated. In this study, the expected value of matrix S can be reached at n=2 because the ranking orders of projects for n=1 and n=2 are similar.

Tab. 2. Matrix S representing fuzzy complexity graph

<table>
<thead>
<tr>
<th>Project</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0.067</td>
<td>0.867</td>
<td>0.967</td>
<td>0.967</td>
</tr>
<tr>
<td>2</td>
<td>0.067</td>
<td>0</td>
<td>0.9</td>
<td>0.067</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.1</td>
<td>0.033</td>
<td>0</td>
<td>0.067</td>
<td>0.033</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0.033</td>
<td>0.9</td>
<td>0</td>
<td>0.733</td>
</tr>
<tr>
<td>5</td>
<td>0.033</td>
<td>0.9</td>
<td>0.433</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 3 presents the expected value of matrix S. Using S’ in Table 4, the degree of membership of relative complexity for a project complexity denoted by α.

Tab. 3. Matrix S

<table>
<thead>
<tr>
<th>Project</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>Ratio</th>
<th>Normalized</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0.067</td>
<td>0.867</td>
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<td>0.967</td>
<td>1</td>
</tr>
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<td>0</td>
<td>0.9</td>
<td>0.067</td>
<td>0.967</td>
<td>0.967</td>
<td>0.967</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>0.1</td>
<td>0.033</td>
<td>0</td>
<td>0.067</td>
<td>0.033</td>
<td>0.9</td>
<td>0.931</td>
<td>0.103</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0.033</td>
<td>0.9</td>
<td>0</td>
<td>0.733</td>
<td>0.9</td>
<td>0.931</td>
<td>0.103</td>
</tr>
<tr>
<td>5</td>
<td>0.033</td>
<td>0.9</td>
<td>0.433</td>
<td>0</td>
<td>0</td>
<td>0.967</td>
<td>1</td>
<td>0.103</td>
</tr>
</tbody>
</table>

Tab. 4. Matrix S’

<table>
<thead>
<tr>
<th>Project</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>Ratio</th>
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<th>Ranking</th>
</tr>
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<tbody>
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<td>1</td>
<td>0</td>
<td>0.067</td>
<td>0.9</td>
<td>0.067</td>
<td>0.733</td>
<td>0.967</td>
<td>0.967</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0.067</td>
<td>0</td>
<td>0.9</td>
<td>0.067</td>
<td>0.733</td>
<td>0.9</td>
<td>0.931</td>
<td>0.103</td>
</tr>
<tr>
<td>3</td>
<td>0.1</td>
<td>0.033</td>
<td>0</td>
<td>0.067</td>
<td>0.033</td>
<td>0.9</td>
<td>0.931</td>
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</tr>
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<td>0.9</td>
<td>0</td>
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<td>0.931</td>
<td>0.103</td>
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<td>0.433</td>
<td>0</td>
<td>0</td>
<td>0.967</td>
<td>1</td>
<td>0.103</td>
</tr>
</tbody>
</table>

Based on Table 4, Figure 5 shows the mapped scaled Cartesian diagram that indicates project 1 is the most complex project. Also, project 3 and 4 are the least complex projects. Since the first projection values for projects 3 and 4 are similar, normalized α and β can be used for ranking projects 3 and 4. Assuming required budget and resources for project 3 are known, the required evaluation, budgeting and resource allocation for the other projects can be performed by using their relative complexity. For example, in Figure 6, the relative complexities for the degrees of membership of projects 1 and 3 are 0.967 and 0.1 that are corresponding to the degrees of membership 0.967 and 0.1 for projects 1 and 3, respectively.

Thus, the coefficient factor, Δ, is 1 in scale 1 to 10, which means 87% difference between relative complexities of projects 1 & 3 can be translated to 87% difference in their budgets and resources.
Fig. 5. Mapped complexity graph on \( \mu(i) \times \mu(i') \) axes

Fig. 6. Coefficient factor curve.

6. Conclusions

The complexity of educational projects can be studied through technical and transparency factors. Due to the lack of information, these factors may not be measured precisely. As a result, we proposed a fuzzy graph-based model that resolved conflict of experts’ opinion with respect to the relative complexity of projects in order to compute the complexity. The complexity measure can be used as a yardstick to either evaluate the required budget and resources for projects or as a comparative analysis among projects.

Having the degree of complexity membership function, the relative complexity relations can be presented by a graph and alternatively by a Complexity Matrix. The model employs a pseudo factor (relaxation) in order to map the graph into a scaled Cartesian diagram for a better pictorial view of the complexity relations. Having the degrees of relative complexity, one is able to calculate the coefficient factor that may be used as a yardstick for comparative analysis and estimating the budget and resources of a project with respect to the base project. For future work, one may use multi-layer graphs in which a layer represents one aspect of complexity.

References


