



A Fuzzy Approach For Multi-Objective Supplier Selection

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ABSTRACT

Assessment and selection of suppliers are two most important tasks in the purchasing part in supply chain management. Supplier selection can be considered to be a single or multi-objective problem. From another point of view, it can be a single or multi-sourcing problem. In this paper, an integrated AHP and Fuzzy TOPSIS model is proposed to solve the supplier selection problem. This model makes the decision-maker to be able to solve this problem with different criteria and different weight for each criterion with respect to the purchasing strategy. Finally, the proposed model is illustrated by an example.

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1. Introduction

Since purchasing part in supply chain management has direct affect on reduction of costs and also increasing of advantages and flexibility of organization, therefore the purchasing is one of the main parts in performance of organization [1].

In most industries the purchasing cost of goods and services constitutes the main cost of a product, such that in some cases, it can reach to up to 70% [2]. In high technology companies, costs of materials and services constitute up to 80% of total product cost [3]. Therefore, many experts believe that the supplier selection is the most important activity of a purchasing department.

Because selecting the right suppliers reduces the purchasing cost and improves corporate competitiveness [4], [5]. The main goal of the works in supply chain management is the customer satisfaction that means he/she can buy his/her buyers needs with maximum quality and minimum price and in short time.

Suppliers can effect on some objectives of the organization such as technology, performance and delivery capability.

The main goals in supplier selection are reduction purchase risks and creation long and good relationship between the suppliers and the purchaser [6].

Indeed, the supplier selection includes two issues. First, which suppliers must be selected? And the amount of purchasing from each of them must be determined. Solutions to these two questions reduce costs and improve competitive situation of the organization [7].

Supplier selection is a multiple criteria decision-making (MCDM) problem. Some conflicting factors such as price, quality and delivery capability effect on the supplier selection problem [8]. For the first time, Dickson carried out priority determination of 23 different commonly used criteria for the supplier selection problem based on sending questionnaire to 273 purchasing agents.

He found that the quality, delivery capability and performance history are the most important criteria [9]. Furthermore, Moore and Fearon stated that price, quality and delivery are important criteria for the supplier selection. They represented an approach based on the linear programming that can be applied to this decision making [10].

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Weber et al. reviewed 74 various works. They shown the pure price of products is the most important criteria for the supplier selection problem. They deduced that the supplier selection problem is the multi-criterion problem. In addition, the priority of each criterion depends on purchasing conditions [3]. Furthermore, Ghodsypour et al. investigated the related works and found the number of criteria and weight of each criterion depends on purchasing strategy [11]. There are two kinds of supplier selection problems: Single sourcing, in which a supplier can satisfy all the buyer's requests. And multi-sourcing, in which more than one supplier have to be selected because no supplier can satisfy all buyer's requirements. From another point of view, the supplier selection problem can be divided into two categories: single-objective and multi-objective programming. [12]. Several methods have been developed to solve the supplier selection problem, such as linear programming (LP), non-linear programming, dynamic programming, mixed integer programming, probabilistic programming, decision theory, analytic network process (ANP), neural network (NN), data envelopment analysis (DEA), case based reasoning (CBR) and fuzzy set theory (FST). To use the advantages of these methods and overcome their weaknesses, the integration of different methodologies has been developed [8]. Bellman et al. suggested a fuzzy programming model for decision-making in fuzzy environments [13]. Zimmermann used the Bellman method to solve fuzzy multi-objective linear programming problems [14]. For the first time, Gaballa applied the mathematical programming to the supplier selection in a real case. He formulated a single-objective and mixed-integer programming to minimize the summation of purchasing, transportation and inventory costs by considering multiple items, multiple time periods, and vendors' quality, delivery and capacity [15].

