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Single-Setup-Multiple-Deliveries for a Single Supplier-Single Buyer with Single Product and Backorder

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particle swarm optimization

ABSTRACT

This article investigates integrated production-inventory models with backorder. A single supplier and a single buyer are considered and shortage as backorder is allowed for the buyer. The proposed models determine optimal order quantity, optimal backorder quantity and optimal number of deliveries on the joint total cost for both buyer and supplier. Two cases are discussed: single-setup-single-delivery (SSSD) case and single-setup-multiple-deliveries (SSMD) case. Two algorithms are applied for optimizing SSMD case: Gradient search and particle swarm optimization (PSO) algorithms. Finally, numerical example and sensitivity analysis are provided to compare the total cost of the SSSD and SSMD cases and effectiveness of the considered algorithms. Findings show that the policy of frequent shipments in small lot sizes results in less total cost than single shipment policy.

1. Introduction

The JIT concept was first introduced and adopted in Toyota Motor Corporation that led to a higher quality, lower cost and substantially less labour time than what is achieved by Toyota's competitors [1]. Since the importance of just -in-time (JIT) was recognized in the early 1980s, there have been numerous studies discussing implementation of JIT and its effectiveness in the US manufacturing firms from various dimensions. All researchers have a consensus on the notion that JIT is an overall organizational phenomenon and the greatest possible gains from JIT can be achieved when JIT practices operate as an integrated system. Researchers have studied small lot sizing as a means of implementing successful JIT, with the buyer–supplier coordination focusing on material flows with an objective of minimizing supply chain costs. Small lot sizing improves the system's productivity by obtaining lower levels of inventory and scrap, lower inspection costs for incoming parts, and earlier detection of defects, etc., even though possible higher delivery costs and loss of discount rates may be incurred.

The idea of joint optimization for buyer and supplier was initiated by Goyal and later reinforced by Monahan, Lal and Staelin, Lee and Rosenblatt, Banerjee, Joglekar, and Dada and Srikanth[2]-[9]. Goyal and Gupta provided a review of many integrated models for buyer–supplier coordination [10]. While these studies focused on joint lot sizing and buyer–supplier coordination, the issue of frequent deliveries in small quantities was overlooked. Taking a different path, Pan and Liao, Larson, and Ramasesh developed EOQ-based models to discuss the effect of frequent deliveries on total costs [11], [13]. Their studies, however, failed to consider the issue of coordination from an integrated standpoint. In 1995, Lu proposed an optimal solution procedure determining production and delivery lot sizes simultaneously in a single-manufacturer–single retailer system under an assumption that the production lot size is an integer

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multiple of the delivery lot size. Actually, he considered a joint economic lot sizing (JELS) problem, which determines production and delivery lot sizes simultaneously in a manufacturer–retailer supply chain [14].

Goyal and Hill considered unequal-sized delivery lot policies postulating that time intervals between successive deliveries and delivery quantities may vary according to the predetermined delivery pattern [15]-[17]. Kim and Ha proposed a model for frequent deliveries taking into consideration the aggregate total relevant cost of both buyers and suppliers. Later, in 2003, they developed a novel cooperative model for enhancing the linkage between buyer and supplier [18] and [19]. References [20] and [21] considered successive shipment sizes increased by a fixed factor when the vendor's holding cost is larger than the buyer's. Ben-Daya and Zamin, Huang, and Lin considered unreliability process on JELS [22]-[24]. Under the single-vendor and single-buyer environment along with the assumption that there are imperfect items with identical quantities delivered from supplier to buyer, Huang was able to develop a method to find an optimal solution for an integrated production-inventory model that minimizes the total joint annual cost for a just-in-time (JIT) manufacturing system [23]. In 2010, Lin presented a new inventory model by considering multiple deliveries for items with imperfect quality and quantity discounts where buyer has exerted power over its supplier has developed. The order quantity is manufactured at one setup and is shipped over multiple deliveries [24]. Lin *et al.* and Ho also considered Single-vendor and single-buyer for a single product inventory model.

They presented these models for JIT-lot splitting and products with imperfect quality [25]- [26]. Ben-Daya and Zamin considered a JELS problem under equal-shipment policy with stochastic demand [27]. In 2003, Kelle *et al.,* and David and Eben-Chaime explored the partnership and the negotiation mechanism between the manufacturer and the retailer in terms of lot sizing and delivery scheduling in the same supply chain structure [28] and [29]. In the case where multiple items are produced in a single facility, Kim *et al.* proposed a joint production–delivery policy that determines an optimal common production cycle and delivery lot sizes under the assumption that the production lot size of each item is an integer multiple of its corresponding delivery lot size[30]. Diponegoro and Sarker developed an ordering policy for raw materials and determined an economic batch size for a product in a manufacturing system that supplies finished products to customers for a finite planning horizon. Fixed quantities of finished products are delivered to customers frequently at a fixed interval of time [31].

