*IUST International Journal of Engineering Science,*
**Vol. 19, No.5-1, 2008, Page 23-38**

# DETERMINATION OF ULTIMATE PIT LIMITS IN OPEN MINES USING REAL OPTION APPROACH

**A.D. Akbari, M. Osanloo & M.A. Shirazi**

***Abstract:*** *Planning and design procedure of an open pit mining project just can be started after ultimate pit determination. In the carried out study in this paper it was shown that the most important factor in ultimate pit determination and in consequence in the whole planning and design procedure of an open pit mine is the metal price. Metal price fluctuations in recent years were exaggerated and imposed a high degree of uncertainty to the mine planning procedure while none of the existent algorithms of the pit limit determination consider the metal price uncertainty. Real Option Approach (ROA) is an efficient method of decision making in the condition of uncertainty. This approach usually used for evaluation of defined natural resources projects up to now. This study considering the metal price uncertainty used real option approach to prepare a methodology for determining the Ultimate Pit Limits (UPL). The study was carried out on a non-ferrous metallic cylindrical ore deposit but the achieved methodology can be adjusted for all kinds of the deposits. The achieved methodology was comprehensively described through the examples in a way that can be used by the mine planners.*

***Keywords:*** *Open pit mining; Ultimate Pit Limits (UPL); Price uncertainty; Decision making; Real Option Approach (ROA)*

## 1. Introduction

Open pit mine planning is a procedure that can be started just after ultimate pit determination and cut-off grade calculation which both of them directly depend on final product price of the mine. Ultimate pit determination in each period of time is a function of financial affairs. This function is well defined by Break-Even Stripping Ratio (BESR). Systematic studies on this approach were carried out by Lilico [1] and Koskiniemi [2]. Lerchs and Grossman had presented their 3D graph theory before them, but the Lerchs and Grossman theory was a methodology for ultimate pit determination by computer and through a block model of the deposit. They modeled the block model of a mine by a weighted directed graph in which each vertex represents for blocks and each arc represents for the blocks interdependency from extraction point of view [3, 4]. The direction of the arcs from a vertex to the other vertex shows the extraction priority of the second block to the first block and the weights come from the blocks economic values. They assumed the Ultimate Pit Limits (UPL) determination problem is equal to finding the maximum weight of the aforesaid weighted directed graph. Their theory is constructed on the basis of blocks economic values which calculated regardless to the price uncertainty. Zhao and Kim tried to improve the Lerchs and Grossmann algorithm by considering just the arcs which are defined in the ore-waste interfaces [5], but again Zhao and Kim developed their algorithm regardless to price uncertainty. Similar to Zhao and Kim, Yamtomi et al. in 1995 tried to improve an old idea by modifying the floating cone algorithm [6], but they also didn’t consider the price uncertainty again. Johnson in 1968 proposed using network flow analysis for determining the UPL, but it was Picard who dealt with the subject and made it well-documented [7]. A network flow analysis model consists of a source node and a terminal node, one node for each block in the model, links with capacities equal to the values of the corresponding blocks for connecting source node to each positively valued block, same links for connecting each non-positively valued block to terminal node, and links of infinite capacities connecting positively valued blocks to the blocks with zero or negative values which

---

***Paper first received Feb. 10, 2007, and in revised form Feb. 12, 2009.***

***A.D. Akbari*** is with the Department of Mining Engineering Department, Azad University, Science and Research Branch, Tehran, Iran, afshinakbari@ parsonline.net
***M. Osanloo*** is with the Department of Mining, Metallurgical and Petroleum Engineering Department, Amirkabir University of Technology, Tehran, Iran
***M.A. Shirazi*** is with the Department of Industrial Engineering Department, K.N. Toosi University of Technology, Tehran, Iran

must be removed in order to facilitate mining of the positively valued blocks. The aim of network flow analysis algorithm is maximizing the amount of flow from the source node to the terminal node in the mentioned model. There are reports about another UPL determination algorithm which proposed by Krobov. This algorithm operates by putting an inverse cone on every positive block in the pit and allocating the positive values within the cone against the negative values within the cone until no negative values remain, so that the positive blocks pay for the negative blocks. David et al. say Korbov algorithm suffers from an inability to process overlapping cones correctly [8], but Dowd and Onur claim to overcome this limitation and being capable of finding the true optimum solution by Krobov algorithm [9] and [10]. By the way, the Krobov algorithm neglects the price uncertainty like the other ones.

