



## A Normative Dynamic Model of Regional Economy

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### KEYWORDS

Dynamic model,  
regional economy,  
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### ABSTRACT

*This paper presents a normative balance mathematical model of regional economy that contains a lot of unspecified parameters which are not defined directly by the data of economic statistics. A method for estimation of the model parameters by application of parallel computations on multi-processors systems is presented. It is determined the unknown parameters of economic model by indirect way, comparing time series for macro indexes calculated by model with statistical time series for these indexes. Use of the method is illustrated by the parameter estimation of a macroeconomic regional model by statistical data of Vyatka Region of Russia. The each production sector shadow money stock grows due to sale of shadow final product to households and as intermediate product to other sectors. Identified model is used for estimation of the Regional Government economic politics.*

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### 1. Introduction

Parallel calculations on modern multiprocessors computers open new opportunities in application of mathematical models for research of regional transition economy. Application of normative economic models was limited by complexity of parameter estimation for a lot of unknown parameters in such models. This work offers a technique for economic model parameter estimation and presents criteria of closeness and similarity for time series of economic parameters. They use these criteria for indirect estimation of unknown economic model parameters by comparison of calculated macroindexes with their statistical analogues. The simulation model of regional economy enables to receive a quantitative estimation of dynamics of macroindexes for regional economy [1]. It is required much less time for construction of normative balance dynamic models than for dynamic models based on solution of optimal problems. Presented here three sector normative balance mathematical model of a regional economy considers

the taxation and a shadow turnover. Dynamics of material and financial balances of the model is set by change of product stocks, production factors and money. This regional model has eight economic (representative) agents: three Production sectors ( $X$ ,  $Y$ ,  $Z$ ), Households ( $H$ ); Regional bank system ( $B$ ), Trade intermediary ( $T$ ), Regional Government ( $G$ ), Outside world ( $O$ ).

Production sectors in the regional economic model are presented by three sectors: ( $X$ ) the timber industry complex including forestry; ( $Y$ ) a complex of new innovation industries in biotechnology and the chemistry, including science and education; ( $Z$ ) combination of the remained industries.

The each production sector official stock of money grows due to credits, due to sale of the goods on a outside market and in the inner markets, due to transfers from the budget and receipts of the washed money from a shadow turn. It decreases due to payment to households, intermediate consumption, credits payments and transfers of taxes to the consolidated regional budget.

However the model contains about hundred equations and more than eighty parameters which cannot be defined directly from statistical data. It would seem that normative models of such type are absolutely

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senseless, as for carrying out of the intelligent qualitative and quantitative calculations on them it is necessary to estimate their external parameters, but the most part of the parameters cannot be estimated directly from economic statistical data. Even in a case when we have all statistical data the quality of them is as a rule such that they suffice only for determination of intervals for the model parameters. Besides, the initial values of some model variables are unknowns and consequently should be considered as such parameters as well. Moreover, even if all necessary statistics is available the quality of the data isn't always good.

Here the new technology of estimation for external parameters of the model is offered. It based on use of high-speed parallel calculations on multiprocessing systems. For parameter estimation of the model time series of regional economic macroindexes calculated by the model are compared to corresponding statistical time series of them. As criteria of affinity of time series the Theil index of discrepancy and recently constructed similarity criterion for nonlinear time series based on wavelet coefficients are used.

At construction of the model and its parameter estimation they solve problems of the analysis and structurization of initial data in conformity with model structure. So, at time of model construction the problem of a choice and the system analysis of initial statistical data is solved. Conclusions and the forecasts received by scenario calculations on simulation model basically can be experimentally denied, and hypotheses on which the model is based, can be challenged. It gives the opportunity to modify model in future and receive more proved forecasts by constantly improved model and make more proved analytical conclusions.

This paper gives an introduction to mathematical modeling of regional economy by normative balance dynamic model, and presents an application of the technique to economic analysis. The main purpose of the paper is to illustrate the method, rather than present a state-of-the-art analysis. Math modeling approach makes it possible to pursue the analysis further than possible with analytic methods, and it can yield qualitative as well as quantitative results.