Yang and Wee, *Law and Wee,* Lo *et al.,* Jong and Wee, Yan *et al.* and Wang *et al.* built integrated inventory model subject to multiple deliveries for deteriorated products. They found that the cost could be significantly reduced via integration and lot-splitting effects with JIT implementation [32]-[37]. In 2011, Glock studied the coordination of a supplier network in an integrated inventory model. Specifically, they consider a buyer sourcing a product from heterogeneous suppliers and tackle both the supplier selection and lot size decision with the objective to minimize total system costs by considering multiple deliveries[37]. Chang *et al.* developed the model of Kim and Ha (2003) by considering multiple buyer [38]. Recently, particle swarm optimization (PSO) algorithm has been introduced by Russell Eberhart and James Kennedy in 1995[39]. This algorithm is a population-based search algorithm based on the simulation of the social behaviour of birds within a flock. In PSO, individuals, referred to as particles, are "flown" through hyper-dimensional search space. Changes to the position of particles within the search space are based on the social-psychological tendency of individuals to emulate the success of other individuals. The changes to a particle within the swarm are therefore influenced by the experience, or knowledge, of its neighbours.

In this study, we extend Kim and Ha (2003)'s model by permitting backorder for the buyer. The purpose of this study is to develop an integrated JIT lot-splitting model, which determines the optimal order and backordering quantities and number of deliveries. For determining these quantities, we use Gradient search and particle swarm optimization (PSO) algorithms. We limit our discussion to a simple JIT environment, i.e., single buyer and single supplier, under deterministic conditions for a single product with backordering for buyer. By comparing integrated total costs, we examine the benefits of the proposed JIT lot-splitting policy of facilitating multiple deliveries over the lot-for-lot delivery policy. We show that the policy of frequent shipment in small lot size results in less total cost than single shipment policy. The study is organized as follows: In Section 2, assumptions and notations are provided. Section 3 and 4 develop a lot-splitting (single setup, single delivery and single setup, multiple deliveries) models and how the optimal policy for buyer and supplier can be achieved for each case. In Sections 5 and 6, numerical example and sensitivity analysis are presented. Conclusions are summarized in Section 7.

2. Assumptions and Notations

An integrated approach allows the buyer and the supplier to reduce their total costs as compared to non-integrated approach. Cost savings accrued through the integration can be shared by both parties in some equitable fashion. In this study, we developed Kim and Ha (2003)'s model by allowing backorder for the buyer. When the buyer orders the quantity Q, the supplier can pursue one of the following policies: (1) single setup and single delivery (SSSD). (2) Single setup and multiple deliveries (SSMD). In SSMD case, the vendor actually holds the buyer's inventory, which is pushed back to him due to the small delivery lot size. Upon setting up the long-term agreements between a buyer and a supplier, the annual demand for the buyer is known to the supplier.

2.1. Notations

The following notations are considered:

A = ordering cost for buyer

S = setup time for supplier

C = hourly cost for supplier's setup time

F = fixed transportation cost per delivery

V =unit variable cost for order handling or receiving each item

H_B = holding cost/unit/yr, for buyer

H_S = holding cost/unit/yr, for supplier ($H_B \succ H_S$)

π = back ordering cost /unit/yr

D = annual demand rate for buyer

P = annual production rate for supplier ($P \succ D$)

N = number of deliveries per production lot (integer number and Decision variable)

Q= order quantity for buyer/Production lot size (Decision variable)

q = Shipment lot size ($q = Q/N$)

B =backorder quantity for buyer (Decision variable)

b = backorder quantity per shipment for buyer ($b = B/N$)

2.2. Assumptions

1. There is a single supplier and single buyer for a single product.
2. The demand rate, production rate and delivery time are constant and deterministic.
3. It is assumed that $P \succ D$. If $P \prec D$, then the problem would be infeasible since we cannot satisfy the demand in full.
4. All cost parameters are known and constant.
5. It is never optimal to send any shipments while the buyer has some inventory, since we assume that $H_B \succ H_S$.
6. Unit price is fixed and thus no quantity discount is assumed.
7. Shortage is acceptable and completely backordered for the buyer.
8. There are no constraints on the number and size of transportation vehicles.
9. The buyer is assumed to pay transportation and order handling costs in order to facilitate frequent deliveries.
10. The transportation and receiving cost is considered to be a linear function of the shipped quantities at a fixed cost.
11. Order quantity and backorder quantity are continuous real numbers. Therefore this model is applicable for products such as sugar, tea, cereal and so on.
12. Number of deliveries is an integer number.
13. In the SSSD case, the supplier makes the production set up every time the buyer places an order and supplies on a lot for lot basis.
14. In the SSMD case, the buyer places an order and the supplier splits the order quantity into small lot sizes and delivers them to the buyer in equal shipments.
15. In each setup, supplier manufactures Nq product.
16. Lead time is zero.

3. Single-Setup-Single-Delivery Model

We present in this section the conventional lot for lot inventory model as a benchmark for our proposed model. In lot for lot, the optimal lot size is produced at one setup and delivered at one time.

