A GENETIC ALGORITHM AND A MODEL FOR THE RESOURCE CONSTRAINED PROJECT SCHEDULING PROBLEM WITH MULTIPLE CRUSHABLE MODES

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Abstract: This paper presents an exact model and a genetic algorithm for the multi-mode resource constrained project scheduling problem with generalized precedence relations in which the duration of an activity is determined by the mode selection and the duration reduction (crashing) applied within the selected mode. All resources considered are renewable. The objective is to determine a mode, the amount of continuous crashing, and a start time for each activity so that all constraints are obeyed and the project duration is minimized. Project scheduling of this type occurs in many fields; for instance, predicting the resources and duration of activities in software development projects. A key feature of the model is that none of the typical models can cope with the continuous resource constraints. Computational results with a set of 100 generated instances have been reported and the efficiency of the proposed model has been analyzed.

Keywords: Crashable modes; Multi-mode; Time/resource trade-off; Time windows.

1. Introduction

For many real-life applications of project scheduling, it is possible to perform the individual activities in alternative ways (modes). These modes are different in processing time, time lags to other activities, and resource requirements. They reflect time/cost, time/resource and resource/resource trade-offs [10]. Such real-life projects can be modeled as instances of the Multi-Mode Resource-Constrained Project Scheduling Problem or briefly MRCPSP which is denoted by MPS | prec | C_{max}. MRCPSP in the situations involving minimum and maximum time lags or Generalized Precedence Relations called MRCPSP-GPR [7: Page 512] and denoted by MPS | temp | C_{max} [16: Page 160, 9: Ch 6 Page 103]. MRCPSP is a generalized version of the standard well-known Resource-Constrained Project Scheduling Problem or briefly RCPSP (PS | prec | C_{max}) which is in GPR denoted by RCPSP-GPR or PS | temp | C_{max} [16, Page 22].

There is a general case of MRCPSP-GPR in which the duration/cost of an activity is treated as a function of both the resource requirements (mode selection) and the amount of crashing (duration reduction), applied within the selected mode. This complex case has been introduced by Ahn and Erenguc [1] and called Resource-Constrained Project Scheduling Problem with Multiple Crashable Mode or briefly RCPSPMCM. They provide this example: Activity i can be done by "worker A using machine X (mode 1)" or by "worker B using machine Y (mode 2)". Assuming 8 hours of work per day, worker A, using machine X can finish activity i in 10 working days at the price of $400 and worker B using machine Y can complete the activity in 8 working days at the price of $500. Furthermore, workers A and B can shorten the activity duration by working additional hours each day. For example, worker A can finish the activity in 8 days by working 10 hours a day. Duration reduction, i.e. crashing, can be done in various ways: using overtime or additional shift(s), or allocating more resources that might be acquired easily by incurring additional expenditures.

Their objective function involves the minimization of the total project cost which is the sum of the activity execution cost and the tardiness cost. In the absence of resource constraints, the problem reduces to the Time/Cost Trade-off Problem or briefly TCTP. In the absence of crashing within a mode, the problem reduces to the MRCPSP.

The complexity of the problem is increased when the crashing applied within a selected mode is continuous and minimum and maximum time lags (time windows) between a pair of activities are considered. In such a case denoted by RCPSPMCM-GPR, due to the continuous crashing, no exact solution method has been reported [1: Page 255].
The problem is strongly \textit{NP-hard}. Moreover, the problem \textit{RCPSP-GPR} is also \textit{NP-hard}, and even the question whether a problem instance has a feasible solution is \textit{NP-complete} in strong sense [3]. As a generalization of the problem \textit{RCPSP-GPR}, the problem \textit{MRCPSP-GPR} and its corresponding feasibility problem belong to the same complexity class. [9: Page 8, 16: Page 165].

The remainder of this paper is organized as follows: In Section 2, we discuss the relevant literature review. Section 3 gives a conceptual formulation based on the previous works. A brief reason why this specific formulation is tackled by the authors is discussed in Section 4. New mathematical formulation for \textit{RCPSP-GPR}, \textit{MRCPSP-GPR}, \textit{MRCPSP-GPR with Mode-Dependent time lags} and \textit{RCPSPMCM-GPR} as the contributions of this paper are presented in Subsections 5.1, 5.2, 5.3 and 5.4 respectively. Genetic algorithm implementation as another contribution is discussed in Section 6. Computational results are reported in Section 7. Section 8 is reserved for our conclusions.

2. Relevant Literature Review

\textit{MRCPSP} without maximum time lag has been treated by several authors since the early eighties [16, P. 160]. \textit{RCPSPMCM} without \textit{GPR} in which resource/resource, time/resource and time/cost trade-off are considered, has been introduced by Ahn and Erenguc [1]. They have presented a heuristic procedure for \textit{RCPSPMCM} but no exact solution method has been reported. A linear approximation of duration function has been carried out by Deckro and Hebert [6] called modeling diminishing returns but they have not considered resource constraints in their model.

Exact algorithms for the case of \textit{MRCPSP} have been reviewed and their performance has been tested by Hartmann and Drexl [12]. The most efficient method for solving \textit{MRCPSP} known thus far is the branch-and-bound algorithm of Sprecher and Drexl [19]. The best heuristic procedure for \textit{MRCPSP} at present is a genetic algorithm published by Hartmann [16: Page 160].

Since we are going to present a mathematical formulation for \textit{RCPSPMCM}, the focus of remaining literature review will be on the exact procedures especially mathematical formulations. In order to be able to specify the resource constraints in the correct and solvable form, linear programming based approaches for \textit{RCPSP} (not \textit{MRCPSP}) have been presented by several authors [7].