Gradually the concept of ultimate pit determination just be used in interaction with production planning and becomes pale against determining an optimum production plan with regard to maximizing the NPV. It was because of the relative metal price certainty during past decades. Obviously if the metal price follows a known trend, having some forecasts for the pit limits during the mine life won't be so difficult. Hence all efforts of a mine planner will be concentrated on explaining a good production planning within the forecasted limits of the open pit mine or determining the pit limits while defining the production planning and through the production planning, in order to maximize the NPV of the operation. Linear Programming approaches are good examples for these points of view. Some methodologies of this type were presented by Gershon [11], Cai, and Huttagosol and Cameron [12] regardless to the problem of price uncertainty. Also most of the artificial intelligence techniques are accounted in the methodologies of this group which deals with the UPL determination and production planning jointly. Denby and Schofield [13] used the genetic algorithm but the most successful method of this type was presented by Tolwinski and Underwood before in 1992 which currently is in use trough NPV Scheduler software by some mine planers [14]. Tolwinski and Underwood combined concepts from both stochastic optimization and artificial neural networks to produce their algorithm for estimating the optimal evaluation of an open-pit mine.

The limitation of the most of artificial intelligence techniques in general is that their results are not reproducible from one run to the next and from this research point of view is that they consider price of the final product as a fixed variable. Gershon in 1987 presented his heuristic method just for production planning [15], but Wang and Sevim modified it for determination of UPL and production planning simultaneously [16]. Gershon utilized the concept of block positional weight. The positional weight for a block is derived by generating a cone downwards from the block to the edge of the predetermined UPL and considering the values of all the blocks in that cone. The resulting block positional weight provides a measure of the desirability of removing a given block at that specific time. It reflects the quality of ore, position of the block and the quality of ore under the block. Wang and Sevim utilized Gershon's downward cone concept in their heuristic, however their approach doesn't need the ultimate pit to be determined first. Their Heuristic begins by determining the largest pit that will both satisfy the slope requirements and contain whole of the deposit. It then proceeds to order an array of suitable candidate cones by their average grades and removes enough of the lowest grade cones to satisfy the required pit size increment. This procedure is repeated until there are no blocks remaining to be extracted. In this way, a series of incremental pits will be generated. The mentioned heuristics suffer from some defects such as the problem of overlapping cones and inability of maximizing the NPV in some cases, in addition to neglect the problem of price uncertainty like the other discussed algorithm.

Lerchs and Grossmann in 1965 just after presenting their 3D graph theory recognized that having an optimum final contour for a pit was not of much use without having a good production planning. To satisfy this requirement, they introduced the concept of parametric analysis, in which the development of a pit is characterized by the gradual modification of one or more key parameters.

In doing so Lerchs and Grossmann were seeking to produce a production planning which maximize the NPV through maximizing the integral of cash flow with respect to the total volume mined. Their selected parameter was an amount by which the economic value of each block in the model would be reduced.

When the amount is zero, the normal ultimate pit is produced. As it increases, when it passes the critical values, the ultimate pit contour jumps to enclose a smaller volume. If there are not too many interdependencies between the ore and waste regions in the deposit and there is sufficient variation in the economic values of the blocks, the end result is a series of nested pits which can be used as production planning or can be defined as the UPLs suit different time conditions. This technique is referred to as the Nested Lerchs-Grossmann algorithm. Whittle used this algorithm in his Whittle programming 4D packages to generate a series of nested pits in which each pit is optimal for a different set of economic conditions [17, 18]. Therefore this technique could be referred as the first technique in which the economic condition containing price, considered unstable and uncertain. But the amplitude of considered uncertainty of this technique won't be enough in current economic climate and the technique cannot be useful now as it will be discussed in this paper later.