The buyer's total cost consists of an ordering cost, a holding cost, a backordering cost, and a transportation cost:

$$TC(Q,B,N)_{Buyer} = A\frac{D}{Q} + H_B\frac{(Q-B)^2}{2Q} + \pi\frac{B^2}{2Q} + (F+VQ)\frac{D}{Q} \quad (1)$$

The supplier's cost function includes a set up cost and a holding cost:

$$TC(Q,B,N)_{Supplier} = CS\frac{D}{Q} + H_S\frac{QD}{2P} \quad (2)$$

The total cost function for an integrated inventory model includes all costs from both buyer and supplier; so, by Summing equations (1) and (2), we find the aggregate total cost as follows:

$$TC(Q,B,N)_{Aggregated} = (A+CS)\frac{D}{Q} + H_B\frac{(Q-B)^2}{2Q} + H_S\frac{QD}{2P} + \pi\frac{B^2}{2Q} + (F+VQ)\frac{D}{Q} \quad (3)$$

From the aggregate total cost in Eq. (3), we now determine the optimal order quantity and the optimal back order quantity. By taking the first derivatives of Eq. (3) with respect to B and Q, setting them equal to zero, and solving for B and Q simultaneously, we obtain the following formulas:

$$Q^* = \sqrt{\frac{2D(A+F+CS)(\pi+H_B)}{H_S(\pi+H_B)\frac{D}{P} + \pi H_B}} \quad (4)$$

$$B^* = \frac{H_B Q^*}{\pi + H_B} \quad (5)$$

4. Single-Setup-Multiple-Deliveries Model

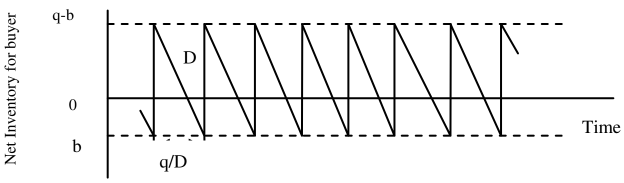
In SSMD model, the order quantity, Q, is produced at one setup but delivered in small quantities, q, over N times, i.e., $Q = N.q$. Small lot sizing is a means of implementing successful JIT.

Without loss of generality, we assume the multiple deliveries are to be arranged in such a way that each succeeding delivery arrives at the time that buyer has **b** backorder quantity. All The total cost for buyer is:

$$TC(Q,B,N)_{Buyer} = A\frac{D}{Q} + H_B\frac{(Q-B)^2}{2QN} + \pi\frac{B^2}{2QN} + (F+\frac{VQ}{N})\frac{DN}{Q} \quad (6)$$

The supplier's total cost is the sum of the setup cost and the holding cost.

a.



b.

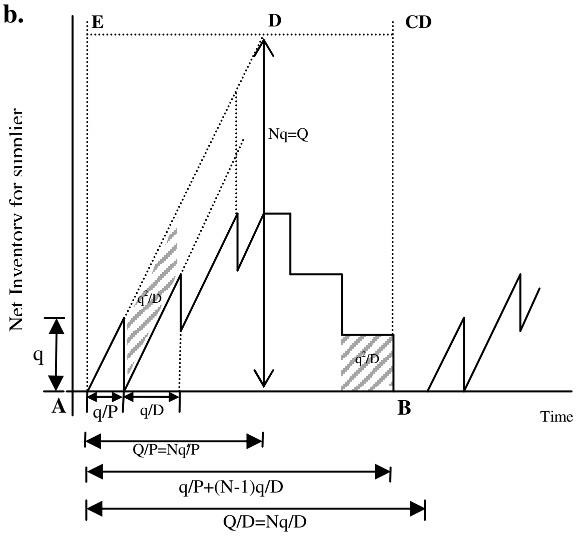


Fig. 1. Inventory time plot for SSMD model (one setup six deliveries)

From Fig.1.b the holding cost of supplier is derived as following:

$$H_S\frac{D}{Nq}\left\{S_{ABCE} - S_{ADE} - \left(\frac{q^2}{D} + 2\frac{q^2}{D} + ... + (N-1)\frac{q^2}{D}\right)\right\} =$$

$$H_S\frac{D}{Nq}\left\{Nq\left(\frac{q}{P} + (N-1)\frac{q}{D}\right) - \frac{N^2q^2}{2P} - \left(\frac{q^2}{D}(1+2+...+(N-1))\right)\right\}$$

$$= H_S\frac{D}{Nq}\left\{\frac{Nq^2}{2D}\left((2-N)\frac{D}{P} + N - 1\right)\right\} = H_S\frac{Q}{2N}\left\{(2-N)\frac{D}{P} + N - 1\right\}$$

Therefore, the supplier's total cost is:

$$TC(Q,B,N)_{Supplier} = CS\frac{D}{Q} + \frac{QH_S}{2N}\left\{(2-N)\frac{D}{P} + N - 1\right\} \qquad N \geq 2 \quad (7)$$

And, the aggregate total cost function is:

$$TC(Q,B,N)_{Aggregated} = (A+CS)\frac{D}{Q} + H_B\frac{(Q-B)^2}{2QN} + \frac{QH_S}{2N}\left\{(2-N)\frac{D}{P} + N - 1\right\} + \pi\frac{B^2}{2QN} + (F+\frac{VQ}{N})\frac{DN}{Q} \quad (8)$$

Note that if the number of deliveries, N, in Equations (6)-(8) is one, the SSMD model becomes identical to Equations (1)-(3) for SSSD policy.

