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Relax and Fix Heuristics for Simultaneous Lot Sizing and Sequencing the Permutation Flow Shops with Sequence-Dependent Setups

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KEYWORDS

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ABSTRACT

This paper proposes two relax and fix heuristics for the simultaneous lot sizing and sequencing problem in permutation flow shops involving sequence-dependent setups and capacity constraints. To evaluate the effectiveness of mentioned heuristics, two lower bounds are developed and compared against the optimal solution. The results of heuristics are compared with the selected lower bound.

1. Introduction

Flow shop is one of the most widely investigated production and scheduling problem of the literature [1-9]. It comprises a series of machines that perform operations on a production as it progresses down the line. A special case of flow shop that assumes the same order of products in all machines is called permutation flow shop.

Traditionally the problem of scheduling jobs on a flow shop is decomposed into the sub-problems of lotsizing and sequencing. This is an approximate way of solving the problem because in general the lotsizing decision is dependent on the sequencing decision. Nevertheless, there have been several heuristics developed for each of the above two problems. For a more detailed review of the relevant work done in this area, please refer to [18-21].

Researchers have also investigated the problem of integrating lotsizing and sequencing decisions in the flow shop. Sikora et al. [18] considered a variation with limited intermediate buffer space and deadlines, and they studied the objectives of minimizing C_{max} and inventory holding costs. They integrated the Silver-Meal lotsizing heuristic [19], which they modified to deal with lot splitting, with Palmer's flow shop heuristic [16], which they augmented with an improvement procedure, and demonstrated the effectiveness of their approach by scheduling an actual assembly line. In another paper, Sikora [17] presented a GA that used separate crossover and mutation operators for lotsizing and sequencing decisions. He compared this GA with the integrated approach presented in Sikora et al. [18] and found that the GA that used a population size of ten prescribed much better schedules with significantly less run time than the integrated approach. However, the performance of the GA was sensitive to the selection of parameter values and it was difficult to determine effective values. Lee et al. [12] presented a hybrid GA to minimize C_{max} in the case of a finite buffer space. This hybrid GA incorporates SA in its mutation operation and pairwise-exchange improvement procedures in an attempt to avoid the local optima at which GAs frequently stop. They evaluated the performance of this hybrid GA in an application to an actual assembly plant and their computational tests showed that it performs better than the pure GA, specially when the problem size is large.

Recently, simultaneous lot-sizing and scheduling in non-permutation capacitated flow shop with sequence-dependent setups has been considered by Mohammadi et al. [13-14]. They proposed a mathematical formulation and MIP-based heuristics for the problem. Involving capacity constraints, setup carry over and variable lotsizes in production stages are the main feature of their model. Because of restriction in computation times, the quality of solutions was poor specially for large instances of the problem.

To solve larger instances of problem, Mohammadi et al. [15] also proposed a new algorithmic approach based on a simplified mathematical model. In this

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paper, instead of solving a succession of smaller MIPs, they would relax all binary variables of the problem. The resulting problem would be solved through a T-iteration based algorithm. In a specific iteration *k*, relaxed binary variables of period k would be divided into two groups where members of the first group would get value 1 and members of the second group would get value 0.

Current paper proposes two relax and fix heuristics for the simultaneous lotsizing and sequencing problem in permutation flow shops involving sequence-dependent setups and capacity constraints.

The paper has the following structure. Section 2 introduces a detailed description of the problem and its underlying assumptions. Section 3 deals with the development and comparison of lower bounds in detail and section 4 provides heuristics. Section 5 reports the numerical experiments and finally section 6 discusses the concluding remarks and recommendations for future studies.

2. Problem Formulation

2.1. Assumptions

Several products are produced on serially-arranged machines. Order of products in all machines is the same.