Weber et al. applied a multi-objective approach to systematically analyze the trade-off between conflicting criteria in supplier selection problems [16]. Ghodsypour et al. developed a decision support system (DSS) for reducing the number of suppliers. They used an integrated analytical hierarchy process (AHP) with mixed-integer programming. They considered suppliers' capacity constraint and the buyers' limitations on budget and quality and etc. [17]. Ghodsypour et al. developed an integrated AHP and linear programming model to consider both qualitative and quantitative factors in purchasing activity [12]. Kumar et al. developed a "fuzzy multi-objective integer programming vendor selection problem" (F-MIP-VSP) model [7]. In addition, Ghodsypour et al. developed a fuzzy multi-objective linear model to enable the decision makers to assign different weights to various criteria [18]. Chen et al. developed fuzzy TOPSIS method with trapezoidal fuzzy numbers. They first applied linguistic values to assess the weights of each criterion. Then they used a hierarchy multiple-criteria decision-making (MCDM) model based on fuzzy set theory. They calculated the distances between the candidate suppliers and the fuzzy positive and negative ideal solutions (FPIS & FNIS). To determine the priority of all suppliers, the closeness coefficient was defined [19]. Ha et al. developed a hybrid method including AHP, DEA and NN methodologies [20]. Guneri et al. presented an integrated fuzzy-lp approach for the supplier selection that can be easily applied to multiple sourcing supplier selection problems including vagueness and uncertainties in practice [8]. Lately, Chamodrakas et al. proposed an approach based on satisficing and fuzzy AHP to solve the supplier selection problem in electronic marketplaces [21].

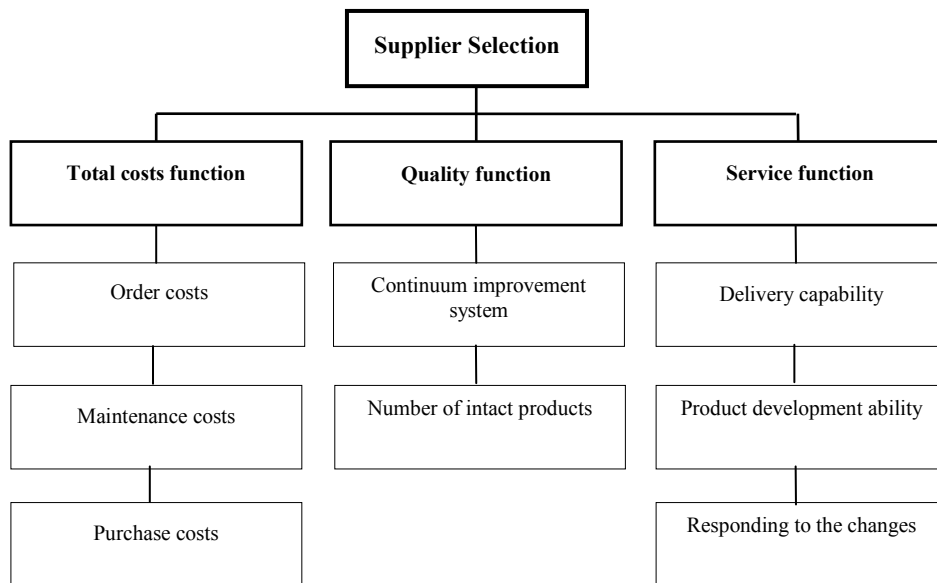


Fig. 1. Hierarchical structure of decision problem.

The rest of the paper is organized as follows: The cost, quality and service functions are described in section 2. In section 3, the fuzzy membership function is determined. The proposed model is presented in section 4. The model is illustrated with an example in section 5 and finally, conclusions are drawn in the last section.

2. Proposed Method

In the model presented by Ghodsypour [1] purchaser wants select the best suppliers among m suppliers. Capacity of each supplier is finite. Three goal functions and three limitation functions are considered in this model. The goal functions are total costs function, quality function and service function. The limitation functions are request limitation, capacity limitation and 0/1 limitation functions.

Each goal function includes some criteria. The hierarchical structure of the goal functions is presented in Figure 1.

In this paper, request limitation is not considered. The parameters which are used in the following sections are briefly described in Table 1.

2.1. Total Costs Function

Total costs function considers all logistic costs in purchasing stage, such as pure price, maintenance costs, transportation costs and order costs. In this model the purchaser accepts the transportation costs. Therefore, the total cost function can be calculated by summation all the three costs categories:

- yearly order costs (include transportation costs)
- yearly maintenance costs
- yearly purchase costs

Purchase from one supplier can be obtained as:

$$Q = \sqrt{2DA/rP} \tag{1}$$

Purchasing process from (i+1)-th supplier occur only when all products purchased from i-th supplier are finished. The Purchasing process is presented in Figure 2.