## 2. Model Description

Regional economy production sectors ( $X$ ,  $Y$ ,  $Z$ ) use labor, the capital and intermediate products of adjacent production sectors. Production sectors deliver product on domestic market, on outer market, and on the market of intermediate product. It is considered that the prices are formed in each market of each product and change of the prices is in inverse proportion to change of stocks of corresponding products. Households  $L$  offer labor and consume final production.

Trading intermediary  $T$  redistributes material and financial flows. Bank system  $B$  emits money resources, gives out credits to production sectors. The regional

government  $G$  accumulates taxes from production sectors (the profit tax  $n_1$ , the value-added tax  $n_2$  and excises  $n_3$ , the uniform social tax  $n_4$ , the customs duties on export  $n_5$ ) and households (the customs duties on import  $n_6$ , the surtax  $n_7$ ) and adjusts charges of the budget.

To account of a shadow turnover in model, we assume, that production sectors divide the product on official and shadow one and that the last is not taxed. As a result each production sector has two kinds of money - "white" and "black". "Black" money can be washed, and the stock of not washed money is exposed to penal sanctions ("taxation" of shadow sector). All money of the consumer are considered "white", and the consumer divides the income by norms of consumption of official and shadow products of all sectors.

For simplicity standard designations will be used. Macroindexes and parameters of the model are supplied by the top and bottom indexes, and the top indexes are used to point agents, and bottom to point the goods. Thus the same symbols can be used as top or bottom index or can be stayed at normal place but they designate the different things. In the given work parameters and intensive indexes will be designated by small letters, and extensive macroindexes and the called indexes by capital letters. It is considered that distribution of a stock of each good is made by norm:  $a_i^{nm}$  - a share of a stock of the good  $i$  going from economic agent  $n$  to economic agent  $m$  ( $a_i^{nm} = a_i^n$ ). It is considered that distribution of money is made also under some norm:  $b_i^{mn}$  - a share of money stock going from agent  $m$  to agent  $n$  for a product  $i$  ( $b_i^{mn} = b_i^n$ ). Capital intensities also are set by some norms:  $c_{im}$  - agent  $m$  norm of expenses on product  $i$  for creation of one unit of capital product. Parameters of production functions of sectors are set by constants. For example, output  $Y_X(t)$  of product  $X$  produced by economic agent  $X$  (timber industry complex of the Vyatka Region, sector  $X$ ) is described by Cobb-Douglas production function of used production factors (stocks  $Q$ ): labor  $L$ , capital  $K$  and intermediate products from sectors  $Y$  and  $Z$ .

$$Y_X = (a_L^X Q_L^X)^{\delta_L^X} \cdot (a_K^X Q_K^X)^{\delta_K^X} \cdot (a_Y^X Q_Y^X)^{\delta_Y^X} \cdot (a_Z^X Q_Z^X)^{\delta_Z^X}, \quad (1)$$

where  $\delta_L^X + \delta_K^X + \delta_Y^X + \delta_Z^X = 1$ . Hereinafter, all indexes  $Y_X(t)$ ,  $Q_L^X(t)$ ,  $Q_K^X(t)$ ,  $Q_Y^X(t)$ ,  $Q_Z^X(t)$  are considered as functions of time  $t$ , therefore this argument falls in formulas. All parameters, as a rule, are considered as constants, as here  $a_L^X, a_K^X, a_Y^X, a_Z^X, \delta_L^X, \delta_K^X, \delta_Y^X, \delta_Z^X \in (0, 1)$ . Agent  $X$  produces official product  $X$  and shadow product  $V$  by the common capital, the common labor and the common stocks of intermediate products, and made product (output)  $Y_X(t)$  shares in fix proportion  $(1 - q_X) : q_X$  on open ("white") output  $X$  and shadow ("black") output  $V$ , where  $q_X$  is a shadow share in the common output of the product  $Y_X(t)$ . The shadow

product is used for sale to population and other sectors. It is assumed in the model that investments can be official only. The stock of open product  $Q_X^X(t)$  increases due to production and decreases due to shipment to agents  $Y, Z, L$  (ships to the intermediary by the norm) and for investments  $I_X(t)$ . It is considered that charges on investments from the own product coincide with incomes of them. There is a fix share of own product stock that goes on outer market  $X_X^{XO}(t)$ .

