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IMAGE SEGMENTATION USING GAUSSIAN MIXTURE MODEL

Rahman Farnoosh, and Behnam Zarpak

Abstract: *Stochastic models such as mixture models, graphical models, Markov random fields and hidden Markov models have key role in probabilistic data analysis. In this paper, we used Gaussian mixture model to the pixels of an image. The parameters of the model were estimated by EM-algorithm.*
In addition pixel labeling corresponded to each pixel of true image was made by Bayes rule. In fact, a new numerically method was introduced for finding the maximum a posterior estimation by using EM-algorithm and Gaussians mixture distribution. In this algorithm, we were made a sequence of priors, posteriors were made and then converged to a posterior probability that is called the reference posterior probability. Maximum a posterior estimated can determine by the reference posterior probability which can make labeled image. This labeled image shows our segmented image with reduced noises. We presented this method in several experiments.

Keywords: *Bayesian Rule, Gaussian Mixture Model (GMM), Maximum a Posterior (MAP), Expectation- Maximization (EM) Algorithm, Reference Analysis.*

1 Introduction

Automatically image segmentation means dividing an image into different types of regions or classes, recognizing the objects, and detecting of edges by machine. All of these can be obtained after segmentation of a picture.
In addition noise removing and noise reduction of pictures also are important in classical image problems. There are many mathematical methods for noise reduction, image segmentation, objects recognition and edge detection. However this paper argues probabilistic approach via Gaussian mixture model as a general probability distribution for doing these problems. Nevertheless, Bayesian data analysis have many difficulties and it uses complex MCMC algorithms or variational methods (VM) with high computing to get Maximum a posterior (MAP). These have worked well in the last decades [1 ,2].
In this article, Bayesian rule for finding MAP estimation was used by EM-algorithm numerically. This paper organized as follows.
GMM and its properties was first reviewed, then EM-MAP algorithm for learning parameters in a given image training data was introduced for construction of a suitable pixels based GMM model. In addition, this article made a sequence of priors and the posteriors probability for MAP finding. The initial values of EM-algorithm were chosen such that one can get convergence. Finally, some experiments by simulated images were shown.

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Rahman Farnoosh, & Behnam Zarpak, Department of Applied Mathematics, Iran University of Science and Technology, rfarnoosh@iust.ac.ir, *zarpak@iust.ac.ir*

2. Gaussian Mixture Model

Image is a matrix within which each element is a pixel. The value of the pixel is a number that shows intensity or color of the image. Let X be a random variable that takes these values. For a probability model determination, we can suppose to have mixture of Gaussian distribution as the following form:

$$f(x) = \sum_{i=1}^{k} p_i N(x \mid \mu_i, \sigma_i^2) \qquad (1)$$

Where k is the number of components or regions and $p_i > 0$ are weights such that $\sum_{i=1}^{k} p_i = 1$,

$$N(\mu_i, \sigma_i^2) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(\frac{-(x-\mu_i)^2}{2\sigma_i^2}\right) \qquad (2)$$

where μ_i, σ_i^2 are mean and standard deviation of class i.
For a given image X, the lattice data are the values of pixels and MoG is our pixel based model. However, the parameters are $\theta=(p_1,\dots,p_k,\mu_1,\dots,\mu_k,\sigma_1^2,\dots,\sigma_k^2)$ and we can guess the number of regions in MoG by histogram of lattice data.

3. EM-MAP Algorithm

There are several published articles about EM algorithm for GMM [3, 4]. In the present paper the EM algorithm was modified and renamed to EM-MAP algorithm. The process of E-MAP algorithm can be defined:

0. Input: Observed Image in a vector $x_j, j = 1, 2, \dots, n$ and $i \in \{1, 2, \dots, k\}$ labels set

1. Initialize:
$$\theta^{(0)} = (p_1^{(0)}, \dots, p_k^{(0)}, \mu_1^{(0)}, \dots, \mu_k^{(0)}, \sigma_1^{2\,(0)}, \dots, \sigma_k^{2\,(0)})$$

2. (E-step)
$$p_{ij}^{(r+1)} = P^{(r+1)}(i \mid x_j) = \frac{p_i^{(r)} N(x_j \mid \mu_i^{(r)}, \sigma_i^{2(r)})}{f(x_j)} \qquad (3)$$