In formulation by Pritsker et al. [17], the binary decision variable \( x_{it} \) is defined to be 1 if activity \( i \) finishes at time instant \( t \), and to be 0 otherwise.

For the \textit{MRCPSP} without \textit{GPR} (\textit{MPS} | \textit{prec} | \( C_{\text{max}} \)), the best 0-1 programming model based on an extension of the formulation by Pritsker et al., has been presented by Talbot [20]. The model of Talbot have been developed by Reyck and Herroelen [8] for the case of \textit{GPR} (\textit{MPS} | \textit{temp} | \( C_{\text{max}} \)).

Since lower bound calculations play the most important role in branch-and-bound procedures, it has been addressed by recent papers [2]. The best known lower bound for \textit{RCPSP} and \textit{MRCPSP} is currently based on the resolution of several large linear programs [4, 13]. Computational experience by Mothering et al. [16: Page 75] has shown that already for medium-sized projects of \textit{RCPSP} with 100 activities, the resulting linear program cannot be solved in an acceptable amount of computational time. Broker and Knits [4] strengthen one of this relaxation by taking into account time windows for the activities and use column generation to deal with the large number of variables.

3. Conceptual Formulation for \textit{RCPSPMCM-GPR}

Let:

- \( v_{im} \) binary decision variable; 1 if activity \( i \) is performed in mode \( m \), 0 otherwise.
- \( s_i, f_i \) starting/finishing time of activity \( i \)
- \( d_i \) duration of activity \( i \)
- \( ss_{ij}^{\text{min}}, ss_{ij}^{\text{max}} \) minimal/maximal time lag between start to...
- \( sf_{ij}^{\text{min}}, sf_{ij}^{\text{max}} \) minimal/maximal time lag between start to...
- \( fs_{ij}^{\text{min}}, fs_{ij}^{\text{max}} \) minimal/maximal time lag between finish to...
- \( ff_{ij}^{\text{min}}, ff_{ij}^{\text{max}} \) minimal/maximal time lag between finish to...
- \( ss_{imij}^{\text{min}} \) elements of transformed matrix of minimal time lags, if activities \( i \) and \( j \) are performed in modes \( m_i \) and \( m_j \) respectively.
- \( R_k \) availability or the maximum number of the \( k^{\text{th}} \) resource type
- \( r_{imk} \) resource requirement of type \( k \) for activity \( i \) in mode \( m \)
- \( K \) number of resource types required for the project
- \( \overline{T} \) upper bound on the shortest project duration, where \( \overline{T} \geq es_{n+1} \) (earliest start)
- \( D \) project due date


\[ \overline{P} \] per period penalty cost incurred if the project is delayed beyond \( \overline{D} \) 

\[ c_{i,m} \] marginal crashing cost of activity \( i \) using mode \( m \)

\[ d_{i,m}/\overline{d}_{i,m} \] rashed (minimum) / normal (maximum) duration of activity \( i \) in mode \( m \)

\[ \tau_{i,m}/\overline{\tau}_{i,m} \] crashed (maximum) / normal (minimum) cost of activity \( i \) in mode \( m \)

Assuming an AoN network \( N(v) \) in standardized form with minimal start to start precedence relations using the transformation rules [3], the problem can be modeled conceptually as a mathematical programming model in the following way [1, 16]:

\[
\begin{align*}
\text{Min} & \quad \sum_{i=1}^{n} \left( c_{i,m} \times (d_{i,m} - \overline{d}_{i,m}) \right) \times \nu_{i,m} + F \max \{0, S_{m,j} - \overline{D}\}
\end{align*}
\]

Subject to:

\[
\sum_{i=1}^{n} \nu_{i,m} = 1 \quad i = 1, \ldots, n
\]

\[
s_j - s_i \geq \sum_{i_{\text{in}}}^{M} \sum_{j_{\text{in}}=\text{min}} \nu_{i_{\text{in}},j_{\text{in}}} \quad i < j, (i, j) \in E_{\text{proj}}
\]

\[
\sum_{i_{\text{in}}=1}^{M} \nu_{i_{\text{in}}} \leq d_{i_{\text{in}}} \quad i = 1, \ldots, n
\]

\[
\sum_{i_{\text{in}}}^{M} \nu_{i_{\text{in}}} \leq d_{i_{\text{in}}} \quad i = 1, \ldots, n
\]

\[
\sum_{i_{\text{in}}=1}^{M} \nu_{i_{\text{in}}} \leq R_k \quad k = 1, \ldots, K ; \quad t = 1, 2, \ldots, T
\]

\[
s_j \geq 0
\]

\[
s_0 = 0
\]

\[
\nu_{i,m} \in \{0,1\} \quad (i \in V, \quad m_j = 1, \ldots, M_j)
\]

Eq. (1) minimizes the project cost including crashing cost within each mode and penalty cost incurred if the project is delayed beyond tardiness of the project, computed as \( \max \{0, s_{n+1} - \overline{D}\} \). \( s_0 \) and \( s_{n+1} \) are the starting times of the non-real first and end activities respectively and \( s_j, d_i \) and \( \nu_{i,m} \) are decision variables to be determined. Eq. (2) ensures that only one of the modes is selected. Eq. (3) denotes the GPRs in standardized form.

The duration variable of the activity, is bounded between \( d_{i,m} \) and \( \overline{d}_{i,m} \) by Eq. (4) if activity \( i \) is performed in \( m_j \). Eq. (5) which is a conceptual statement of the resource constraints expresses that at no time instant of \( t \), during the project horizon between 0 and \( \overline{T} \) the resource availability may be violated. Moreover, we define:

\[ A(s,t,v) := \{i \in V|s_i \leq t < s_i + d_i\}, (t \geq 0) \]

which is the set of real activities in progress at time \( t \), depending on starting time \( s \) and assignment \( v \).