In our study, we assume that the values of X_i and Q_i are not changed in different periods and we have:

$$Q = \sum_{i=1}^m Q_i, \tag{2}$$

$$Q_i = X_i Q, \quad i = 1, 2, \dots, m \tag{3}$$

$$T_i = X_i T, \quad i = 1, 2, \dots, m \tag{4}$$

$$0 \leq X_i \leq 1, \quad i = 1, 2, \dots, m \tag{5}$$

$$\sum_{i=1}^m X_i = 1 \tag{6}$$

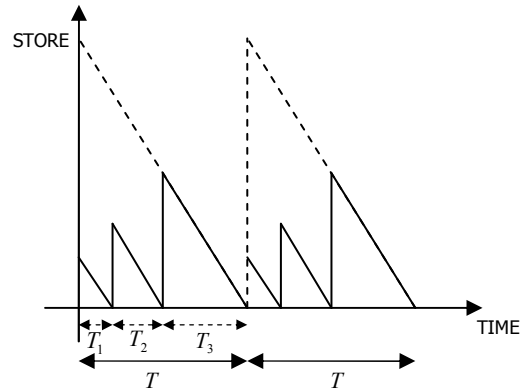


Fig. 2. Store amount in the case of three suppliers and the comparison with the case of one supplier

Fundamental costs of TAPC are defined as:

- Yearly order costs (AOC)
- Yearly maintenance costs (AHC)
- Yearly purchasing costs

2.1.1. Yearly Order Costs (AOC)

Order costs in each period can be calculated as [1]:

$$OCP = \sum_{i=1}^m A_i Y_i \tag{7}$$

$$Y_i = \begin{cases} 0 & \text{if } X_i = 0 \\ 1 & \text{if } X_i > 0 \end{cases}, \quad i = 1, 2, \dots, m \tag{8}$$

Order costs in one year are obtained by multiplying order cost in each period (OCP) and number of periods in each year:

$$AOC = OCP \times \frac{1}{T} = \left(\sum_{i=1}^m A_i Y_i \right) \frac{1}{T} = \left(\sum_{i=1}^m A_i Y_i \right) \frac{D}{Q} \tag{9}$$

2.1.2. Yearly Maintenance Costs (AHC)

The average store of each supplier in its period and maintenance costs are calculated and shown in Table 2. Therefore, the maintenance cost for each period (THCP) is calculated as [1]:

$$THCP = \frac{X_1 Q}{2} r P_1 T_1 + \frac{X_2 Q}{2} r P_2 T_2 + \dots + \frac{X_m Q}{2} r P_m T_m \tag{10}$$

where

$$T_i = \frac{X_i Q}{D} \quad i = 1, 2, \dots, m \quad (11)$$

Therefore

$$THCP = \frac{X_1 Q r P_1}{2} \times \frac{X_1 Q}{D} + \frac{X_2 Q r P_2}{2} \times \frac{X_2 Q}{D} + \dots + \frac{X_m Q r P_m}{2} \times \frac{X_m Q}{D} \quad (12)$$

$$THCO = \frac{r Q^2}{2D} \left(\sum_{i=1}^m X_i^2 P_i \right) \quad (13)$$

Since yearly maintenance cost (AHC) is equal with multiplying the maintenance cost in each period and the number of periods in a year then:

$$AHC = (THCP) \frac{1}{T} = \frac{(THCP)D}{Q} \quad (14)$$

Therefore

$$AHC = \frac{r Q^2}{2D} \left(\sum_{i=1}^m X_i^2 P_i \right) \frac{D}{Q} = \frac{r Q}{2} \left(\sum_{i=1}^m X_i^2 P_i \right) \quad (15)$$