3. (M-step)
$$\hat{p}_i^{(r+1)} = \frac{1}{n} \sum_{j=1}^{n} p_{ij}^{(r)} \qquad (4)$$
$$\hat{\mu}_i^{(r+1)} = \frac{\sum_{j=1}^{n} p_{ij}^{(r+1)} x_j}{n \hat{p}_i^{(r+1)}} \qquad (5)$$
$$\hat{\sigma}_i^{2(r+1)} = \frac{\sum_{j=1}^{n} p_{ij}^{(r+1)} (x_j - \hat{\mu}_i^{(r+1)})^2}{n \hat{p}_i^{(r+1)}} \qquad (6)$$

4. Iterate steps 2 and 3 until an arbitrary error i.e. $\sum_i e_i^2 < \varepsilon$

5. Compute $p_{lj} = Arg\ Max_i\ p_{ij}^{(final)}$
$j = 1, 2, \dots, n$

6. Construct labeled image corresponding of each image**.**

This EM-MAP algorithm is a pixel labeling method such that the labeled image shows each segment or object by different type of labels. Note that formula (3) is Bayes rule. p_i^r is discrete prior probability in stage r and p_{ij}^{r+1} is discrete posterior probability in the next stage.

In this algorithm, a sequence of priors and then posteriors were made until to get convergence. The labeled images chooses with MAP of the final posterior.

4. The Entropic Prior

In EM-MAP algorithm, there are some difficulties in choosing the following:

1) the number of classes
2) the weights
3) the means
4) the variances

In practice we can guess the prior of parameters for drawing the histogram of the observed image. Not only image histogram gives us the above four parameters for using initial values in the EM algorithm, but also this extracted information usually has maximum entropy. We we plan to come back to this clame later. Besides, the final posterior probability will get stable entropy.

We also compute the number of misclassification of final picture. This shows that how much our algorithm is well.

5. Practical Experiments

In first example, we make three boxes in an image and add to it white noise. The observed image and its histogram are presented in Figs. 1and 2.

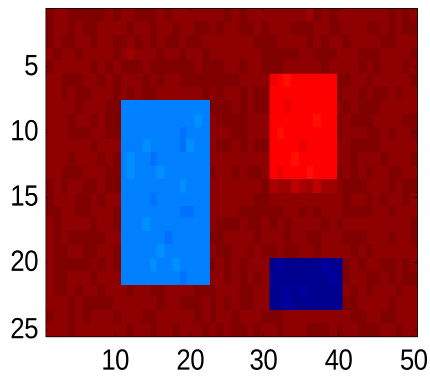


Fig 1. The observed image

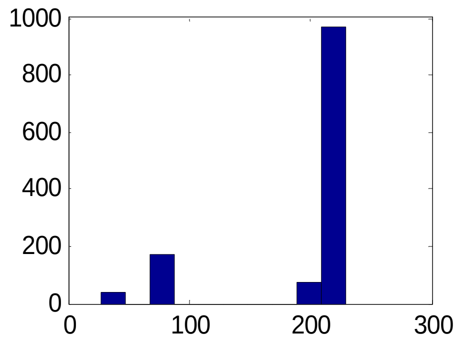


Fig 2. The histogram of observed image

From the histogram (Fig 2) we can extracted the following informations:

k=4, the empirical probability is $p^{(0)} = (0.0320 \quad 0.1344 \quad 0.0576 \quad 0.7760)$ and the entropy of $p^{(0)}$ is 0.7411, $\mu^{(0)} = (40\ 75\ 210\ 220)$ and $\sigma^{2\,(0)} = (100\ 100\ 100\ 100)$**.**

The stopping time occurs when L-2 norm of absolute errors have very small value. After running EM-MAP, we had six-time iteration and this figure 3.

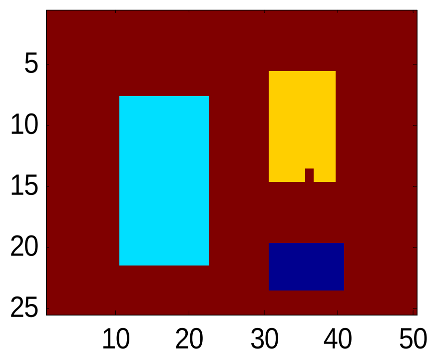


Fig 3. The observed image after six times iteration

Where ‘Blue=1’, ‘Cyan=2’, ‘Yellow=3’ and ‘Red=4’. There is percentage 0.0008 misclassification that is only one red instead of yellow pixel. In this example, pixel labeling and noise removing are well done.
In the second example, some different shapes such as circle, triangle, rectangle and etc.
Consider In the following image Fig 4.