### 4. Difficulties of Modeling Resource Constraints

The mathematical program above cannot be solved directly because it is necessary to translate the set \( A(s,t,v) \), that is used in Eq. (5) into the solvable constraints. Hence 0-1 programming formulations have to be used in order to be able to specify the resource constraints in the correct and solvable form [7: P 208]. In formulation by Talbot [20] which is for the case of MRCPP, the binary decision variable \( v_{i,m,t} \) is defined to be 1 if activity \( i \) is performed in mode \( m \) and started at time \( t \), and to 0 otherwise. Thus the Eq. (5) can be stated as follows:

\[
\sum_{i_{\text{in}}}^{M} \sum_{i=1}^{n} \nu_{i_{\text{in}},j_{\text{in}}} \leq R_k \quad k = 1, \ldots, K ; \quad t = 1, 2, \ldots, T
\]

The variable \( v_{i,m,t} \) can only be defined over the interval between the earliest (\( e_{i_{\text{in}}} \)) and latest (\( l_{i_{\text{in}}} \)) starting time of the activity in question. The difficulty increases when dealing with different and smaller duration units (e.g., hour, minute, second) is necessary. Consequently, the number of decision variables will be increased exponentially.

### 5. New Mathematical Formulation

The formulation which is presented here has been inspired by one of the rectangle packing problems models [5].

#### 5.1. Mathematical Formulation for RCPSP-GPR

##### 5.1.1. Main Idea

In RCPSP-GPR we have a single mode so \( m_j \) must be omitted from the notations defined above. For visualizing the problem in three dimensions, imagine the number of resource types required for the project are two (K=2).

It can, however, be beyond the confines of two, i.e. any integer value (K=1, 2, 3, ...). Moreover, it must be noted that the geometrical model is applicable in the situation involving the uniformly distributed resource needs over processing times which is not the case about nonrenewable resources. Thus renewable resources which usually have this property are considered.

In such a case, there is a certain correspondence between boxes to be packed, and activities to be
scheduled. Each box would correspond to an activity, with a duration equal to the length and a resource request of type \( k (k=1, 2) \) equal to width and height respectively.

An empty box \( B_0 \) of width \( W_0 \) equal to time horizon \( T \), length \( L_0 \) equal to \( R_1 \), the resource capacity available of type 1 and height \( H_0 \) equal to \( R_2 \), the resource capacity available of type 2 is given. There is a series of boxes \( B_i \) (or Activities \( A_i \) ) \( (i = 1, \ldots, n) \), of width \( w_i = d_i \), length \( l_i = r_{i1} \) and height \( h_i = r_{i2} \) to be packed in which \( m_i \) has been omitted from both \( d_{in} \) and \( r_{in,k} \) for the case of single mode.

Furthermore, the constraint that activity preemption is not allowed corresponds to the natural requirement that boxes must be packed as a whole.

The bottom left behind corner of the big empty box \( B_0 \) is supposed to be at point \((0,0,0)\) so the top right front corner is \((T,R_1,R_2)\). Let:

\[
(x_i, y_i, z_i) : \text{The bottom left behind coordinates of activity } i ,
\]

\[
(x_i + d_i, y_i + r_{i1}, z_i + r_{i2}) : \text{The top right front coordinates of activity } i .
\]

### 5.1.2. GPRs Constraints

The x-coordinate of the bottom left behind corner of activity \( i \) is given by the activity starting time:

\[
x_i = s_i \quad n = 1, 2, \ldots, n
\]

and is the most important decision variable to be determined. Thus, GPRs can be formulated as follows:

\[
x_i + s_{ij}^{\text{min}} \leq x_j \leq x_i + s_{ij}^{\text{max}} \quad (i, j) \in E_a ,
\]

\[
x_i + s_f^{\text{min}} \leq x_j + d_j \leq x_i + s_f^{\text{max}} \quad (i, j) \in E_o ,
\]

\[
x_i + d_i + f_i^{\text{min}} \leq x_j \leq x_i + d_i + f_i^{\text{max}} \quad (i, j) \in E_p ,
\]

\[
x_i + d_i + f_j^{\text{min}} \leq x_j + d_j \leq x_i + d_i + f_j^{\text{max}} \quad (i, j) \in E_g ,
\]

in which \( f_i = x_i + d_i \) and \( f_j = x_j + d_j \) are the finishing time of activity \( i \) and \( j \) respectively.

### 5.1.3. Constraint of Makespan

The finishing time of end activities \( f_j = x_j + d_j \) should not be exceeded from \( T \), i.e.,

\[
T - x_i - d_i \geq 0. \quad i \in \text{end activities}
\]

### 5.1.4. The Objective Function

The minimization of makespan for RCPSP-GPR which is linear and the most popular objective in the project scheduling problems can be used as follows:

\[
\text{Minimize } T .
\]

### 5.1.5. No Overlapping Constraints

The constraints for packing boxes are as follows [5]:

1. Since the activity boxes may not be rotated, each edge of an activity box should be parallel to a specific edge of the main box \( B_0 \).

2. There should be no overlapping for any two small boxes, i.e., the overlapping area is zero.

In the situations involving single type renewable resources in project \((K=1)\), the problem can be formulated as the same as rectangle packing problem. The difficulties start when the number of resource types are two or more \((K \geq 2)\). In this case which is one of the contributions of this paper, the problem must be formulated totally different from packing problem.