4.1. Gradient Search Algorithm

From the aggregate total cost in Eq. (8), we now determine the optimal order quantity and the optimal back ordering with regarding this fact that $TC(Q,B,N)$ is a convex function. By taking the first derivatives of Eq. (8) with respect to B, Q and N, setting them equal to zero, and solving for B, Q and N simultaneously, we obtain the following formulas:

$$N = \sqrt{\frac{2D(A+CS)\left[H_S(\pi+H_B)(\frac{2D}{P}-1)+\pi H_B\right]}{2DFH_S(1-\frac{D}{P})(\pi+H_B)}} \quad (9)$$

$$Q = \sqrt{\frac{2DN(A+NF+CS)(\pi+H_B)}{H_S\left\{(2-N)\frac{D}{P}+N-1\right\}(\pi+H_B)+\pi H_B}} \quad (10)$$

$$B = \frac{H_B Q}{\pi + H_B} \quad (11)$$

$TC(Q,B,N)$ is a convex function in N>0, B>0 and Q>0. Since N is a positive integer, let N* denote the optimum integer value of N, and Q^*, B^* are the optimum values of Q, B which are obtained by substituting the N* into equations (10) and (11).

If N in Eq.(9) is not an integer number, we choose N*; which yields $TC(Q^*,B^*,N^*) = \min\{TC(Q^-,B^-,N^-), TC(Q^+,B^+,N^+)\}$ in Eq. (8) with regarding this fact that $TC(Q,B,N)$ is a convex function, where N^+ and N^- represent the nearest integers larger and smaller than the optimal N; then Q^+, B^+ and Q^-, B^- are determined by substituting N^+ and N^- in equations (10) and (11).

Note that if $\left[H_S(\pi+H_B)(\frac{2D}{P}-1)+\pi H_B\right]$ in equation (9) is smaller than zero, we can't compute N by equation (9), so we consider $N^*=1$.

After computing Q^*, B^* and N^*, We can determine q^* and b^* as follow:

$$q^* = \frac{Q^*}{N^*} \;,\; b^* = \frac{B^*}{N^*}$$

4.2. Particle Swarm Optimization Algorithm

Particle swarm optimization (PSO) algorithm has been introduced by Russell Eberhart and James Kennedy in 1995 [39]. This algorithm is a population-based search algorithm based on the simulation of the social behaviour of birds within a flock. In PSO, individuals, referred to as particles, are "flown" through hyper-dimensional search space. Individuals in a particle swarm follow a very simple behaviour: to emulate the success of neighbouring individuals and their own successes. In simple terms, the particles are "flown" through a multidimensional search space, where the position of each particle is adjusted according to its own experience and that of its neighbours. Let $x_i(t)$ denote the position of particle i in the search space at time step t. The position of the particle is changed by adding a velocity, $v_i(t)$, to the current position.

$$v_i(t)=wv_i(t-1)+r_1c_1(x_{pbest_i}-x_i(t-1))+r_2c_2(x_{gbest}-x_i(t-1)) \quad (12)$$

$$x_i(t)=x_i(t-1)+v_i(t) \quad (13)$$

where

$x_i(t)$ **:** The position of particle i at time step t

$v_i(t)$: The velocity of particle i at time step t

x_{pbest_i} : Best "remembered" position of particle i

x_{gbest} : Best "remembered" swarm position

r_1, r_2 : Random numbers between 0 and 1

c_1, c_2 : Cognitive and social parameters

w : An inertial constant.

Following, the pseudo code of PSO algorithm is written:

1. **Initialization**
   - Determine swarm size, n
   - Determine $x_{\min}$ and $x_{\max}$
   - Randomly initialize $x_i(0)$ for all i, $i=1,...,n$
   - Randomly initialize $v_i(0)$ for all i, $i=1,...,n$
   - $pbest_i = f(x_i(0)), x_{pbest_i} = x_i(0)$, $i=1,...,n$
   - $gbest = \min_{x_i} f(x_i(0)), x_{gbest} = \arg\min_{x_i} f(x_i(0))$
   - Set constant c_1, c_2 and w
   - Set iteration number of algorithm
2. **Optimization**
   for t=1 to iteration number
   - Create random vectors r_1, r_2
   - Update the particle velocities by equation (12)
   - Update the particle positions equation (13)
   - Check and set particle positions, $x_i(t)$, in interval[$x_{\min}, x_{\max}$]
   - Evaluate function value $f(x_i(t))$
   - Update the local bests: If $f(x_i(t)) \prec pbest_i = \; pbest_i = f(x_i(t)), x_{pbest_i} = x_i(t)$ Update the global best: If $f(x_i(t)) \prec gbest = \; gbest = f(x_i(t)), x_{gbest} = x_i(t)$ end
3. **Final Solution**
   - x_{gbest} is the optimal solution with fitness $gbest$.