$$\frac{dQ_X^X}{dt} = (1 - q_X)Y_X - (a_X^{XL} + a_X^{XY} + a_X^{XZ} + a_X^{XO})Q_X^X - c_X^X I_X, \quad (2)$$

Where

$$I_X = (b_K^X W^X) / (p_X^X c_X^X + p_Y^X c_Y^X + p_Z^X c_Z^X), p_X^X = \min\{p_X^Y, p_X^Z\}, X_X^{XO} = a_X^{XO} Q_X^X \quad (3)$$

The shadow share  $q_X$  in total output defines a gain of stocks both opened  $Q_X^X(t)$  and shadow  $Q_V^X(t)$  products. The stock of shadow product  $V$  in sector  $X$  decreases due to deliveries to households and allied sectors.

$$\frac{dQ_V^X}{dt} = q_X Y_X - (a_V^{XL} + a_V^{XY} + a_V^{XZ})Q_B^X \quad (4)$$

The stock of open ("white") money  $W_X(t)$  of economic agent grows when agent  $X$  takes bank credits, sales the goods on outer market, sales the goods on domestic markets, takes transfers from regional consolidated budget  $T^{GX}(t)$  and receipts of the "washed" money from a shadow turnover  $b_B^{XB}(t)$ . It decreases due to wages payments to households  $L$ , due to consumption of intermediate products of sectors  $Y$  and  $Z$ , due to payments on credits and transfers of taxes to the consolidated budget.

$$\frac{dW^X}{dt} = wp_X^O X_X^{XO} + C^{BX} + (p_X^L a_X^{XL} + p_X^Y a_X^{XY} + p_X^Z a_X^{XZ})Q_X^X - (b_Y^{XY} + b_Z^{XZ} + b_W^{XY} + b_U^{XZ} + b_L^{XL} + b_H^{XB})W^X - T^{XG} + T^{GX} + b_B^{XB} W^X, \quad (5)$$

where  $w(t)$  is Ruble/USD exchange rate,  $TXG(t)$  is transfer payments to the consolidated regional budget (equal the sum of taxes),  $TGX(t)$  is transfer from the budget to sector (VAT returns plus grants and so on). Deductions of agent  $X$  to the consolidated budget  $TXG(t)$  develop from the profit tax  $T1XG(t)$ , the added value tax  $T2XG(t)$ , the excises  $T3XG(t)$ , the uniform social tax  $T4XG(t)$ , to customs duties on export  $T5XG(t)$ .

$$T^{XG} = T_1^{XG} + T_2^{XG} + T_3^{XG} + T_4^{XG} + T_5^{XG}, \quad (6)$$

Where

$$T_5^{XG} = n_5 wp_X^O X_X^{XO}, T_4^{XG} = n_4 b_L^{XL} W^X, \quad (7)$$

$$T_4^{XG} = n_4 b_L^{XL} W^X$$

$$T_3^{XG} = n_3^X [wp_X^O X_X^{XO} + (p_X^L a_X^{XL} + p_X^Y a_X^{XY} + p_X^Z a_X^{XZ})Q_X^X] \quad (8)$$

$$(b_Y^{XY} + b_Z^{XZ} + b_H^{XB})W^X - T_3^{XG} - T_4^{XG} - T_5^{XG} \} \\ T_2^{XG} = n_2 \{wp_X^O X_X^{XO} + (p_X^L a_X^{XL} + p_X^Y a_X^{XY} + p_X^Z a_X^{XZ})Q_X^X - \quad (9)$$

$$T_1^{XG} = n_1 \{wp_X^O X_X^{XO} + (p_X^L a_X^{XL} + p_X^Y a_X^{XY})Q_X^X - \\ -(b_Y^{XY} + b_Z^{XZ} + b_H^{XB} + b_L^{XL})W^X - T_2^{XG} - T_3^{XG} - T_4^{XG} - T_5^{XG} \} \quad (10)$$