- Each machine is constrained in capacity.
- When the machines are setup, sequence-dependent setup costs and times occur.
- The setting-up of a machine must be completed in a period.
- There must be precisely *N* (number of products) setups in each period on each machine, even if a setup is just from a product to itself. Since a setup time (and cost) from a product to itself is zero, note that the model does not force a machine to have exactly *N* positive-time (and cost) setups but rather up to *N* such setups. The remaining zero-time (cost) setups are modeling phantoms and do not exist in reality ([4] & [5]).
- External demand exists for final products and is satisfied at the end of each period.
- There are no lead times between the different production levels for transportation or cooling the products.
- Shortages are not permitted.
- A component cannot be produced earlier in a period than the production of its required component is finished. In other words, production on a production level can only be started if a sufficient amount of the product from the previous production level is available; this is called vertical interaction.
- To guarantee the vertical interaction, idleness between each setup and its production is defined with the help of shadow product ([10], [13]-[15]).
- There are no demand and no storage costs for shadow products.
- At the beginning of each period, machines are not setup for each of products.
- The triangle inequality holds, i.e., it is never faster to change over from one product to another by means of a third product. In other words, a direct changeover is at least as capacity efficient as going via another product.

2.2. Mathematical Model

The following notations are used in the model formulation:

Indices:

i, j, k	Index of product type
n, n'	Designation for a specific setup number
m	Index of production's level
t	Index of planning period

Parameters:

T	Planning horizon
N	Number of different products
M	Number of production levels / number of machines
$bigM$	A large real number
$C_{m,t}$	Available capacity of machine *m* in period *t* (in time units)
$d_{j,t}$	External demand for product *j* at the end of period *t* (in units of quantity)
$h_{j,m}$	Storage costs unit rate for product *j* in level *m*
$b_{j,m}$	Capacity of machine *m* required to produce a unit of product (or shadow product) *j* in time units per quantity units
$p_{j,m,t}$	Production costs to produce one unit of product *j* on machine *m* in period *t* (in money unit per quantity unit)
$S_{i,j,m}$	Sequence-dependent setup time at machine *m* when switching from product *i* to *j* in time units; (for *i≠j*, $S_{i,j,m} \geq 0$ and for *i=j*, $S_{i,j,m}$ =0)
$W_{i,j,m}$	Sequence-dependent setup cost at machine *m* when switching from product *i* to *j* in money units; (for *i≠j*, $W_{i,j,m} \geq 0$ and for *i=j*, $W_{i,j,m}$ =0. We also assume that the relation between setup costs and times can be considered as: $W_{i,j,m} = f_w \cdot S_{i,j,m}$ where f_w is opportunity cost per unit of setup time
0	The setup configuration on machines at the beginning of each period

Decision Variables:

$I_{j,m,t}$	Inventory level of product *j* at level *m* at the end of period *t*
$y^n_{i,j,t}$	1, if the *n*th setup on machines is performed at period *t* when switching from product *i* to *j*; 0, otherwise
$x^n_{j,m,t}$	Quantity of product *j* produced after *n*th setup on machine *m* at period *t*

$q^{n}_{j,m,t}$ Shadow product indicating the gap (in quantity units) between *nth* setup (to product *j*) on machine *m* at period *t* and its related production in order to ensure that direct predecessor of this product (production of product *j* on machine *m-1* at period *t*) has been completed. In other words, it denotes the idle time (in quantity units) before production of product *j* on machine *m* in period *t* in order to guarantee vertical interaction.

According to the above notation, the proposed mathematical formulation for the problem can be written as follows:

$$Min \sum_{n=1}^{N}\sum_{j=1}^{N}\sum_{i=0}^{N}\sum_{m=1}^{M}\sum_{t=1}^{T} w_{i,j,m}\cdot y^{n}_{i,j,t} + \sum_{n=1}^{N}\sum_{j=1}^{N}\sum_{m=1}^{M}\sum_{t=1}^{T} p_{j,m,t}\cdot x^{n}_{j,m,t} + \sum_{j=1}^{N}\sum_{m=1}^{M}\sum_{t=1}^{T} h_{j,m}\cdot I_{j,m,t} \tag{1}$$