Note the following about the above algorithm:

- Swarm size is usually determined between 10 and 50.
- Usually, the positions of particles are initialized to uniformly cover the search space. It is important to note that the efficiency of the PSO is influenced by the initial diversity of the swarm, i.e. how much of the search space is covered, and how well particles are distributed over the search space. An efficient initialization method for the particle positions is $x(0)=x_{\min}+r_j(x_{\max}-x_{\min})$ where $r_j \sim U(0,1)$.
- The initial velocities can be initialized to zero, i.e. $v(0)=0$; While it is possible to also initialize the velocities to random values, it is not necessary, and it must be done with care.
- c_1 and c_2 are constants that say how much the particle is directed towards good positions. They represent a "cognitive" and a "social" component, respectively, in that they affect how much the particle's personal best and the global best (respectively) influence its movement. Usually we take $c_1, c_2 \approx 2\ (c_1 + c_2 = 2)$ Or they could be randomly initialized for each particle.
- w is an inertial constant. Good values are usually slightly less than 1. Or it could be randomly initialized for each particle [39].

5. Numerical Examples

In this section, we want to solve the suggested model and we use the test problem of Kim and Ha's article (2003) for analyzing our model. So, the value of parameters is similar to Kim and Ha's article. Consider a buyer who is currently using an EOQ policy with a single delivery assumption, seeking short-term price breaks. The buyer wants to change the current ordering practice toward a development of a long-term relationship with a supplier for successful JIT implementation. The buyer currently has the annual demand of 4800 units and the order cost is $25 per order. For order shipments, the buyer pays the fixed transportation cost of $50 per trip as well as the unit variable cost for order handling and receiving of $1.00/ unit. We further assume that the supplier uses 25% of its annual production capacity of 19,200 units in order to fulfill the buyer's order requirements. The supplier currently spends 6 hours with five workers to set up the production system. With the hourly wage of $20 per worker, the onetime setup cost is $600 ($20/hour×5 workers×6 hours).We assume that the current H_B and H_S are $7 per unit per year and $6 per unit per year. In summary, A=25, D=4800, P=19200, $C\times S$=600, F=50, V=1, H_B=7, H_S=6 and π=8. The Q^*, B^*, N^* and TC^* for SSMD model are computed respectively 1195.9, 558.1, 2 and 10620.

6.Sensitivity Analysis

To study the effects of changes in the system parameters D, P, A, C.S, H_B, H_S, F, V and π on the optimal order quantity, shortage quantity, number of deliveries and optimal joint total cost, a sensitivity analysis is performed. The sensitivity analysis is performed by changing (increasing or decreasing) the parameters by 10%, 20%, 30% taking one at a time, keeping the remaining parameters at their original values. The effects of changes of the parameters on proposed SSSD and SSMD models and also on the Kim and Ha (2003)'s model are investigated. Therefore we calculate the following ratios for different quantity of these parameters:

$$r_1 = \frac{TC^*_{\text{Gradient Search}}(Q,B,N) - TC^*_{PSO}(Q,B,N)}{TC^*_{PSO}(Q,B,N)} \times 100$$

$$r_2 = \frac{TC^*_{\text{Gradient Search}}(Q,B,N) - TC^*(Q,B)}{TC^*(Q,B)} \times 100$$

$$r_3 = \frac{TC^*_{\text{Gradient Search}}(Q,B,N) - TC^*_{Kim\&Ha}(Q,N)}{TC^*_{Kim\&Ha}(Q,N)} \times 100$$

Which r_1 shows the deviation of PSO solution from Gradient Search solution, r_2 determines the difference of optimal cost function in proposed SSMD and SSSD models and r_3 compares our SSMD model with Kim and Ha (2003)'s model.

Tab. 1. Changing the parameter D

Change in parameter (%)	P	SSSD Q^*	SSSD B^*	SSSD $TC^*(Q,B)$	SSMD Gradient Search Q^*	Gradient Search B^*	Gradient Search N^*	Gradient Search $TC^*(Q,B,N)$	SSMD PSO Q^*	PSO B^*	PSO N^*	PSO $TC^*(Q,B,N)$	r_1	r_2	$TC^*_{Kim\&Ha}(Q,N)$	r_3
-30%	3360	973.8	454.4	8018	973.8	454.4	1	8018	973.8	454.4	1	8018	0	0	8942.7	-10.3
-20%	3840	1025.1	478.4	8897.1	1025.1	478.4	1	8897.1	1025.1	478.4	1	8897.1	0	0	9782.7	-9
-10%	4320	1071.1	499.8	9764.8	1071.1	499.9	1	9764.8	1071.1	499.9	1	9764.8	0	0	10597	-8.5
0%	4800	1112.8	519.28	10623	1195.9	558.1	2	10620	1154.7	538.9	2	10624	-0.04	-0.03	11389	-6.7
10%	5280	1150.7	537	11475	1254.3	585.3	2	11384	1231.8	574.8	2	11385	-0.009	-0.8	12158	-6.7
20%	5760	1185.5	553.2	12320	1310	611.3	2	12135	1309.3	611	2	12135	0	-1.5	12892	-6.2
30%	6240	1217.5	568.1	13159	1363.5	636.3	2	12876	1387.8	647.6	2	12877	-0.008	-2.1	13610	-5.7