In packing problem, boxes must be packed to a container in which no overlapping between a pair of boxes coordinates is permitted, i.e., one of y-z, x-z or x-y overlapping is allowed at a time.

The no overlapping constraints for project scheduling must be changed as: There should be no overlapping for any two boxes between x- and y-coordinates as well as x- and z-coordinates, i.e., it is not important to have overlapping between y- and z-coordinates.

For RCPSP-GPR with single type renewable resource in which no precedence relation of type \( f_{ij}^{\text{min}} \geq 0 \) between two activities \( i \) and \( j \) exists, one of the following constraints must be held:

\[
(x_j \geq x_i + d_i) \lor (x_j \geq x_i + d_j) \lor (y_j \geq y_i + r_i) \lor (y_j \geq y_i + r_j)
\]

Let:

\[
t_{ij}^x : \quad 0-1 \text{ integer variable; } 0 \text{ if activity } i \text{ to be located at the left hand side of activity } j \text{ (activity } i \text{ precedes activity } j \text{) without any overlapping between them, } 1 \text{ otherwise.}
\]

\[
t_{ij}^y : \quad 0-1 \text{ integer variable; } 0 \text{ if activity } j \text{ to be located at the left hand side of activity } i \text{ (activity } j \text{ precedes activity } i \text{) without any overlapping between them, } 1 \text{ otherwise.}
\]

We use the same definitions for notations \( t_{ij}^x \) and \( t_{ij}^y \) toward y-coordinate, \( t_{ij}^x \) and \( t_{ij}^y \) toward z-coordinate. Using the binary decision variables above, these constraints can be stated as follows:

\[
x_j - x_i - d_i + M \ast t_{ij}^x \geq 0 \quad i < j, i = 1, \ldots, n-1, \quad f = 2, \ldots, n
\]
where $M$ is a big constant. Equations above ensure that there should be no overlapping for any two boxes, between $x$- and $y$-coordinates. No overlapping constraints for $x$- and $z$-coordinates which are applicable in double types of renewable resources, are defined as follows:

$$z_j - z_i - r_z + M * t_{ji} \geq 0 \quad i, j = 1, \ldots, n$$

$$z_j - z_i - r_z + M * t_{ji} \geq 0 \quad i, j = 1, \ldots, n$$

$$t_{ij} + t_{ji} + t_{ij} + t_{ij} = 3 \quad i, j = 1, \ldots, n$$

The same equations as (24), (25) and (26) can be developed in situations involving $K > 2$.

### 5.1.6. Resource Constraints

The $y$- and $z$-coordinates of activities should not exceed from $R_1$ and $R_2$ respectively. This can be stated in the form of constraints as follows:

$$R_1 - y_i - r_y \geq 0 \quad i = 1, \ldots, n$$

$$R_2 - z_i - r_z \geq 0 \quad i = 1, \ldots, n$$

### 5.2. Mathematical Formulation for MRCPSP-GPR

In the case of MRCPSP-GPR individual activities can be executed in alternative ways (modes). Activity $i$ ($i = 1, \ldots, n$) when performed in mode $m$, ($m = 1, \ldots, M$) has a duration $d_{im}$ and requires $r_{im,k}$, a constant amount of resource $k$ over duration. To describe the mathematical formulation, Let:

$$d_i = \sum_{m=1}^{M} d_{im} \nu_{im} \quad i = 1, \ldots, n$$

$$r_{ik} = \sum_{m=1}^{M} r_{im,k} \nu_{im} \quad i = 1, \ldots, n \quad k = 1, \ldots, K$$

MRCPSP-GPR can be formulated by replacing Eq. (29), (30) and (2) into the RCPSP-GPR model.

### 5.4. Mathematical Formulation for RCPSPMCM-GPR

The RCPSPMCM-GPR involves MRCPSP-GPR plus the continuous crashing in each mode. Let:

- $K$ number of resources types required for each mode different from other modes
- $d_{im} (r_{im,1}, \ldots, r_{im,K})$ duration of activity $i$ in mode $m$, as a function of resources requirements
- $\epsilon_{im,k}$ lower/upper bound on resource requirement of type $k$ for activity $i$ in mode $m$
- $\Delta r_{im,k}$ resource increment of type $k$ above the lower bound in mode $m$, for Activity $i$
- $c_k$ procurement cost per unit of resource of type $k$
- $\beta_i$ regression coefficient

All other notations remain as previously defined. As illustrated in Fig. 1, an approximation of an activity duration in each mode is made by the function defined over the range of possible values of the upper and lower bounds on the resources requirements of any type in that mode as follows:

$$d_{im} = d_{im} (r_{im,1}, \ldots, r_{im,K}) \leq r_{im,k} \leq \epsilon_{im,k} \quad i = 1, \ldots, n$$

So $\overline{d}_{im}$ and $\underline{d}_{im}$ are constants and can be calculated by Eqs. (32) and (33) respectively.