Subject to:

$$d_{j,t} = I_{j,M,t-1} + \sum_{n=1}^{N} x^{n}_{j,M,t} - I_{j,M,t} \; ; j=1,\dots,N,\ t=1,\dots,T \tag{2}$$

$$I_{j,m,t-1} + \sum_{n=1}^{N} x^{n}_{j,m,t} = I_{j,m,t} + \sum_{n=1}^{N} x^{n}_{j,m+1,t} \; ; \quad j=1,\dots,N,\ m=1,\dots,M-1,\ t=1,\dots,T \tag{3}$$

$$\begin{aligned} &\sum_{n=1}^{n'}\sum_{i=0}^{N}\sum_{j=1}^{N} y^{n}_{i,j,t}\cdot S_{i,j,m} + \sum_{n=1}^{n'}\sum_{j=1}^{N} b_{j,m}\cdot q^{n}_{j,m,t} + \sum_{n=1}^{n'}\sum_{j=1}^{N} b_{j,m}\cdot x^{n}_{j,m,t} \le \sum_{n=1}^{n'}\sum_{i=0}^{N}\sum_{j=1}^{N} y^{n}_{i,j,t}\cdot S_{i,j,m+1} \\ &+ \sum_{n=1}^{n'}\sum_{j=1}^{N} b_{j,m+1}\cdot q^{n}_{j,m+1,t} + \sum_{n=1}^{n'-1}\sum_{j=1}^{N} b_{j,m+1}\cdot x^{n}_{j,m+1,t}\, , \ \mathbf{n'=1,\dots,N,\ m=1,\dots,M\text{-}1,\ t=1,\dots,T} \end{aligned} \tag{4}$$

$$\sum_{n=1}^{N}\sum_{i=0}^{N}\sum_{j=1}^{N} y^{n}_{i,j,t}\cdot S_{i,j,m} + \sum_{n=1}^{N}\sum_{j=1}^{N} b_{j,m}\cdot x^{n}_{j,m,t} + \sum_{n=1}^{N}\sum_{j=1}^{N} b_{j,m}\cdot q^{n}_{j,m,t} \le C_{m,t} \tag{5}$$

$m=1,\dots,M,\ t=1,\dots,T$

$$x^{n}_{j,m,t} \le (C_{m,t}/b_{j,m}).\sum_{i=0, i\ne j (for\, n>1)}^{N} y^{n}_{i,j,t} \tag{6}$$

n=1,…,N, j=1,…,N, m=1,…,M, t=1,…,T

$$q^{n}_{j,m,t} \le (C_{m,t}/b_{j,m}).\sum_{i=0}^{N} y^{n}_{i,j,t} \tag{7}$$

n=1,…,N, j=1,…,N, m=1,…,M, t=1,…,T

$$y^{1}_{j,i,1} = 0 \; ; j\ne 0\ , i=1,\dots,N \tag{8}$$

$$\sum_{i=1}^{N} y^{1}_{0,i,1} = 1 \tag{9}$$

$$\sum_{j=0}^{N} y^{n}_{j,i,t} = \sum_{k=1}^{N} y^{n+1}_{i,k,t} \; ; n=1,\dots,N-1,\ i=1,\dots,N,\ t=1,\dots,T \tag{10}$$

$$y^{n}_{i,j,t} = 0 \text{ or } 1 \tag{11}$$

$$I_{j,m,t}\, ,\, x^{n}_{j,m,t}\, ,\, q^{n}_{j,m,t} \ge 0 \tag{12}$$

$$I_{j,m,0} = 0 \; ; j=1,\dots,N,\ m=1,\dots,M \tag{13}$$

In this model, equation (1) represents the objective function which minimizes the sum of the sequence-dependent setup costs, the storage costs and the production costs. Equation (2) ensures the demand supply in each period. Equation (3) shows that in a network, total of in-flows to each node is equal to out-flows from that node.

Equation (4) guarantees within one period each typical product *j* on machine *m* is produced before its direct successor (product *j* on machine *m+1*).

Equation (5) represents the capacity constraints of machines during periods.
Equation (6) indicates that setup is considered in production process.
Equation (7) indicates the relationship between shadow products and setups.
Equations (8) and (9) guarantee that for each machine, the first setup at the beginning of the planning horizon is from a defined product.
Equation (10) represents the relationship between successive setups.
Equations (11) and (12) represent the type of variables.
Equation (13) indicates that at the beginning of planning horizon there is no on-hand inventory.

3.Development of Lower Bounds

So far, we have successfully formulated the problem. However, the formulation presented in the previous section is not a practical approach to solve large instances of the problem.

In this section we obtain two lower bounds on the problem. First lower bound is achieved by solving model M1 that is obtained from the initial model by relaxing all binary variables. The second lower bound is obtained by solving a new model M2 that is derived from M1, adding the following equation :

$$\sum_{i=0}^{N} y_{i,j,t}^{1} + \sum_{i=1,i\neq j}^{N} \sum_{n=2}^{N} y_{i,j,t}^{n} = a_{j,t} \tag{14}$$

$a_{j,t}$ is a binary variable.