Tab. 1. Nomenclature

| Nomenclature | | | |
|--------------|--|-------|---|
| CC_i | Closeness coefficient of each supplier | A_i | Order cost for i -th supplier |
| X_i | Rate of order quantity for i -th supplier | T | Length of each period |
| D | Total Demand | T_i | Part of period for using Q_i |
| Q_i | order quantity for i -th supplier | r | Rate of maintenance cost |
| Q | Order quantity for all supplier | m | Number of suppliers |
| P_i | Unit price of product of i -th supplier | n | Number of criteria |
| X_{ij} | Score of i -th supplier against j -th criteria | C_i | Production capacity for i -th supplier in each period |
| | | q | Number of functions |

Tab. 2. The store and maintenance of each supplier

| supplier | Period length | Average of store | Average maintenance T_i cost in |
|----------|---------------|------------------|-----------------------------------|
| 1 | T_1 | $X_1 Q/2$ | $(X_1 Q/2)rP_1$ |
| 2 | T_2 | $X_2 Q/2$ | $(X_2 Q/2)rP_2$ |
| ... | ... | ... | ... |
| i | T_i | $X_i Q/2$ | $(X_i Q/2)rP_i$ |
| ... | ... | ... | ... |
| m | T_m | $X_m Q/2$ | $(X_m Q/2)rP_m$ |

2.1.3. Yearly Purchasing Costs

Finally, yearly purchasing costs (TAPC) can be calculated as [1]:

$$TAPC = \left(\sum_{i=1}^m A_i Y_i \right) \frac{D}{Q} + \frac{rD}{2} \left(\sum_{i=1}^m X_i^2 P_i \right) + \sum_{i=1}^m X_i P_i D \quad (16)$$

2.2. Quality Function

The quality and service functions formulas are calculated by using the method which was proposed in [8,19]. In this method, the distance between alternative suppliers and fuzzy positive and negative ideal solution are first calculated. Then, the formulas are found from the result of a linear programming model [8,19].

In our paper, two criteria are used for computing the quality function:

- continuum improvement system
- number of intact products

\tilde{D} is a fuzzy matrix that presents the aggregated fuzzy rating of alternative suppliers, $A = \{A_1, A_2, \dots, A_m\}$, with respect to each criterion $C = \{C_1, C_2, \dots, C_n\}$.

$$\tilde{D} = \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \dots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \dots & \tilde{x}_{2n} \\ \dots & \dots & \dots & \dots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \dots & \tilde{x}_{mn} \end{bmatrix}_{m \times n} \quad (17)$$

Where X_{ij} is a trapezoidal fuzzy number which shows the score of i -th supplier in j -th criteria.

Then, to detect the best supplier in each criterion, the decision matrix is normalized and expressed as:

$$\tilde{R} = [\tilde{r}_{ij}]_{m \times n}, \quad (18)$$

Where:

$$\tilde{r}_{ij} = \left(\frac{a_{ij}}{d_j^*}, \frac{b_{ij}}{d_j^*}, \frac{c_{ij}}{d_j^*}, \frac{d_{ij}}{d_j^*} \right), \quad j = 1, \dots, n \quad (19)$$

$$d_j^* = \max_i d_{ij}, \tag{20}$$

Where $(a_{ij}, b_{ij}, c_{ij}, d_{ij})$ are the four parameters of a trapezoidal fuzzy number (X_{ij}) .

Then, the weighted normalized fuzzy decision matrix can be computed as:

$$\tilde{V} = [\tilde{v}_{ij}]_{m \times n}, \quad i=1,2,\dots,m, \quad j=1,2,\dots,n \tag{21}$$

where $\tilde{v}_{ij} = \tilde{r}_{ij}(\cdot)\tilde{w}_j$.

\tilde{w}_j is weight of j -th criterion. Fuzzy positive and negative ideal solution can be calculated as:

$$A^* = (\tilde{v}_1^*, \tilde{v}_2^*, \dots, \tilde{v}_n^*), \tag{22}$$

$$A^- = (\tilde{v}_1^-, \tilde{v}_2^-, \dots, \tilde{v}_n^-), \tag{23}$$

where $\tilde{v}_j^* = \max_i \{v_{ij4}\}$ and $\tilde{v}_j^- = \min_i \{v_{ij1}\}$. Finally, the distances of each supplier to fuzzy positive and negative ideal solution must be calculated.