$$\overline{d}_{im} = d_{im} (\epsilon_{im,1}, \ldots, \epsilon_{im,K})$$

$$\underline{d}_{im} = d_{im} (\epsilon_{im,1}, \ldots, \epsilon_{im,K})$$

The duration reduction (crashing) for each activity between $\overline{d}_{im}$ and $\underline{d}_{im}$ is allowed by increasing the amount of resources committed to it. $d_{im} (r_{im,1}, \ldots, r_{im,K})$ may be a constant, a linear expression, or a non-linear function such as hyperbolic function and can be different from activity to activity. Eq. (33) can be replaced by Eq. (36) as follows:

$$d_{im} = d_{im} (\epsilon_{im,1} + \Delta r_{im,k}, \ldots, \epsilon_{im,K} + \Delta r_{im,k}) \quad i = 1, \ldots, n$$

$$m_i = 1, \ldots, M_i$$

$$k = 1, \ldots, K_m$$

where:

$$r_{im,k} = \epsilon_{im,k} + \Delta r_{im,k} \quad i = 1, \ldots, n \quad m_i = 1, \ldots, M_i$$

$$k = 1, \ldots, K_m$$

$$0 \leq \Delta r_{im,k} \leq \epsilon_{im,k} \quad i = 1, \ldots, n \quad m_i = 1, \ldots, M_i$$

$$k = 1, \ldots, K_m$$
incorporating concerning items as follows:

\[
\text{Minimize } \sum_{i=1}^{M} \left( \sum_{k=1}^{K} c_{ik} \Delta_{m_k} \right) \times \nu_{m_i} + T \max \left( \beta, T - D \right) \quad (42)
\]

where

\[
\Delta_{m_k} = \sum_{k=1}^{K} c_{ik} \Delta_{m_k},
\]

\[
\sum c_{ik} \Delta_{m_k} = c_{m_k} \times (d_{m_k} - d_i) \quad (44)
\]

In which Eq. (1) has been modified by replacing the duration costs with the resource requirements costs. As a special case, when \( c_i = D = 0 \) and \( T = 1 \), Eq. (42) is converted to Eq. (17). The model is a non-linear programming due to \( \Delta_{m_k} \) but in the absence of discrete multi-mode, and using linear regressions such as Eqs. (39) and (41), the problem is converted to a linear programming. In the absence of crashing within a mode the problem reduces to the MRCPS-P-GR and the linear model as well.

5.4.4. An Example

Fig.2 shows an example in the form of an AON network for a project with eight real activities, each of which can be executed in two discrete mode and a single type of resources required for each mode. Crashing in each mode can be performed using Eq. (39) by increasing \( \Delta_{m_k} \).

![Fig. 2. An example in the form of an AON network](image)

Fig. 2. An example in the form of an AON network

Relevant data of Fig. 2 are shown in Tables 1 and 2.

<table>
<thead>
<tr>
<th>Activity</th>
<th>( P_{i11} )</th>
<th>( R_{i11} )</th>
<th>Resource required ( r_{i11} )</th>
<th>Resource increment ( 0 \leq \Delta r_{i11} \leq r_{i11} - P_{i11} )</th>
<th>( \bar{d}_{i1} )</th>
<th>( d_{i1} )</th>
<th>Linear approximation ( d_i(\Delta r_{i11}) = \bar{d}<em>{i1} - \beta_i \Delta r</em>{i11} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>5</td>
<td>1 + ( \Delta r_{a11} )</td>
<td>( 0 \leq \Delta r_{a11} \leq 4 )</td>
<td>9</td>
<td>4</td>
<td>9 - 1.25( \Delta r_{a11} )</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>2</td>
<td>1 + ( \Delta r_{b11} )</td>
<td>( 0 \leq \Delta r_{b11} \leq 1 )</td>
<td>4</td>
<td>2.4</td>
<td>4 - 1.60( \Delta r_{b11} )</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>1</td>
<td>1 + ( \Delta r_{c11} )</td>
<td>( r_{c11} = 0 )</td>
<td>3</td>
<td>3</td>
<td>3 - 0.00( \Delta r_{c11} )</td>
</tr>
<tr>
<td>D</td>
<td>1</td>
<td>3</td>
<td>1 + ( \Delta r_{d11} )</td>
<td>( 0 \leq \Delta r_{d11} \leq 2 )</td>
<td>4</td>
<td>2</td>
<td>4 - 1.00( \Delta r_{d11} )</td>
</tr>
<tr>
<td>E</td>
<td>1</td>
<td>3</td>
<td>1 + ( \Delta r_{e11} )</td>
<td>( 0 \leq \Delta r_{e11} \leq 2 )</td>
<td>9</td>
<td>1</td>
<td>9 - 4.00( \Delta r_{e11} )</td>
</tr>
<tr>
<td>F</td>
<td>1</td>
<td>2</td>
<td>1 + ( \Delta r_{f11} )</td>
<td>( 0 \leq \Delta r_{f11} \leq 1 )</td>
<td>4</td>
<td>3</td>
<td>4 - 1.00( \Delta r_{f11} )</td>
</tr>
<tr>
<td>G</td>
<td>1</td>
<td>2</td>
<td>1 + ( \Delta r_{g11} )</td>
<td>( 0 \leq \Delta r_{g11} \leq 2 )</td>
<td>9</td>
<td>1</td>
<td>4 - 3.00( \Delta r_{g11} )</td>
</tr>
<tr>
<td>H</td>
<td>1</td>
<td>6</td>
<td>1 + ( \Delta r_{h11} )</td>
<td>( 0 \leq \Delta r_{h11} \leq 5 )</td>
<td>9</td>
<td>1.5</td>
<td>9 - 1.50( \Delta r_{h11} )</td>
</tr>
</tbody>
</table>
shows one resource will be working half-time addition to 5 full-time. Moreover, assume \( c_1 = c_2 = 1, D = 0 \) and \( P = 1000 \). The problem has been solved using LINGO 8. Table 3 shows the amount of important decision variables.