Tab. 2. Changing the parameter P

Change in parameter (%)	P	SSSD Q^*	SSSD B^*	SSSD $TC^*(Q,B)$	SSMD Gradient Search Q^*	Gradient Search B^*	Gradient Search N^*	Gradient Search $TC^*(Q,B,N)$	SSMD PSO Q^*	PSO B^*	PSO N^*	PSO $TC^*(Q,B,N)$	r_1	r_2	$TC^*_{Kim\&Ha}(Q,N)$	r_3
-30%	13440	1050.1	490.1	10971	1281.5	598	3	10606	1247.2	582	3	10608	-0.02	-3.3	11204	-5.3
-20%	15360	1074.9	501.6	10828	1195.9	558.1	2	10620	1206.1	562.8	2	10620	0	-2	11287	-5.9
-10%	17280	1095.4	511.2	10715	1195.9	558.1	2	10620	1176.7	549.1	2	10621	-0.009	-0.89	11352	-6.4
0%	19200	1112.8	519.28	10623	1195.9	558.1	2	10620	1154.7	538.9	2	10624	-0.04	-0.03	11389	-6.7
10%	21120	1127.5	526.2	10547	1127.5	526.2	1	10547	1127.5	526.2	1	10547	0	0	11414	-7.6
20%	23040	1140.3	532.1	10483	1140.3	532.2	1	10483	1140.3	532.2	1	10483	0	0	11435	-8.3
30%	24960	1151.5	537.4	10428	1151.5	537.4	1	10428	1151.5	537.4	1	10428	0	0	11453	-8.9

Tab. 3. Changing the parameter A

Change in parameter (%)	P	SSSD Q^*	SSSD B^*	SSSD $TC^*(Q,B)$	SSMD Gradient Search Q^*	Gradient Search B^*	Gradient Search N^*	Gradient Search $TC^*(Q,B,N)$	SSMD PSO Q^*	PSO B^*	PSO N^*	PSO $TC^*(Q,B,N)$	r_1	r_2	$TC^*_{Kim\&Ha}(Q,N)$	r_3
-30%	17.5	1106.6	516.4	10591	1189.7	555.2	2	10590	1147.8	535.7	2	10594	-0.038	-0.009	11357	-6.75
-20%	20	1108.6	517.4	10602	1191.8	556.2	2	10600	1108.6	517.3	1	10602	-0.019	-0.02	11368	-6.75
-10%	22.5	1110.7	518.3	10613	1193.8	557.1	2	10610	1152.4	537.8	2	10614	-0.038	-0.028	11379	-6.76
0%	25	1112.8	519.28	10623	1195.9	558.1	2	10620	1154.7	538.9	2	10624	-0.04	-0.03	11389	-6.7
10%	27.5	1114.8	520.2	10634	1197.9	559	2	10630	1114.8	520.2	1	10634	-0.04	-0.038	11400	-6.75
20%	30	1116.9	521.2	10645	1200	560	2	10640	1159.3	541	2	10643	-0.028	-0.05	11411	-6.76
30%	32.5	1118.9	522.2	10656	1202.1	561	2	10650	1118.9	522.2	1	10656	-0.06	-0.056	11421	-6.75

Tab. 4. Changing the parameter C_S

Change in parameter (%)	P	SSSD			SSMD Gradient Search				SSMD PSO				r_1	r_2	$TC^*_{Kim\&Ha}(Q,N)$	r_3
		Q^*	B^*	$TC^*(Q,B)$	Q^*	B^*	N^*	$TC^*(Q,B,N)$	Q^*	B^*	N^*	$TC^*(Q,B,N)$				
-30%	420	952.9	444.7	9786.9	952.9	444.7	1	9786.9	952.9	444.7	1	9786.9	0	0	10573	-7.4
-20%	480	1009	470.9	10080	1009	470.9	1	10080	1009	470.9	1	10080	0	0	10856	-7.1
-10%	540	1062.1	495.7	10359	1062.1	495.7	1	10359	1062.1	495.7	1	10359	0	0	11128	-6.9
0%	600	1112.8	519.28	10623	1195.9	558.1	2	10620	1154.7	538.9	2	10624	-0.04	-0.03	11389	-6.7
10%	660	1161.2	541.9	10877	1244.4	580.7	2	10856	1208.9	564.1	2	10859	-0.03	-0.02	11638	-6.7
20%	720	1207.6	563.5	11120	1291.1	602.5	2	11083	1260.7	588.3	2	11085	-0.02	-0.3	11865	-6.6
30%	780	1252.4	584.4	11354	1336.1	623.5	2	11302	1310.7	611.6	2	11304	-0.02	-0.5	12085	-6.5