Distance between two trapezoidal fuzzy numbers can be calculated by using vertex method as [19]:

$$d_v(\tilde{m}, \tilde{n}) = \sqrt{\frac{1}{4}[(m_1 - n_1)^2 + (m_2 - n_2)^2 + (m_3 - n_3)^2 + (m_4 - n_4)^2]} \tag{24}$$

The closeness coefficient of each supplier can be constructed as:

$$CC_i = \frac{d_i^-}{d_i^* + d_i^-}, \quad i = 1, 2, \dots, m \tag{25}$$

Quality function can be shown as:

$$Max(Quality) = \sum_{i=1}^m (CC_i)_Q X_i \tag{26}$$

2.3. Service Function

Three criteria are used for computing the service function:

- delivery capability
- product development ability
- responding to the changes

The service function is obtained in the same way that the quality function is obtained:

$$Max(service) = \sum_{i=1}^m (CC_i)_S X_i \tag{27}$$

3. Fuzzy Membership Function Determination

The shape of the fuzzy membership functions is considered to be linear. To determine fuzzy membership functions the following procedure must be completed [18]:

Step I: To solve the multi-objective problem, one objective is considered and the other ones are eliminated. Consequently, we face to a single-objective problem and then the best possible values for the objectives are obtained (f^*).

Step II: The previous step is repeated to obtain the worst possible values (f^-).

Step III: Top and bottom acceptable values of each function can be calculated as:

Maximization goal function:

$$\mu_{f_j}(x) = \begin{cases} 1 & f_j \geq f_j^* \\ (f_j(x) - f_j^-) / (f_j^* - f_j^-) & f_j^- \leq f_j(x) \leq f_j^* \\ 0 & f_j \leq f_j^- \end{cases} \tag{28}$$

Minimization goal function:

$$\mu_{f_i}(x) = \begin{cases} 1 & f_i \geq f_i^* \\ (f_i^* - f_i(x)) / (f_i^* - f_i^-) & f_i^- \leq f_i(x) \leq f_i^* \\ 0 & f_i \leq f_i^- \end{cases} \tag{29}$$

These functions can be shown in Figure 3 [18].

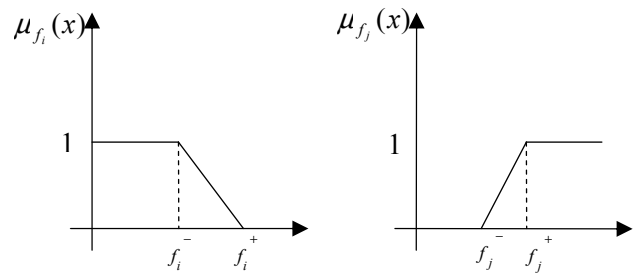


Fig. 3. Generic shapes of fuzzy functions [18]

4. The Proposed Model

Final model can be shown as:

Max λ

S.t:

$$w_i \lambda \leq (f_i^* - f_i(x)) / (f_i^* - f_i^-) \tag{30}$$

$$w_j \lambda \leq (f_j(x) - f_j^-) / (f_j^* - f_j^-) \tag{31}$$

$$\lambda \in [0,1]$$

$$X_1 + X_2 + \dots + X_m = 1 \tag{32}$$

$$X_i \geq 0, i = 1, 2, \dots, m \tag{33}$$

$$0 \leq X_i D \leq C_i \tag{34}$$

$$Y_i = 0, 1 \tag{35}$$

$$\sum_{j=1}^q w_j = 1 \quad w_j \geq 0 \tag{36}$$

$$f_1(x) = \sqrt{2Dr(\sum_{i=1}^m A_i Y_i)(\sum_{i=1}^m X_i^2 P_i) + \sum_{i=1}^m P_i X_i D} \tag{37}$$

$$f_2(x) = \text{Max(Quality)} = \sum_{i=1}^m (CC_i)_Q X_i \tag{38}$$

$$f_3(x) = \text{Max(service)} = \sum_{i=1}^m (CC_i)_S X_i \tag{39}$$

Then, this model must be solved.