### Table 3. Optimum solution of example 1

<table>
<thead>
<tr>
<th>Activity</th>
<th>( v_{i1} )</th>
<th>( v_{i2} )</th>
<th>( x_i )</th>
<th>( d_i )</th>
<th>( y_i )</th>
<th>( r_{j1} )</th>
<th>( \Delta r_{j1} )</th>
<th>( z_i )</th>
<th>( r_{j2} )</th>
<th>( \Delta r_{j2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>1</td>
<td>0.0000</td>
<td>2.4500</td>
<td>------</td>
<td>------</td>
<td>0.0000</td>
<td>3.0000</td>
<td>2.0000</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>0</td>
<td>2.4500</td>
<td>3.0769</td>
<td>3.9230</td>
<td>1.5769</td>
<td>0.5769</td>
<td>------</td>
<td>------</td>
<td>------</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>0</td>
<td>2.5269</td>
<td>3.0000</td>
<td>2.9230</td>
<td>1.0000</td>
<td>0.0000</td>
<td>------</td>
<td>------</td>
<td>------</td>
</tr>
<tr>
<td>D</td>
<td>1</td>
<td>0</td>
<td>2.4500</td>
<td>2.0769</td>
<td>0.0000</td>
<td>2.9230</td>
<td>1.9230</td>
<td>------</td>
<td>------</td>
<td>------</td>
</tr>
<tr>
<td>E</td>
<td>1</td>
<td>0</td>
<td>5.6519</td>
<td>2.2907</td>
<td>------</td>
<td>------</td>
<td>1.5842</td>
<td>1.6657</td>
<td>1.4157</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>0</td>
<td>1</td>
<td>5.5269</td>
<td>2.4157</td>
<td>------</td>
<td>------</td>
<td>0.0000</td>
<td>1.5842</td>
<td>0.0842</td>
<td></td>
</tr>
<tr>
<td>G</td>
<td>0</td>
<td>1</td>
<td>4.5269</td>
<td>1.0000</td>
<td>0.9230</td>
<td>2.0000</td>
<td>1.0000</td>
<td>------</td>
<td>------</td>
<td>------</td>
</tr>
<tr>
<td>H</td>
<td>0</td>
<td>1</td>
<td>7.9427</td>
<td>1.5400</td>
<td>------</td>
<td>------</td>
<td>0.0000</td>
<td>3.2500</td>
<td>2.2500</td>
<td></td>
</tr>
</tbody>
</table>

\[ T = 9.48 \]

#### 6.2. Genetic Operators

**For exploring of solution space, we use the single and double point crossovers and two specialized versions of mutation named 'Mode Mutation' (MM) and 'Resource Mutation' (RM). Mutation operators enable the proposed GA to explore the search space which can not be reachable by the crossover operators [11].**

#### 6.2.1. Single Point Crossover

It is the same classical crossover in which two selected parents are recombined by a single cross point. An instance of single point crossover implementation for the example of Figure 2 is shown in Figure 5. The bold line is cross point.
6.2.2. Double Points Crossover

It is an extended version of single point crossover in which two selected parents are recombined by two cross points. Figure 6 shows an instance of double point crossover implementation in which offspring 1 and 2 are created when information related to activities "D", "E", and "F" is swapped between parents 1 and 2.

![Fig 5. Implementing single point crossover](image)

![Fig 6. Implementing double point crossover](image)

6.2.3. Mode Mutation

It is implemented on the first row of chromosome as follows: an activity is randomly selected then the current mode is randomly changed into other mode (i.e., 1,…,M_f). In Figure 7, the mode assigned to activity "D" is changed into 2.

![Fig 7. Implementing mutation](image)

6.2.4. Resource Mutation

It is implemented on the other rows of chromosome as follows: an activity is randomly selected then Δr_{ijk} in the current solution is randomly changed between 0 ≤ Δr_{ijk} ≤ L_{m_{ik}}. In Figure 7, Δr_{i11} and Δr_{i22} are changed.
6.3. Selection Strategy

The selection strategy means that how to choose the individuals in the current population that will create offspring for the next generation. Generally, it is better that best solutions in the current generation selected as parents for creating offspring. The most common method for the selection mechanism is the "roulette wheel" sampling, in which each individual is assigned a slice of a circular "roulette wheel" and the size of the slice is proportional to the individual’s fitness.

The wheel is spun \( \text{Pop} \text{Size} \) times, where \( \text{Pop} \text{Size} \) is the number of individuals in the population. On each spin, the individual under the wheel's marker is selected to be in the pool of parents for the next generation. This method can be performed as follows:

a. Let \( CF \) be the sum of the Cost Function of all solutions in the current population as follows:

\[
CF = \sum_{p=1}^{\text{Pop} \text{Size}} (cf_p) \tag{45}
\]

b. Where \( cf_p \) is the cost function or fitness values of solution \( p \) and \( \text{Pop} \text{Size} \) is equal to the number of chromosomes in the population.

c. Let \( \rho_p \) be the relative probability related to chromosome \( p \) as follows:

\[
\rho_p = \frac{cf_p}{CF}, \quad p = 1, ..., \text{Pop} \text{Size}\tag{46}
\]

d. Let \( P_p \) be the cumulative probability related to chromosome \( p \) as follows:

\[
P_p = \sum_{j=1}^{p} \rho_j, \quad p = 1, ..., \text{Pop} \text{Size}\tag{47}
\]

e. Generate a random number, say \( r \), in the range of \([0, 1]\). If \( r < P_1 \), then the first chromosome is selected. Otherwise, the \( p^{th} \) chromosome is selected where,

\[
P_{p-1} < r < P_p, \quad 2 \leq p \leq \text{Pop} \text{Size}\tag{48}
\]

The fitness of each solution is directly obtained by Eq. (42). The initial population is randomly created in terms of a continuous uniform distribution.

6.4. Stoppage Conditions

We use three criteria for stopping the proposed GA as follows: 1) the maximum number of the established generation, 2) an arbitrary minimum value of variance of the last generation, and 3) the maximum run time. The algorithm is finished if one of the three mentioned criteria satisfied.