Equation (14) is similar to the right hand side of equation (6), $\sum_{i=0,i\neq j(for\,n>1)}^{N} y_{i,j,t}^{n}$. In equation (14), we aggregate $\sum_{i=0,i\neq j(for\,n>1)}^{N} y_{i,j,t}^{n}$ by summing over all *n*.

Lemma 1. Equation (14) is valid to M1.

Proof. Suppose that equation (14) is not valid to M1, it means that there is at least one couple (j,t), where $\sum_{i=0}^{N} y_{i,j,t}^{1} + \sum_{i=1,i\neq j}^{N} \sum_{n=2}^{N} y_{i,j,t}^{n} > 1$.

Suppose that $\sum_{i=0}^{N} y_{i,j,t}^{1} + \sum_{i=1,i\neq j}^{N} \sum_{n=2}^{n'-1} y_{i,j,t}^{n} = 1$ and $y_{k,j,t}^{n'} = 1$, (k≠j). For n = n' to N, all j would be changed to the k. By this modification, for all couple (j,t), $\sum_{i=0}^{N} y_{i,j,t}^{1} + \sum_{i=1,i\neq j}^{N} \sum_{n=2}^{N} y_{i,j,t}^{n} = 0$ or 1.

According to the fact that setup costs from a product to itself is zero and considering the triangle inequality :

$$(y_{k,k,t}^{n'} \cdot W_{k,k,m} + y_{k,i,t}^{n'+1} \cdot W_{k,i,m}) \leq (y_{k,j,t}^{n'} \cdot W_{k,j,m} + y_{j,i,t}^{n'+1} \cdot W_{j,i,m})$$

Therefore, by assuming that equation (14) is not valid to M1, there would be a solution better than or equal to the optimum of M1 and it is impossible.

4. Relax and Fix Heuristics

One important approach to find feasible solusions for larger instances of MIPs is using fix and relax method [7]. Dillenberger et al. [8] formulated a lot sequencing and sizing model with representation of sequence-independent setup times on multiple machines. The resulting mixed integer programming (MIP) model is difficult to solve optimally for industrial problems, and so the authors resorted to the fix-and-relax method [7], more widely known as relax-and-fix [20]. This involves the solution of a series of partially relaxed MIPs, each with a number of binary variables that is small enough to be quickly and optimally solved by conventional branch-and-bound methods. As the series progresses, each set of binary variables is permanently fixed at their solution values, and the relaxed variables are reduced in number, eventually disappearing. The procedure is broadly similar to a depth-first identification of an initial integer solution for a MIP model in a branch-and-bound search. Speed is its major advantage. Araujo et al. [2] and Beraldi et al .[3] are two recent applications of relax-and-fix method to solve MIPs.

4.1. The First Relax and Fix Heuristic (H1)

In this heuristic, demand of each period is satisfied by producing during that period, this is guaranteed using equation (15) fo current period.

$$\sum_{n=1}^{N} \sum_{i=0,i\neq j(for\,n>1)}^{N} y_{ijt}^{n} = 1 \tag{15}$$

The formal procedure contains the following steps.

Step1. The first setup in period 1 is from product 0, fix the values of $y_{0,j,1}^{1}$ to be 1 and the values of $y_{i,j,1}^{1}$ to be 0 for j=1,…,N and i≠0.

Step2. To identify to which product the second setup is in period 1, solve the partial linear programming relaxation, other than those fixed in step 1, where the values of $y_{i,j,1}^{2}$ are constrained to be 0 or 1. Step3. For n=3,…,N, solve the partial linear programming relaxation, with the $y_{j,i,1}^{1}$ and $y_{j,i,1}^{2}$ fixed at their 0 or 1 solution values from steps 1 and 2, with the values of $y_{j,i,1}^{3}$ to $y_{j,i,1}^{n-1}$ fixed at their 0 or 1 solution values from the previous applications of step 3, and with the values of $y_{j,i,1}^{n}$ constrained to be 0 or 1, while the remaining y-variables may vary continuously between 0 and 1.