4-1. Solving the Model

Solving the model includes the following steps:

- 1-List the number of various cases of Y_i s.
- (2^m cases)
- 2-Eliminate the cases which can not response the requirement limitation.
- 3-Assign the valid Y_i in Eq (37).

$$f_1(x) = \sqrt{2Dr(\sum_{i \in S} A_i)(\sum_{i=1}^m X_i^2 P_i) + \sum_{i=1}^m P_i X_i D} \tag{40}$$

{S} is the set of Y_i s which their values are equal to one.

4-These problems are solved using *Lingo* software. Then, the best suppliers are selected and their optimum quantities are calculated.

5. Numerical Example

A hypothetical textile company is considered as an example in our study. In this example we want to find the best suitable case in which the suppliers are determined with the amount of purchasing yarn for a new product from each one.

As discussed before, three goal functions (total costs, quality and service functions) are considered in our

model. Total amount of required materials for this company maintenance cost rate are 10000 units and $r = 0.2$, respectively. Table 3 lists total cost function criteria, Order cost and purchase cost for each supplier. By substituting the information in Table 3 in (36):

$$Z_1 = \sqrt{400(9Y_1 + 4Y_2 + 8Y_3)(9X_1^2 + 16X_2^2 + 32X_3^2)} + 1000(9X_1 + 16X_2 + 32X_3) \tag{41}$$

where Z_j is the value of cost function. The information about the fuzzy criteria and weigh of each criterion of quality and service functions are listed in Table 4 and Table 5, respectively. C_1 and C_2 in table 4 are continuum improvement system and number of intact product, respectively. E_1 , E_2 and E_3 in Table 5 are delivery capability, product development ability and responding to the changes, respectively.

Table 6 and Table 7 show the normalized fuzzy criteria matrix for quality and service functions, respectively. Weighted normalized fuzzy criteria matrixes for these two functions are reported in Table 8 and Table 9.

Fuzzy positive and negative ideal solutions for quality function are:

$$A_q^* = [(0.9, 0.9, 0.9, 0.9), (1, 1, 1, 1)]$$

$$A_q^- = [(0.42, 0.42, 0.42, 0.42), (0.48, 0.48, 0.48, 0.48)]$$

and for service function we have:

$$A_s^* = [(1, 1, 1, 1), (0.9, 0.9, 0.9, 0.9), (1, 1, 1, 1)]$$

$$A_s^- = [(0.42, 0.42, 0.42, 0.42), (0.42, 0.42, 0.42, 0.42), (0.42, 0.42, 0.42, 0.42)]$$

Distance between FPIS and suppliers' rating and between FNIS and suppliers' rating for quality function are given in Table 10 and Table 11, respectively. These distances for service function are presented in Table 12 and Table 13, respectively.

Table 14 and Table 15 are listed computations of d_i^* , d_i^- and CC_i for quality and service functions, respectively.

Tab. 3. Total cost function criteria for suppliers

| suppliers | purchase cost | Order cost |
|----------------|---------------|------------|
| A ₁ | 5 | 9 |
| A ₂ | 6 | 8 |
| A ₃ | 2 | 4 |

Tab. 4. Fuzzy criteria and weigh of each criterion of quality

| | C_1 | C_2 |
|----------------|-------------------|-------------------|
| A ₁ | (7,8.7,9.3,10) | (6,7.3,7.3,9) |
| A ₂ | (6,7.3,7.3,9) | (7,8,8,9) |
| A ₃ | (7,8,8,9) | (7,8,8,9) |
| weight | (0.7,0.8,0.8,0.9) | (0.8,0.9,1.0,1.0) |

Tab. 5. Fuzzy criteria and weigh of each criterion of service

| | E_1 | E_2 | E_3 |
|----------------|---------------------|-------------------|---------------------|
| A ₁ | (6,7.3,7.3,9) | (7,8.3,8.7,10) | (8,9,10,10) |
| A ₂ | (6,7,7,8) | (7,8,8,9) | (6,7.7,7.7,9) |
| A ₃ | (7,8.3,8.7,10) | (6,7.7,7.7,9) | (7,8.3,8.7,10) |
| weight | (0.7,0.83,0.87,1.0) | (0.7,0.8,0.8,0.9) | (0.7,0.87,0.93,1.0) |