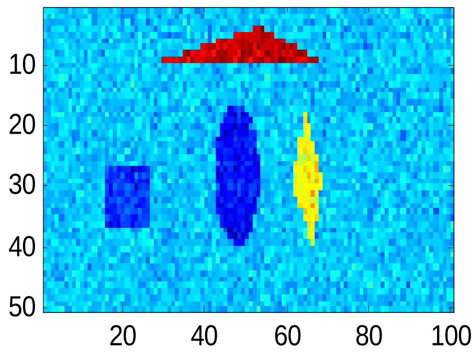


Fig 4. The observed image

For EM-MAP finding, we need to get initial values by histogram of this image. (Fig 5)
We choose k=3, p=(0.0800 0.3400 0.5800) as empirical probability or relative frequency, m=(38 63 88) and d=[12 12 12] with norm of error less than 0.01.

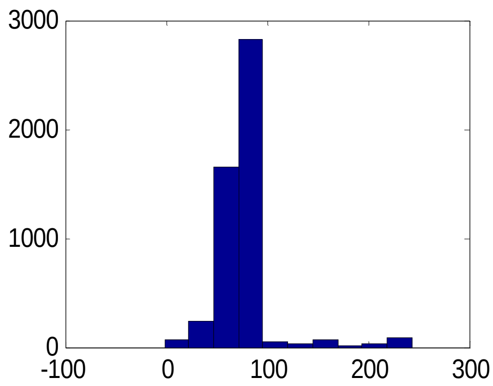


Fig 5. The histogram of observed image

Labeled image after 23 times iteration is the following form Fig 6.

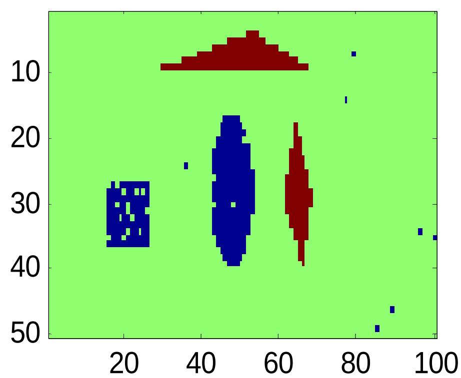


Fig 6. The labeled image

Where ‘Blue=1’, ‘Green=2’ and ‘Red=3’. We have only 25 misclassifications or percentage 0.005.
If in EM-MAP algorithm, we compute the entropy of the posterior probability in
Each stage of iteration, this entropy will be decreasing to reach a stable form Fig 7.

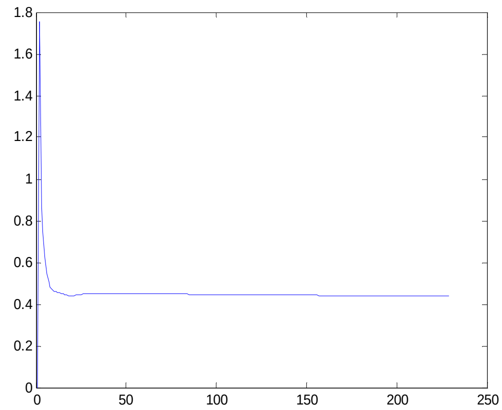


Fig 7. The entropy of the posterior probably

The third example is more complex. The number of components is great. In addition, there is dependent noise in image which noise reduction in this case is more difficult. The true image and observed image have shown as (Fig 8 & 9).