The variance of each generation can be calculated as follow:

\[
\sigma_g^2 = \frac{1}{\text{Pop} \text{Size}} \sum_{p=1}^{\text{Pop} \text{Size}} (CF_{gp} - \overline{CF}_g)^2 \tag{49}
\]

where \( CF_{gp} \) is the fitness of \( p^{th} \) chromosome in generation \( g \). \( \overline{CF}_g \) is average fitness of all chromosomes in generation \( g \) that is calculated as follow:

\[
\overline{CF}_g = \frac{1}{\text{Pop} \text{Size}} \sum_{p=1}^{\text{Pop} \text{Size}} CF_{gp} \tag{50}
\]

7. Computational Results

In order to show that the model serves to solve instances of practical size, ProGen/max [14] is used to generate 100 RCPSPMCM-GPR instances in 20 categories. Table 4 shows the control parameters used for ProGen/max to generate instances of the MRCPSP-GPR and Table 5 gives the necessary adaptations for obtaining instances of RCPSPMCM-GPR.

The Order Strength \( OS \) is a \([0, 1]\)-normalized measure defined as the number of precedence relations, which is minimum for parallel and maximum for series digraphs [14]. The Resource Factor \( RF \) reflects the average portion of resources requested by each activity [14]. Setting \( RF \) at 1 leads to the most complex resource-constrained project scheduling problem instances. The Resource Strength \( RS \) measures the scarcity of the resource availability to the respective requirements [14].

Table 4. The parameter settings for instances of MRCPSP-GPR

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Important Control Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N )</td>
<td>number of non-dummy activities</td>
<td>10,20,30,40,50</td>
</tr>
<tr>
<td>( M_i )</td>
<td>number of Modes per activity</td>
<td>2, 3</td>
</tr>
<tr>
<td>( d_{m} )</td>
<td>duration of each mode</td>
<td>([5, 10])</td>
</tr>
<tr>
<td>( \text{im} )</td>
<td>number of initial and terminal activities</td>
<td>([2, 3])</td>
</tr>
<tr>
<td>( \text{OS} )</td>
<td>order strength</td>
<td>0.5</td>
</tr>
<tr>
<td>( \text{OS} )</td>
<td>percentage of backward arcs</td>
<td>20%</td>
</tr>
<tr>
<td>( \text{OS} )</td>
<td>degree of redundancy</td>
<td>0.0</td>
</tr>
<tr>
<td>( \text{OS} )</td>
<td>number of cycle structures</td>
<td>([1, 3])</td>
</tr>
<tr>
<td>( \text{OS} )</td>
<td>number of activities per cycle structure</td>
<td>([1, 5])</td>
</tr>
<tr>
<td>( \text{OS} )</td>
<td>coefficient of cycle structure density</td>
<td>0.3</td>
</tr>
<tr>
<td>( \text{OS} )</td>
<td>deviations of minimal time lags from duration</td>
<td>0.5</td>
</tr>
<tr>
<td>( \text{OS} )</td>
<td>tightness of maximal time lags</td>
<td>0.5</td>
</tr>
<tr>
<td>( \text{OS} )</td>
<td>slack factor</td>
<td>0.0</td>
</tr>
<tr>
<td>( \text{K} )</td>
<td>number of renewable resource types</td>
<td>2, 3, 4, 6</td>
</tr>
<tr>
<td>( \text{rise} )</td>
<td>renewable resource demand</td>
<td>([1, 5])</td>
</tr>
</tbody>
</table>
The degree of redundancy is computed by dividing the number of redundant arcs in the network by their theoretical maximal value.

The coefficient of cycle structure density is a measure of the amount of precedence relations in a cycle structure. The tightness of maximal time lags determines how the values of the maximal time lags relate to their theoretical minimal value which results in time- and resource feasibility.

The slack factor determines how the minimal time lags depend on the activity modes [14]. Upper bound for resource demand of type \( k \) can be obtained using the following equation:

\[
\mathcal{P}_{imi,k} = \mathcal{P}_{imi} + \text{ran}_i
\]  

(58)

In order to find \( d_{imi} \), let: \( w_{imi} \) be the amount of total work required to complete activity \( i \) in mode \( m_i \) and can be stated in the following form when Microsoft office Project is used [15].

\[
w_{imi} = \sum_{k=1}^{\mathcal{K}} r_{imi,k}
\]  

(59)

### Table 5. Adaptations of the data to obtain instances of RCPSPMCM-GPR

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Important Control Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_m )</td>
<td>number of resources types required for each mode different from other modes</td>
<td>1, 2</td>
</tr>
<tr>
<td>( \bar{d}<em>{imi} = d</em>{imi} )</td>
<td>normal/maximum duration of each mode</td>
<td>([5, 10])</td>
</tr>
<tr>
<td>( \mathcal{P}<em>{imi,k} = r</em>{imi,k} )</td>
<td>lower bound renewable resource demand</td>
<td>([1, 5])</td>
</tr>
<tr>
<td>( \text{ran}_i )</td>
<td>a random real number for range between lower and upper bound of resource demand</td>
<td>([0, 3])</td>
</tr>
<tr>
<td>( \bar{\delta} )</td>
<td>due date of project for instances</td>
<td>0</td>
</tr>
<tr>
<td>( \mathcal{P} )</td>
<td>per period penalty cost of project</td>
<td>100</td>
</tr>
</tbody>
</table>