Tab. 6. Normalized fuzzy criteria matrix for quality function

| | C_1 | C_2 |
|----------------|---------------------|---------------------|
| A ₁ | (0.7,0.87,0.93,1.0) | (0.6,0.73,0.73,0.9) |
| A ₂ | (0.6,0.73,0.73,0.9) | (0.7,0.8,0.8,0.9) |
| A ₃ | (0.7,0.8,0.8,0.9) | (0.7,0.8,0.8,0.9) |

Tab. 7. Normalized fuzzy criteria matrix for service function

| | E_1 | E_2 | E_3 |
|----------------|---------------------|---------------------|---------------------|
| A ₁ | (0.6,0.73,0.73,0.9) | (0.7,0.83,0.87,1.0) | (0.8,0.9,1.0,1.0) |
| A ₂ | (0.6,0.7,0.7,0.8) | (0.7,0.8,0.8,0.9) | (0.6,0.77,0.77,0.9) |
| A ₃ | (0.7,0.83,0.87,1.0) | (6,7.7,7.7,9) | (7,8.3,8.7,10) |

Tab. 8. Weighted normalized fuzzy criteria matrix for quality function

| | C_1 | C_2 |
|----------------|-----------------------|----------------------|
| A ₁ | (0.49,0.7,0.74,0.9) | (0.48,0.66,0.73,0.9) |
| A ₂ | (0.42,0.58,0.58,0.81) | (0.56,0.72,0.8,0.9) |
| A ₃ | (0.49,0.64,0.64,0.81) | (0.56,0.72,0.8,0.9) |

Tab. 9. Weighted normalized fuzzy criteria matrix for service function

| | E_1 | E_2 | E_3 |
|----------------|----------------------|-----------------------|----------------------|
| A ₁ | (0.42,0.61,0.64,0.9) | (0.49,0.66,0.7,0.9) | (0.56,0.78,0.93,1) |
| A ₂ | (0.42,0.58,0.61,0.8) | (0.49,0.64,0.64,0.81) | (0.42,0.67,0.72,0.9) |
| A ₃ | (0.49,0.69,0.76,1) | (0.42,0.62,0.62,0.81) | (0.42,0.72,0.81,1) |

Tab. 10. Distance between FPIS and suppliers' rating for quality function

| | C_1 | C_2 |
|----------------|-------|-------|
| A ₁ | 0.24 | 0.34 |
| A ₂ | 0.33 | 0.28 |
| A ₃ | 0.28 | 0.28 |

Tab. 11. Distance between FNIS and suppliers' rating for quality function

| | C_1 | C_2 |
|----------------|-------|-------|
| A ₁ | 0.32 | 0.26 |
| A ₂ | 0.23 | 0.29 |
| A ₃ | 0.25 | 0.29 |

Tab. 12. Distance between FPIS and suppliers' rating for service function

| | E_1 | E_2 | E_3 |
|----------------|-------|-------|-------|
| A ₁ | 0.4 | 0.26 | 0.25 |
| A ₂ | 0.42 | 0.28 | 0.37 |
| A ₃ | 0.32 | 0.31 | 0.31 |

Tab. 13. Distance between FNIS and suppliers' rating for service function

| | E_1 | E_2 | E_3 |
|----------------|-------|-------|-------|
| A ₁ | 0.28 | 0.3 | 0.43 |
| A ₂ | 0.23 | 0.25 | 0.31 |
| A ₃ | 0.36 | 0.24 | 0.38 |

Tab. 14. Computations of d_i^* , d_i^- and CC_i for quality function

| | d_i^- | d_i^* | $d_i^* + d_i^-$ | $(CC_i)_Q$ |
|----------------|---------|---------|-----------------|------------|
| A ₁ | 0.58 | 0.58 | 1.16 | 0.5 |
| A ₂ | 0.52 | 0.61 | 1.13 | 0.46 |
| A ₃ | 0.54 | 0.56 | 1.1 | 0.49 |

Tab. 15. Computations of d_i^* , d_i^- and CC_i for service function

| | d_i^- | d_i^* | $d_i^* + d_i^-$ | $(CC_i)_S$ |
|----------------|---------|---------|-----------------|------------|
| A ₁ | 1.01 | 0.91 | 1.92 | 0.526 |
| A ₂ | 0.79 | 1.07 | 1.86 | 0.424 |
| A ₃ | 0.98 | 0.94 | 1.92 | 0.51 |