Step 4. For t=2,…,T, repeat steps 1 and 3 for $y_{i,j,t}^{1}$ to $y_{i,j,t}^{N}$, fixing their values at those of the solutions in steps 1 and 3.

Note that each cycle of steps 1 and 2 involves solving N problems with N^2 binary variables each, as N(N-1) of y-variables in each problem will newly have value 0 due to the constraints (9) and (10). Thus, the application of the cyclic fix-and-relax approach involves the solution of $N.T$ MIPs with N binary variables each.

4.2. The Second Relax and Fix Heuristic 2 (H2)

The only difference between this heuristic and the former one is that, H2 permits the demand of a period to be satisfied by producing in last periods. In this heuristic, for current period, equation (15) is replaced by equation (16).

$$\sum_{n=1}^{N} \sum_{i=0, i\neq j (for\, n>1)}^{N} y_{ijt}^{n} = a_{jt} \qquad (16)$$

a_{jt} = 0. or 1.

Thus, the application of the cyclic fix-and-relax approach involves the solution of N.T MIPs with 2N binary variables each.

5. Numerical Experiments

In order to ascertain the accuracy of mentioned lower bounds, we performed some numerical tests. Tables 1 and 2 respectively show the results of such tests in some instances of the problem with *(N=3 , M=2 , T=3)* and *(N=3 , M=3 , T=3)*.

Tab. 1. Comparison of lower bounds and exact optimal solutions in problem size N=3, M=2 and T=3.

The values inside the brackets are the computational time in seconds and the percentage values are the difference between the objective values of the lower bound against the original model.

No.	Original Problem	First lower bound	Second lower bound
	3107.06	2799.56	2990.33
1		9.90%	3.76%
	(12.71)	(0.06)	(0.21)
	3496.35	2907.08	3341.76
2		16.86%	4.42%
	(10.17)	(0.08)	(0.31)
	3205.10	2737.97	3056.67
3		14.58%	4.63%
	(17.22)	(0.13)	(0.29)
	3333.55	2811.88	3231.77
4		15.65%	3.05%
	(13.41)	(0.09)	(0.19)
	3381.63	2813.06	3158.12
5		16.81%	6.61%
	(6.34)	(0.06)	(0.31)

Tab. 2. Comparison of lower bounds and exact optimal solutions in problem size N=3, M=3 and T=3.

The values inside the brackets are the computational time in seconds and the percentage values are the difference between the objective values of the lower bound against the original model.

No.	Original Problem	First lower bound	Second lower bound
	4550.98	3788.75	4373.22
1		16.75%	3.91%
	(131.41)	(0.11)	(0.65)
	4998.95	4301.39	4770.74
2		13.95%	4.57%
	(149.77)	(0.19)	(0.41)
	5143.65	4279.41	4993.17
3		16.80%	2.93%
	(116.53)	(0.16)	(0.49)
	4766.01	4074.29	4610.72
4		14.51%	3.26%
	(186.51)	(0.22)	(0.78)
	5310.58	4496.89	5007.38
5		15.32%	5.71%
	(188.13)	(0.23)	(0.59)

Comparing the results of second columns of Tables 1 and 2 shows that computation time grows exponentially by increasing the dimension of problem. According to Table 1, average computational time for problems with (N=3, M=2, T=3) is 11.97s. According to Table 2, average computational time for problems with (N=3, M=3, T=3) is 154.47s. It means that by increasing one level to production levels, average computational time increases more than 12 times. Tables 1,2 confirm the advantages of the second lower bound, therefore it has been used to compare heuristics. To evaluate and compare the performance of proposed heuristics against the second lower bound, we consider twenty different problem sizes in the range of (N.M.T)=(3*3*3) to (N.M.T)=(15*15*15). List of the problems is shown in Tables 3,4. For each problem size, 5 problem instances are randomly generated.

The exact model, lower bounds and heuristics were coded using GAMS IDE (ver 2.0.19.0) and OSL 2. GAMS models are run on a personal computer with a Pentium 4 processor running at 3.4 GHZ.