The stock of shadow money grows by sale of a shadow product as final product to households  $L$  and, as intermediate product, to sectors  $Y$  and  $Z$ , a part  $b_B^{BX}$  of shadow money stock has time to wash, a part  $b_B^{XG}$  gets as penal sanctions in a profitable part of the regional government consolidated budget, and a part  $b_B^{XL}$  goes to the households as shadow incomes.

$$\frac{dB^X}{dt} = (p_V^L a_V^{XL} + p_V^Y a_V^{XY} + p_V^Z a_V^{XZ})Q_V^X - (b_B^{XL} + b_B^{XG} + b_B^{XB})B^X \quad (11)$$

It is considered that agent  $X$  takes all the credit offered by regional bank system  $B$ , however the volume of given credit  $C^{BX}(t)$  is limited by liquidating cost of production capital which is considered proportional the capital stock:  $C^{BX} = \sigma^X Q_K^X$ ,  $\sigma^X > 0$ . Debts  $Z^X(t)$  of agent  $X$  to bank system  $B$  grows due to delivery of new credits  $C^{BX}(t)$  and charges of current interest rate  $r(t)$  on an available debts, and decreases due to repayment flow  $H^{XB}(t)$ .

$$\frac{dZ^X}{dt} = C^{BX} + rZ^X - H^{XB}, \quad H^{XB} = b_H^{XB} W^X \quad (12)$$

It is easy to write equations for sectors  $Y$  and  $Z$  by analogy to equations (1) - (12) for sector  $X$ .

Change of stock for final product  $X$  of timber industry complex of the Vyatka Region, intended to agent  $L$ ,  $Q_X^L(t)$  defines change of a consumer price index  $p_X^L$  on product  $X$ .

$$\frac{dQ_X^L}{dt} = a_X^{XL} Q_X^X - \frac{b_X^{LX} W^L}{p_X^L}, \\ \frac{dp_X^L}{dt} = \alpha_X^L \left( \frac{b_X^{LX} W^L}{p_X^L} - a_X^{XL} Q_X^X \right). \quad (13)$$

Changes of stocks of all other products in all markets and change of the corresponding prices are similarly defined. Thus we assume, that the product of the same sectors acts in other quality in the different markets and has the other price, that it is, as a matter of fact, other product.

It is considered that increase of the open and shadow wages can occur both at shortage of the labour, and due to increase of consumer prices on production of the sector. For example, the open wage at sector  $X$  is determined by equation

$$\begin{aligned} \frac{ds_L^X}{dt} = & \left[ \alpha_L^X \left( \frac{b_L^{XL} W^X}{s_L^X} - a_L^{LX} Q_L^{LX} \right) \right. \\ & \left. + \frac{\beta_L^X s_L^X}{p_X^L} \left( \frac{b_X^{LX} W^L}{p_X^L} - a_X^{XL} Q_X^{XL} \right) \right]_+ \end{aligned} \tag{14}$$

The stock of money on the regional consolidated budget accounts increases by tax revenues and decreases by transfers to production sectors and households.

$$\begin{aligned} \frac{dW^G}{dt} = & b_V^{XG} B^X + b_W^{YG} B^Y + b_U^{ZG} B^Z + T^{XG} \\ & + T^{YG} + T^{ZG} + T^{LG} \\ & - (b_X^{GX} + b_Y^{GY} + b_Z^{GZ} + b_L^{GL}) W^G \end{aligned} \tag{15}$$

The regional bank system is not closed, and the branches of the Russian banks from other regions play greater role in investment decisions. Therefore as the first step to description of considered regional bank system it was considered that a part gold and exchange currency reserves of the Russian Federation provide reservation for actives of the region.