Therefore, the quality function is:

$$Max(Quality) = Z_2 = 0.5X_1 + 0.46X_2 + 0.49X_3 \quad (42)$$

where Z_2 is the value of quality function. The service function is:

$$Max(Service) = Z_3 = 0.526X_1 + 0.424X_2 + 0.51X_3 \quad (43)$$

where Z_3 is the value of service function. The best and worst possible values of these three functions are calculated with *Lingo* software. Table 16 shows the results.

Tab. 16. Top and bottom acceptable values of each function

| | $\mu = 1$ | $\mu = 0$ |
|----------------------|-----------|-----------|
| Total costs(Z_1) | 17621 | 24245 |
| Quality(Z_2) | 0.469 | 0.4498 |
| Service(Z_3) | 0.496 | 0.5196 |

The weights of decision makers are listed in Table 17. By substituting Z_1 from (40) in $f_i(x)$ in (32) and by substituting Z_2 and Z_3 from (41) and (42) in $f_i(x)$ in (33) the final model is obtained as follows:

Max λ

S.T :

$$0.13\lambda \leq \frac{24245}{6624} - \frac{\sqrt{400(9Y_1 + 4Y_2 + 8Y_3) \times (9X_1^2 + 16X_2^2 + 32X_3^2)}}{6624} - \frac{1000(9X_1 + 16X_2 + 32X_3)}{6624} \quad (44)$$

$$0.21\lambda \leq \frac{0.5X_1 + 0.46X_2 + 0.49X_3 - 46.9}{2.7} \quad (45)$$

$$0.66\lambda \leq \frac{0.526X_1 + 0.424X_2 + 0.51X_3 - 44.98}{6.98} \quad (46)$$

$$X_1 \leq 0.6, \quad \varepsilon Y_1 \leq X_1 \leq Y_1 \quad (47)$$

$$X_2 \leq 0.7, \quad \varepsilon Y_2 \leq X_2 \leq Y_2 \quad (48)$$

$$X_3 \leq 0.5, \quad \varepsilon Y_3 \leq X_3 \leq Y_3 \quad (49)$$

$$D = 1000, \quad 0 \leq 1000X_i \leq C_i \quad (50)$$

$$r = 0.2$$

$$X_1 + X_2 + X_3 = 1 \quad (51)$$

$$X_i \geq 0 \quad Y_i = 0,1 \quad (52)$$

According to the requirement limitation, only some cases are possible to be solved. The cases are shown in Table 18. These cases were solved by *Lingo* software and the best solution occurred in the first case in which X_1 , X_2 and X_3 were 0.479, 0.253 and 0.266.

Tab. 17. Weights of decision makers

| | w |
|-------------|------|
| Total costs | 0.13 |
| Quality | 0.21 |
| Service | 0.66 |

Tab. 18. The possible cases

| | Y_1 | Y_2 | Y_3 |
|--------|-------|-------|-------|
| Case 1 | 1 | 1 | 1 |
| Case 2 | 1 | 1 | 0 |
| Case 3 | 0 | 1 | 1 |
| Case 4 | 1 | 0 | 1 |

6. Conclusion

In this paper, a multi-objective model based on AHP and Fuzzy TOPSIS has been proposed to solve the supplier selection problem. This model has been made the decision-maker to be able to solve the supplier selection problem with different criteria and different weight for each criterion.

Since many of criteria are linguistic in real life, the fuzzy set theory has been used in this model. The cost function was calculated by considering purchasing, maintenance and order costs.

In the quality and service functions, the criteria were presented with trapezoidal fuzzy numbers. Then, the weights were applied to the criteria. Distances between each supplier from positive and negative ideal solution were computed. The closeness coefficient of each supplier was calculated. According to the closeness coefficients, the linear programming model was built to find the best suppliers and their optimum order quantities. The membership function of each objective was obtained. Finally the model was solved by using the *Lingo* software. The proposed model is illustrated by an example.

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