The application of H1 and H2 involves the solution of T.N smaller MIPs. In order to evaluate the quality of heuristics in equal amount of time, the computational time of each MIP is limited to (7200/T.N) seconds for heuristics. Tables 3 and 4 compare the objective functions and cpu times of heuristics and the second lower bound. The required parameters for all numerical experiments are extracted from the following uniform distributions :

$b_{j,m} \approx U(1.5,2)$, $d_{j,t} \approx U(0,180)$, $h_{j,m} \approx U(0.2,0.4)$, $p_{j,m,t} \approx U(1.5,2)$, $W_{i,j,m} \approx U(35,70)$, $S_{i,j,m} \approx U(35,70)$, $C_{m,t} = U(a,b)$,

a=200.N+100.(m-1), b=200.N+200.(m-1).

$C_{m,t}$ is calculated in accordance to satisfy demands of each period on a just-in-time basis with average setups.

Table 3.Comparison of objective functions of the second lower bound and heuristics.

Problem Size (N.M.T)	Number of problem solved	The second lower bound	H1	H2
3*3*3	5	4738.57	5525.07	5327.43
5*3*3	5	7944.38	9269.75	9003.89
3*5*3	5	7887.60	9262.72	8850.62
3*3*5	5	8013.19	9451.46	9234.08
5*5*5	5	22439.33	26420.63	25506.47
7*5*5	5	29611.62	36321.87	35268.54
5*7*5	5	28590.03	35274.49	33655.37
5*5*7	5	29898.77	36735.31	35963.87
7*7*7	5	58602.58	72737.52	69180.66
10*5*5	5	43201.10	52022.13	50415.17
5*10*5	5	41394.32	51025.74	498899.92
5*5*10	5	43088.67	53748.46	51652.23
10*7*7	5	92190.93	113256.65	110596.25
7*10*7	5	89903.07	109742.38	105462.43
7*7*10	5	90236.49	110457.34	106149.51
10*10*10	5	183117.57	230028.75	221181.84
15*10*10	5	289006.90	362070.12	---
10*15*10	5	283046.25	357616.67	350070.96
10*10*15	5	272018.56	354437.45	---
15*15*15	5	674419.93	832505.18	---

* *It means that feasible solution has not been found after 7200 seconds of computing time.*

Tab. 4. Comparison of cpu times of the second lower bound and heuristics.

The values inside the brackets are the computational time in seconds. * means that finding the optimum value for the second lower bound requires more than 7200 seconds and the objective function at this time has been considered.--- means that feasible solution has not been found after 7200 seconds of computing time.

Problem Size (N.M.T)	Number of problem solved	The second lower bound	H1	H2
3*3*3	5	(0.49)	(0.13)	(0.70)
5*3*3	5	(11.61)	(0.44)	(2.98)
3*5*3	5	(2.80)	(0.16)	(1.42)
3*3*5	5	(4.23)	(0.23)	(0.91)
5*5*5	5	(58.37)	(2.19)	(24.51)
7*5*5	5	(149.95)	(15.74)	(208.22)
5*7*5	5	(98.53)	(5.21)	(58.31)
5*5*7	5	(109.21)	(5.28)	(38.81)
7*7*7	5	(288.38)	(77.61)	(1437.27)
10*5*5	5	(912.25)	(132.55)	(2095.89)
5*10*5	5	(150.06)	(11.43)	(127.51)
5*5*10	5	(211.61)	(8.52)	(96.31)
10*7*7	5	(1837.74)	(298.96)	(3807.97)
7*10*7	5	(611.34)	(58.72)	(1411.13)
7*7*10	5	(958.13)	(79.60)	(1983.31)
10*10*10	5	(4306.88)	(823.33)	(5711.96)
15*10*10	5	>7200*	(1831.27)	---
10*15*10	5	>7200*	(1138.51)	(6854.35)
10*10*15	5	>7200*	(1383.41)	---
15*15*15	5	>7200*	(3231.59)	---

6. Concluding Remarks

The contribution of the paper has been to derive and test one exact formulation and two relax and fix heuristics for simultaneous lotsizing and sequencing in permutation flow shops with sequence – dependent setups.

To test the accuracy of proposed heuristics, two lower bounds are developed and compared against the optimal solution. Selected lower bound is used to test the accuracy of proposed heuristics.
Because of the expanding role of meta-heuristic approaches to solve complicated lotsizing problem ([6] & [11]), the application of meta-heuristic approaches to face this complex problem is recommended as an area for future research.

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