Tab. 5. Changing the parameter $H_B\,(H_S \prec H_B)$

Change in parameter (%)	H_B	SSSD			SSMD Gradient Search				SSMD PSO				r_1	r_2	$TC^*_{Kim\&Ha}(Q,N)$	r_3
		Q^*	B^*	$TC^*(Q,B)$	Q^*	B^*	N^*	$TC^*(Q,B,N)$	Q^*	B^*	N^*	$TC^*(Q,B,N)$				
-10%	6.3	1135.6	500.3	10506	1135.6	500.3	1	10506	1135.6	500.3	1	10506	0	0	11442	-8.2
0%	7	1112.8	519.28	10623	1195.9	558.1	2	10620	1154.7	538.9	2	10624	-0.04	-0.03	11389	-6.7
10%	7.7	1093.1	536.1	10728	1184.4	580.9	2	10677	1154.7	566.3	2	10678	-0.009	-0.5	11711	-8.8
20%	8.4	1075.9	551.1	10823	1174.1	601.4	2	10728	1154.7	591.4	2	10729	-0.009	-0.9	11846	-9.4
30%	9.1	1060.9	564.6	10908	1164.9	619.9	2	10775	1154.7	614.5	2	10775	0	-1.2	11957	-9.9

Tab. 6. Changing the parameter $H_S\,(H_S \prec H_B)$

Change in parameter (%)	H_S	SSSD			SSMD Gradient Search				SSMD PSO				r_1	r_2	$TC^*_{Kim\&Ha}(Q,N)$	r_3
		Q^*	B^*	$TC^*(Q,B)$	Q^*	B^*	N^*	$TC^*(Q,B,N)$	Q^*	B^*	N^*	$TC^*(Q,B,N)$				
-30%	4.2	1163.9	543.2	10367	1419.1	662.2	3	10043	1380.1	644	3	10045	-0.02	-3.1	10897	-7.8
-20%	4.8	1146.1	534.8	10454	1277.2	596	2	10249	1291	602.5	2	10250	-0.01	-2	11157	-8.1
-10%	5.4	1129.1	526.9	10539	1234.5	576.1	2	10438	1217.1	568	2	10438	0	-0.98	11371	-8.2
0%	6	1112.8	519.28	10623	1195.9	558.1	2	10620	1154.7	538.9	2	10624	-0.04	-0.03	11389	-6.7
10%	6.6	1097.1	512	10706	1097.1	512	1	10706	1097.1	512	1	10706	0	0	11779	-9.1

Tab. 7. Changing the parameter F

Change in parameter (%)	F	SSSD			SSMD Gradient Search				SSMD PSO				r_1	r_2	$TC^*_{Kim\&Ha}(Q,N)$	r_3
		Q^*	B^*	$TC^*(Q,B)$	Q^*	B^*	N^*	$TC^*(Q,B,N)$	Q^*	B^*	N^*	$TC^*(Q,B,N)$				
-30%	35	1100.3	513.5	10558	1170.9	546.4	2	10498	1154.7	538.8	2	10499	-0.009	-0.6	11155	-5.9
-20%	40	1104.5	515.4	10580	1179.3	550.3	2	10539	1154.7	538.9	2	10540	-0.009	-0.4	11239	-6.2
-10%	45	1108.6	517.4	10602	1187.6	554.2	2	10580	1154.7	538.8	2	10582	-0.02	-0.2	11322	-6.5
0%	50	1112.8	519.28	10623	1195.9	558.1	2	10620	1154.7	538.9	2	10624	-0.04	-0.03	11389	-6.7
10%	55	1116.9	521.2	10645	1116.9	521.2	1	10645	1116.9	512.2	1	10645	0	0	11452	-7
20%	60	1121	523.1	10666	1121	523.1	1	10666	1154.7	538.8	1	10669	-0.03	0	11514	-7.4
30%	65	1125	525	10688	1125	525	1	10688	1125	525	1	10688	0	0	11577	-7.7

Tab. 8. Changing the parameter V

Change in parameter (%)	V	SSSD			SSMD Gradient Search				SSMD PSO				r_1	r_2	$TC^*_{Kim\&Ha}(Q,N)$	r_3
		Q^*	B^*	$TC^*(Q,B)$	Q^*	B^*	N^*	$TC^*(Q,B,N)$	Q^*	B^*	N^*	$TC^*(Q,B,N)$				
-30%	0.7	1112.8	519.3	9183.4	1195.9	558.1	2	9180	1112.8	519.3	1	9183.4	-0.04	-0.04	9949.5	-7.7
-20%	0.8	1112.8	519.3	9663.4	1195.9	558.1	2	9660	1154.7	538.9	2	9663.5	-0.04	-0.035	10429	-7.4
-10%	0.9	1112.8	519.3	10143	1195.9	558.1	2	10140	1154.7	538.9	2	10144	-0.04	-0.03	10909	-7
0%	1	1112.8	519.28	10623	1195.9	558.1	2	10620	1154.7	538.9	2	10624	-0.04	-0.03	11389	-6.7
10%	1.1	1112.8	519.3	11103	1195.9	558.1	2	11100	1154.7	538.9	2	11104	-0.04	-0.027	11869	-6.5
20%	1.2	1112.8	519.3	11583	1195.9	558.1	2	11580	1154.7	538.9	2	11584	-0.04	-0.026	12349	-6.2
30%	1.3	1112.8	519.3	12063	1195.9	558.1	2	12060	1154.7	538.9	2	12064	-0.03	-0.025	12829	-6