### 3. Model Identification

For an application of a method for macroeconomic model parameter estimation based on parallel processing some measure of similarity between two time-series is required and so a new wavelet based measure of similarity was elaborated. Complicated economic models with a lot of parameters can be accurately estimated by the proposed method.

A macroeconomic model usually contains a lot of unspecified parameters. Some variables of the macroeconomic model have initial values that are sometimes unknown and should be considered as parameters as well. In most cases mentioned parameters can't be defined on the basis of economic statistics. Moreover, even if all necessary statistics is available the quality of the data isn't always good. That's why only confidence intervals for the unknown

parameters can be computed from the data. For estimation of the unknown parameters time-series for some macro-indexes (calculated by the model) and statistical time-series for these macro-indexes should be compared by means of some measure of similarity. The unknown parameters can be determined implicitly as those parameters, which provide minimum value of the used measure of similarity. Parallel processing on a cluster of workstations or on a supercomputer enables to perform exhaustive search of the parameters within their confidence intervals (determined either from economic sense or from the available statistical data) and estimate their values for reasonable time. In order to use parallel processing the macroeconomic model should be divided into blocks so that parameter estimation of any block can be done independently of the other blocks. In this case not only real statistical data, but also data calculated from some block can be used as values for external variables of the other blocks.

Estimated values of the parameters give new knowledge about the macroeconomic system under investigation and this knowledge can force a researcher to modify the model. Thus macroeconomic model parameter estimation based on parallel processing becomes a powerful tool for mathematical modeling of economic systems.

It was mentioned that calculated and statistical time-series for some macro-indexes should be compared on the basis of some measure of similarity. As used here, two time-series are considered to be similar if they are close as functions of time (in other words, if there is a strong, possibly nonlinear dependence between two time-series). Convolution of the Theil's index and specially designed wavelet based measure of similarity was used as a characteristic of closeness between two time-series.

With linear trend appropriate rescaling with respect to ordinate axe two completely different time-series can become quite similar and the value of Euclidian distance between these time-series can decrease significantly. Such effect frequently occurs in many real situations.

Thus the similarity between two time-series should be based on certain characteristics of these time-series rather than on the raw values. These characteristics should be calculated in linear time, robust for changes in level, scale and trend of the time-series and the similarity between two time-series is simply given by the (traditional) similarity of the corresponding characteristics.

It is proposed to compute such characteristics on the basis of discrete wavelet transform. In that case these characteristics have all necessary properties. It is recommended to use Daubechies wavelet and scaling filters. If that's the case wavelet coefficients are robust for linear trend. That is if the time-series of a macro-index is simply a linear function of time, all wavelet coefficients are zero.

The described approach of exhaustive search for unknown parameters enables to find the solution, however time of calculations is rather big. A reduction of computation time can be achieved by means of special computation algorithms.

Macroeconomic model should be divided into blocks so that parameter estimation of any block can be done independently. Confidence intervals for parameters should be determined either from economic sense or from statistical estimation based on the real data.

Let  $\tilde{W}_{j,k}^X$  and  $\tilde{W}_{j,k}^Y$  are vectors of spectral characteristics of time series of identical length  $X$  and  $Y$  accordingly. As a measure of affinity between time series it is possible to use cosines of angle between vectors of characteristics [2], namely

$$D(X, Y) = \cos \alpha = \left| \sum_{j,k} \tilde{W}_{j,k}^X \cdot \tilde{W}_{j,k}^Y \right| \quad (16)$$

The introduced similarity index  $D(X, Y)$  characterizes how much strong is nonlinear dependence between time series  $X$  and  $Y$  (in other words, it shows how much fluctuations of time series  $X$  and  $Y$  are similar in relation to their linear trends). The similarity index is a measure of force and an communication orientation between compared time series and, the more it is closer to unit, the more similar is behavior of these series. The Theil  $E(X, Y)$  measures discrepancy of time series  $X_t$  and  $Y_t$  and the more close it to zero, the more close compared numbers. For convenience of calculations, instead of Theil index we shall use an affinity index  $U(X, Y) = 1 - E(X, Y)$ .

$$U(X, Y) = 1 - \frac{\sqrt{\sum_{t=t_0}^T (X_t - Y_t)^2}}{\sqrt{\sum_{t=t_0}^T X_t^2 + \sum_{t=t_0}^T Y_t^2}} \quad (17)$$

For unambiguity of choice for optimal point it is possible to use some convolution of affinity and similarity indexes for time series of macroindexes. For example, if for all macroparameters the adjustment of estimated by model and statistical data has about equal importance, it is possible to maximize compound value of all indexes.