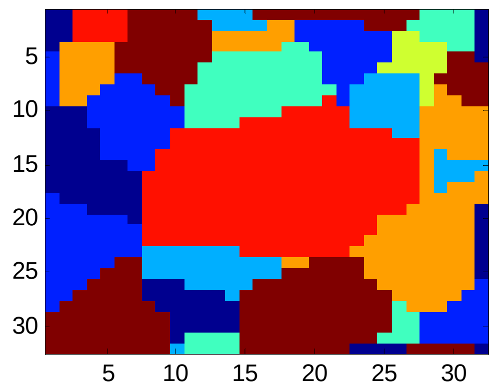


Fig 8. The true image

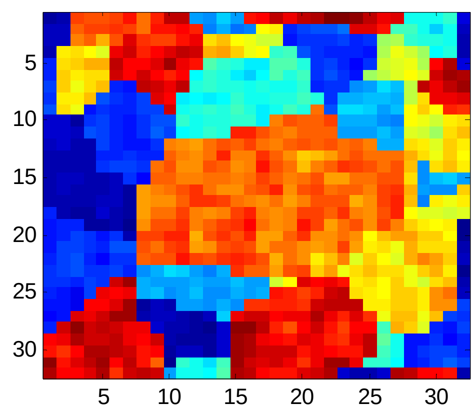


Fig 9. The observed image

Again, information extraction can find by drawing histogram of observed image Fig 10.

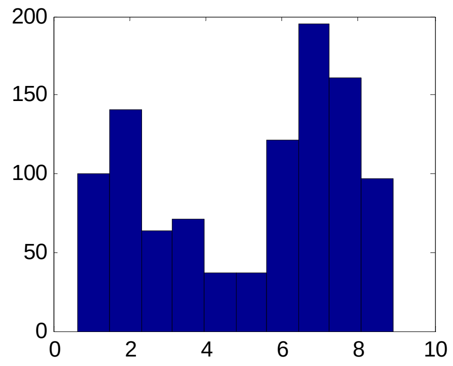


Fig 10. The histogram

So

- k=10
- p=(0.0977 0.1377 0.0625 0.0693 0.0361 0.0361 0.1182 0.1904 0.1572 0.0947)
- m=(1 2 2.5 3.5 4.5 5.5 6 7 7.5 8.5)
- d=(0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5]

After run EM-MAP algorithm, only with seven times iteration and sum of square error $\sum_i e_i^2 < 0.01$, we have Fig 11.

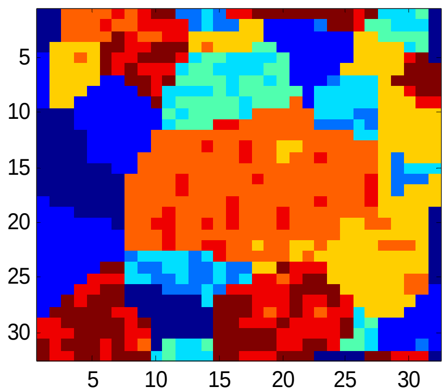
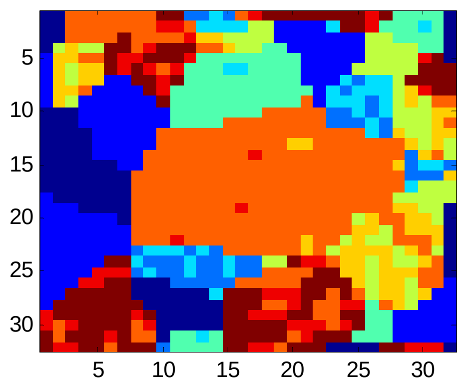


Fig 11. The observe image after seven times iteration

With 1080 times iteration and sum of square error $\sum_i e_i^2 < 10^{-6}$, we have fig 12.



Fige 12. The observed image after 1080 times itération

6. Conclusions

In this paper, we made a new numerical EM-GMM-Map algorithm for image segmentation and noise reduction. This paper used Bernardo's idea [2] about sequence of prior and posterior in reference analysis.

We have used known EM-GMM algorithm and we added numerically MAP estimation. The initial values by histogram of image have suggested is caused to convergence of EM-MAP method. After convergence of our algorithm, we had stability in entropy. Our algorithm is an iteration algorithm of first order, so we had slow convergence. We used acceleration convergence such as Steffensen algorithm to have the second order convergence. But later we noted that in EM-MAP method, the number of classes will reduce to real classes of image. Finally, EM-algorithm is linear iteration method [3], so our method is suitable for simple images. It is important to note that "for segmentation of real images, the results depend critically on the features and feature models used" [4]. But they are not the focus of this paper.

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