Tab. 9. Changing the parameter π

Change in parameter (%)	π	SSSD			SSMD Gradient Search				SSMD PSO				r_1	r_2	$TC^*_{Kim\&Ha}(Q,N)$	r_3
		Q^*	B^*	$TC^*(Q,B)$	Q^*	B^*	N^*	$TC^*(Q,B,N)$	Q^*	B^*	N^*	$TC^*(Q,B,N)$				
-30%	5.6	1185.5	658.6	10266	1185.5	658.6	1	10266	1185.5	658.6	1	10266	0	0	11382	-9.8
-20%	6.4	1156.7	604.2	10402	1156.7	604.2	1	10402	1156.7	604.2	1	10402	0	0	11382	-8.6
-10%	7.2	1132.8	558.4	10520	1132.8	558.4	1	10520	1154.7	569.2	1	10521	-0.009	0	11382	-7.6
0%	8	1112.8	519.28	10623	1195.9	558.1	2	10620	1154.7	538.9	2	10624	-0.04	-0.03	11382	-6.7
10%	8.8	1095.6	485.4	10715	1185.9	524.4	2	10669	1154.7	511.6	2	10671	-0.02	-0.43	11382	-6.3
20%	9.6	1080.7	455.7	10796	1177	496.3	2	10713	1154.6	486.8	2	10714	-0.009	-0.77	11382	-5.9
30%	10.4	1067.7	429.5	10869	1169.1	470.3	2	10753	1154.7	464.5	2	10754	-0.009	-1.1	11382	-5.5

The following inferences can be made from the sensitivity analysis based on Tables 1-9.

1) In SSMD model, we calculated solutions for different values of parameters by two methods, the PSO method, which is a meta-heuristic algorithm and gives an approximate solution and Gradient Search method, which gives an exact solution. With comparing the results of these methods, by using of r_1, we can conclude that the computed values for the total cost are approximately similar in both two methods.
2) Comparing joint total costs of the SSSD and SSMD models for different values of parameters show that $TC^*_{SSSD} \geq TC^*_{SSMD}$, so the policy of frequent shipment in small lot size results in less total cost than single shipment policy. It is noticeable that $TC^*_{SSSD} = TC^*_{SSMD}$ are occurred when N=1, and we know that N=1 changes SSMD to the SSSD model.
3) Taking into account ratio r_2 for different values of each parameter in tables 1-9 demonstrate that on the one hand more values for parameters D, A, CS, H_B, π and on the other hand less values of H_S, F and V results to more values for r_2. Bigger value of r_2 means that SSMD is much better than SSSD.
4) We compared the proposed SSMD model to Kim and Ha (2003)'s SSMD model by computing r_3 ratio. Outcomes prove that our model have better total cost than Kim and Ha (2003)'s model for a given example and considered values for the parameters.
5) When the parameter π increases and other parameters remain unchanged, the optimal joint total cost increases. After comparing the value of r_3 for different values of π, we find that Since Kim and Ha did not permit demand shortage in their model; backordering cost has no effect in the joint total relevant cost of their model. Therefore, it remains unchanged as π varies. However, the effect of backordering cost can be clearly seen in the proposed model as presented in table 9. From this, it is obvious that the proposed model is more advantageous for the lower values of π.
6) Our proposed model is also more beneficial than Kim and Ha (2003)'s model for the lower values of D, CS, V and greater P, H_B, H_S or F.
7) Joint total cost function of the proposed SSMD model increases when the parameter D, A, CS, H_B, H_S, F, V or π increases.
8) Enlarging the parameter F results to lower number of deliveries; it seems reasonable; when you should pay more for each delivery, you will try to decrease the number of shipments in order to minimize the cost. However, increasing the value of parameter V doesn't have any effects on N; because, you pay V$ for each item that you transport and on the whole you should deliver D number, its overall cost will be V.D and For that reason, number of deliveries isn't dependent on V.

7. Conclusions

This study has analyzed the effect of integrated lot splitting strategy in a SCM environment. The proposed model determines optimal order quantity, optimal backorder quantity and optimal number of deliveries on the integrated total relevant cost for the SSSD and SSMD models. The integrated lot-splitting strategy facilitating multiple deliveries in small lot sizes shows a cost-minimizing effect over the conventional approach. Applying different values to the parameters in sensitivity analysis would yield an enlightening look at the issue of the behaviour of the joint total cost function and findings are summarized. After comparing the proposed SSMD model to Kim and Ha (2003)'s SSMD model, outcomes prove that our model which is permitting shortage have better total cost than Kim and Ha (2003)'s model for a given example and considered values for the parameters.

The proposed model can be further extended to multiple products case and multiple buyers and suppliers scenario. It can also be developed for stochastic and/or fuzzy parameters.

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