Assume \( w_{imi} \) is a fixed constant obtained by replacing \( d_{imi} \) and \( r_{imi,k} \) with \( \bar{d}_{imi} \) and \( \mathcal{P}_{imi,k} \), respectively. Accordingly, \( d_{imi} \) can be calculated by the following equation replacing \( r_{imi,k} \) with \( \mathcal{P}_{imi,k} \).

\[
d_{imi} = w_{imi} / \sum_{k=1}^{\mathcal{K}} \mathcal{P}_{imi,k}
\]  

(60)

For the instances of \( K_m = 1 \), \( \beta_{imi} \) is calculated simply by Eq. (40). In the situations involving \( K_m \geq 2 \) which mandate more than two observations, the sufficient observations can be obtained by generating a few random numbers for \( r_{imi,k} \) between \( \mathcal{P}_{imi} \) and \( \mathcal{P}_{imi,k} \), and then calculate \( d_{imi} \) by Eq. (60). At this time a software such as MINITAB or MATLAB can be used to obtain the relevant regression equations. In this paper we have used Eq. (41) for \( K_m = 2 \).

Since the CPU time for solving a mathematical model depends on the number of constraints and variables, to reduce the size of the experiment, we report only on important parameters influencing solution time. We fixed the other parameters which usually influence on the feasible area of the model at specific values rather than varying them over the entire range of complexity. Consequently, the instances are categorized according to the combinations of \( N = \{10, 20, 30, 40, 50\} \) and \((M_i, K_m, K) = (2,1,2), (3,1,3), (2,2,4), (3,2,6)\).

For each category (out of 20), 5 instances have been generated for a total of 100. The proposed GA has been coded in the MATLAB using fmincon(…) and ga(…) functions.

In order to have some benchmarks, the instances have been optimally solved by the Lingo 8.0 software using branch-and-bound (B&B) method as well. Each problem is allowed a maximum of 1000 seconds of CPU time. All the computational experiments have been carried out on an intel® Celeron® mobile 1.3 GHz Personal Computer with 512 Mb RAM. Tables 6 and 7 summarize our findings as average CPU times for B&B and GA.

### Table 6. The average CPU time for solving five instances in each category

<table>
<thead>
<tr>
<th>Number of Activities</th>
<th>( (M_i = 2, K_m = 1, K = 2) )</th>
<th>( (M_i = 3, K_m = 1, K = 3) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B&amp;B</td>
<td>GA</td>
</tr>
<tr>
<td>----------------------</td>
<td>-----</td>
<td>----</td>
</tr>
<tr>
<td>10</td>
<td>59.75</td>
<td>17.21</td>
</tr>
<tr>
<td>20</td>
<td>385.14</td>
<td>51.82</td>
</tr>
<tr>
<td>30</td>
<td>&gt;1000</td>
<td>167.60</td>
</tr>
<tr>
<td>40</td>
<td>&gt;1000</td>
<td>383.04</td>
</tr>
<tr>
<td>50</td>
<td>&gt;1000</td>
<td>925.22</td>
</tr>
</tbody>
</table>
The values below the columns of B&B and GA are the CPU time in second. The quality of GA in compare to B&B has been measured by O.F. Gap% which is the difference between objective functions of two methods. By increasing the number of activities, B&B can not reach to the optimal solution in an acceptable amount of time. The proposed GA could reach to near optimal solutions in shorter time respect to B&B method. As indicated in Tables 6 and 7, the solution time of proposed model is very sensitive to $K_n, K_n$ and $N$.

### 8. Conclusions

In this paper we considered the multi-mode resource constrained project scheduling problem with generalized precedence relations in which the duration of an activity is determined by a discrete mode selection and a continuous function of the resource requirements within the selected mode. This problem called RCPSPMCM-GPR is general case of the problem MRCPSP-GPR. In the absence of discrete multi-mode, the problem reduces to the continuous Time/Cost or Time/Resource Trade-off Problem. In the absence of crashing within a mode, the problem reduces to the RCPSP-GPR.

Although, considering real numbers for durations and resources increases the complexity of the problem, it allows flexibility to achieve an active pattern of resource usage over time as well as a shorter make span. Firstly, we presented an exact model for the RCPSP-GPR and MRCPSP-GPR. Secondly, the model was developed for optimally solving the problem RCPSPMCM-GPR. Thirdly, the genetic algorithm was used to solve the proposed nonlinear model. We modified the objective function by replacing the duration costs with the resource requirements costs. The proposed model has no need for a feasible solution to startup with. No formulation as an exact solution has been reported for the RCPSPMCM-GPR except the one presented in this paper. Furthermore, time horizon can be continuous in this model thus dealing with different processing time units (e.g., hours, days, weeks, and so on), is possible.

Future efforts will be devoted to the application of the other meta heuristic approaches for solving the nonlinear model in the situations involving instances of practical size (more than 100 activities) in a reasonable amount of time.

### Table 7. The average CPU time for solving five instances in each category

<table>
<thead>
<tr>
<th>Number of Activities</th>
<th>$(M_i = 2, K_m = 2, K = 4)$</th>
<th>$(M_i = 3, K_m = 2, K = 6)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B&amp;B</td>
<td>GA</td>
</tr>
<tr>
<td>10</td>
<td>143.19</td>
<td>26.76</td>
</tr>
<tr>
<td>20</td>
<td>&gt;1000</td>
<td>98.91</td>
</tr>
<tr>
<td>30</td>
<td>&gt;1000</td>
<td>268.04</td>
</tr>
<tr>
<td>40</td>
<td>&gt;1000</td>
<td>681.38</td>
</tr>
<tr>
<td>50</td>
<td>&gt;1000</td>
<td>&gt;1000</td>
</tr>
</tbody>
</table>

### References