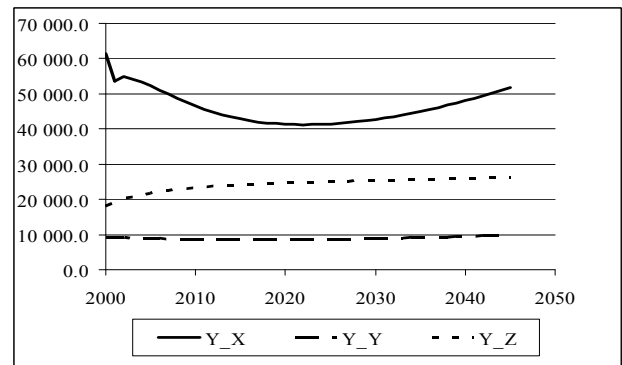
Thus it is necessary to touch only those variants of values of parameters, at which affinity and similarity indexes are above some fix positive values.

Now full search on all unknown parameters, set on an interval, even on the most powerful supercomputers is impossible, as search on a grid from 10 points on each interval on 80 parameters gives 1080 variants, that

approximately is equal to number of nucleons in the Metagalaxy. Means it is necessary to use the directed search, to reduce number of independent parameters due to additional assumptions and to use decomposition of model on rather independent blocks, identification of parameters in which can be made independently, using fixed parameters of other blocks. For reduction of number of independent parameters we shall define parameters of the production functions separately on the basis of statistical data and besides we shall exclude a part of parameters due to the assumption about general equilibrium for initial date. It was possible to reduce number of independent parameters down to 9 if the similar by economic sense parameters of production sectors are equaled to each other. Calculations on supercomputers [3] of Joint Super Computer Centre of Russian Academy of Sciences were made with nine independent parameters.

#### 4. Numerical Experiments with Model

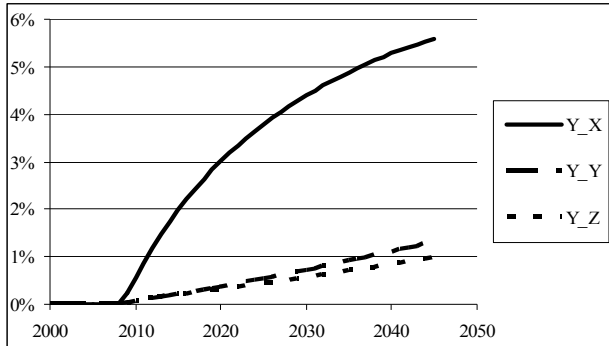
Numerical experiments with model were spent to find the efficient variant qualitatively truly reflecting processes, occurring in economy of the Vyatka Region. Numerical experiments have shown working effectiveness of full model and its separate parts (Figure 1). It means that the model can be used in the further work. External parameters of this variant can be taken for a basis for more exact identification of parameters of model in the future, and to use the variant as base of numerical qualitative scenario calculations with model.



**Fig. 1. Production outputs in production sectors of Vyatka**  
( $Y_X$  – output of sector X,  $Y_Y$  – output of sector Y,  $Y_Z$  – output of sector Z)

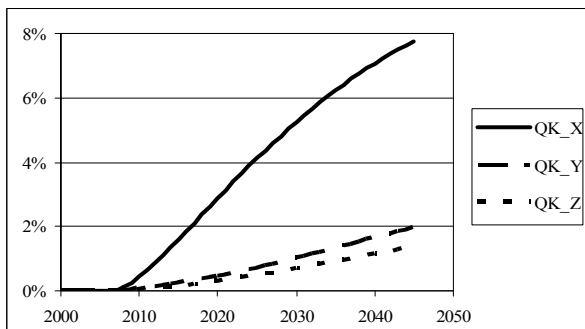
Changes in scenario calculation in comparison with the base variant can be represented by a variation of change of the macroindexes expressed in percentage. Presented here investigation of economic integration by model of regional economy consists of only one scenario (Figures 2- 7). The scenario studies the influence of economic integration on customs duties: custom duties of production sectors on export  $n_5$  and custom duties of households on import  $n_6$  reduce in model approximately twice from the beginning of year

2008 (In scenario numerical experiment with the model of Vyatka Region economy custom duties on export  $n_5$  reduces from 0.12 to 0.08 and custom duties on import  $n_6$  reduces from 0.17 to 0.08).



**Fig. 2. Variations for outputs of regional production sectors ( $Y_X, Y_Y, Y_Z$  – output of sector X, Y, Z)**

For Figure 2 one can see that these changes give increase for all outputs of three production sectors of Vyatka Region economy: output of export oriented sector X increased on 5.5%, but also increased on 1% the outputs for sectors Y and Z dependent on import concurrency. It is connected with increase of demand for intermediate production from sector X.



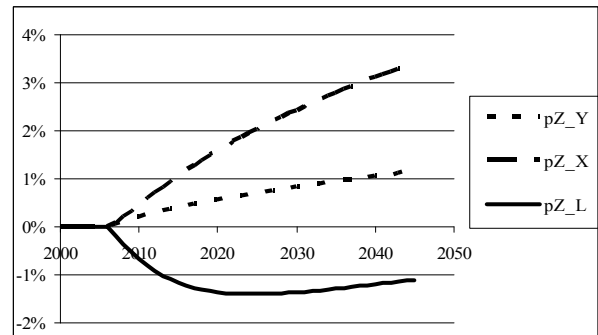
**Fig. 3. Variations for capitals of regional production sectors (capital of sector X, Y, Z)**

Figure 3 show the increase of capital in all production sectors. In Figure 4 one can see that the price indexes vary on a miscellaneous in the different domestic markets. Price index for households in scenario of integration is decreased on 1.5% in comparison with a base variant of calculation and price indexes of intermediate products for sectors are increased. In the Figure 4 one can see that price index on product Z for sector X is increased twice more than price index on product Z for sector Y. Note that changes of price indexes on outer markets are not considered in the scenario. The decrease of consumer price indexes conducts to decrease in all wage indexes on all channels of realization: both on official and on shadow ones (Figure 5). From Figure 5 one can see that the wage indexes for shadow channels decrease not so much as them do for official channels, and differently by sector: the more big decrease is for export sector X (official wage for official product X and shadow wage

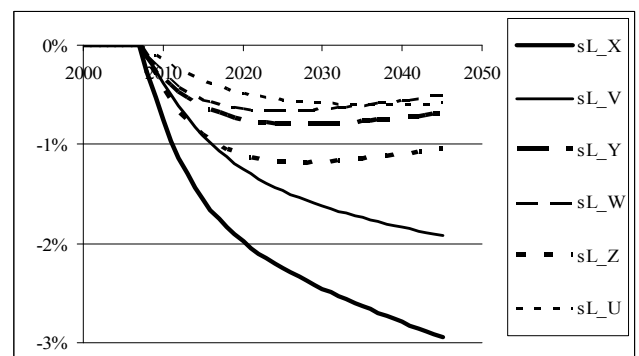
for informal product V), less for innovative sector Y (official wage for official product Y and shadow wage for informal product W), and less for sector Z (official wage for official product Z and shadow wage for informal product U). As you can see from Figure 6 black and white money stocks of sectors are increased. Figure 7 shows that tax revenue of regional budget in absolute prices is decreased.

**5. Conclusion**

The technology of model external parameter estimation offered in work is based on use of high-speed parallel calculations on multiprocessing systems, reduction of number of independent parameters due to the assumption of regional economy equilibrium in the initial moment of calculation and a grouping of parameters by their economic sense. Numerical experiments with the identified model of regional economy have shown, that dynamics of macroindexes of Vyatka Region economy essentially depends on the policy spent by the regional government.



**Fig. 4. Variations for price indexes on product Z markets ( $pZ_Y$  – price index on Z for sector Y,  $pZ_X$  – price index on Z for sector X,  $pZ_L$  – price index on Z for households L)**



**Fig. 5. Variations for wage indexes at production sectors ( $sL_X$  – wage index for formal part of labour force at sector X,  $sL_V$  – wage index for informal part of labour force at sector X,  $sL_Y$  – wage index for formal part of labour force at sector Y,  $sL_W$  – wage index for informal part of labour force at sector Y,  $sL_Z$  – wage index for formal part of labour force at sector Z,  $sL_U$  – wage index for informal part of labour force at sector Z)**

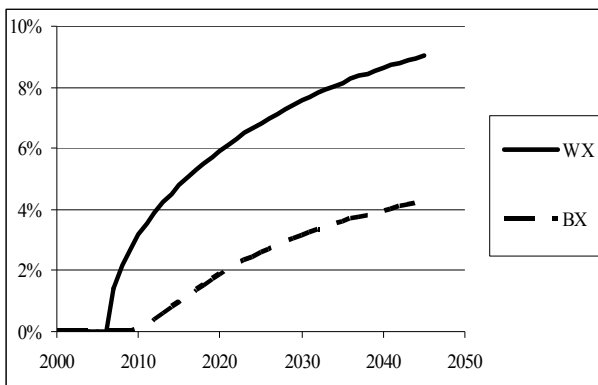
The model constructed here is an innovative product and the experience received on its parameter estimation is those "know-how" which can be used in the adaptation of the given model for concrete regional economic systems. Estimated values of the parameters give new knowledge about the macroeconomic system under investigation.

This knowledge forces a researcher to modify the model. Thus, macroeconomic model parameter estimation based on parallel processing becomes a powerful tool for mathematical modelling of economic systems. These type models of regional economies can be used to study trade interactions of regions. The integration scenario calculated by numerical experiments with presented model convincingly shows that reduction of the customs duties as a result of integration increases outputs of production sectors, reduces a price level, and gives the increase of competitive advantages of region.

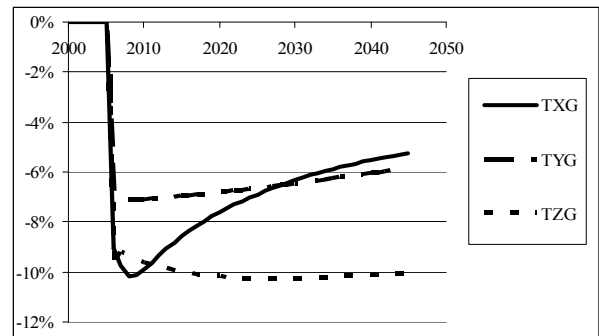
The economic model presented at the paper can be used not only for investigation of this Russian region but also for Iranian economy and economy of its regions.

At first we should separate the Iranian economy or economy of a Iranian Region on three main production sectors, should consider taxation system accepted in Iran. For calibration the economic model on statistical data for Iran or the Iranian region we need in time series for such macroindexes as: gross domestic product (GDP), implicit deflator of GDP, gross production of economic sectors, price indexes for these sectors, consumer price index, demographic macroindexes.

Then we should modify program for identification of this model by parallel calculations on supercomputer. Identified model can be used in research of economic consequences of accepted decisions by Government or Iranian Bank system authority.



**Fig. 6. Variations for money of regional production sector X (WX – white money of sector X, BX – black money of sector X)**



**Fig. 7 Variations for regional government budget income from production sectors (TXG – regional government budget income from sector X, TYG – regional government budget income from sector Y, TZG – regional government budget income from sector Z)**

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