Despite the mining industry, oil industry has paid attention to the concept of price uncertainty from very long time ago. The researcher of oil industry often tried to manage the uncertainty they were facing with, through Real Option Approach (ROA). The root of using ROA belongs to 1930s when oil producers started to store oil in the period of price dropping in order to compensate some of their loss in the period of price raise but in practice they faced with the concept of “Convenience Yield”. Currently there is a similar situation in the metal market because of metal stocks and ore stockpiles. This method in oil industry was in use up to the last years of 1950s and after that it became pale to this extent that in 1990s researcher such as Williams & Wright [19], Deaton & Laroque [20], and Chambers & Bailey [21] actually ignored the concept of convenience yield in their models. But Litzenberger & Rabinowitz [22] tried to balance these two types of ideas through a real option model. The basis of their argumentation was the existing severe uncertainty in some of the price backwardation which should be managed in a production-reserve model using existing managerial flexibilities and considering the uncertainty of the decision making factor which is price. Considine & Larson in 2001 developed a similar model, but they claimed that their model is suitable for all of the natural resources including mineral resources [23].

Dias in 2004 developed a comprehensive real option model for decision making on investment in both exploration and exploitation of natural resources [24], but it was Costa Lima & Suslick who developed a model deals with mineral resources and mining projects directly [25]. Their model has been prepared for evaluating a defined mining project. Dimitrakopoulos & Abdel Sabour in 2007 challenged the capability of the Real Option Valuation (ROV) and after valuating a certain project by ROV and NPV methods concluded that the resulting value by ROV is higher than one achieved by NPV method [26]. This study will use ROA for finding the best UPL for an open pit mine to be developed, whereas in the literature up to now, the use of real option confined to valuation of a defined project with its assumed limitations.

## 2. UPL Determination and Price Uncertainty

There are many computer dependent methodologies for determining the UPL as discussed in introduction before, such as moving cone, Lerchs and Grossman 3D graph, Nested Lerchs-Grossmann algorithm, Krobov algorithm, Wang and Sevim heuristic, Linear Programming by researchers such as Gershon or Cai, and artificial intelligence techniques by researchers such as Denby and Schofield or Tolwinski and Underwood. These techniques depend on the blocks economic values, cut-off grade, and stable slope. But the classical approach of Lilico and Koskiniemi depends on cut-off grade, stable slope and BESR instead of blocks economic values beside. Figure 1 shows the design procedure in an open pit mine with regard to UPL determination.

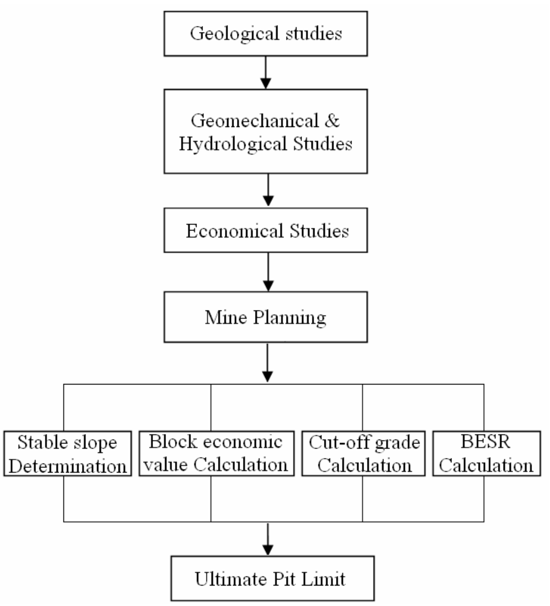

**Fig. 1. Design procedure in an open pit mine with regard to UPL determination**

Slope stability is an imposed geomechanical factor which is fixed for a distinct project, but BESR, cut-off grade and blocks economic values are changeable through the time. In order to study these changeable factors and find which their most sensitive variables are, a sensitivity analysis carried out. The basic models which are used for the analysis are as follows:

$$BESR = \frac{Rg\,(p - c_2) - (b + c_1)}{a} \qquad (1)$$

Where R is overall recovery coefficient of the mineral processing (decimal fraction), g is ore grade (decimal fraction), p is the final product price per tonne, $c_1$ is concentrating cost, $c_2$ is the costs of further treatment such as smelting cost and refining cost per tonne of final product, b is the mining cost per tonne of ore, and a is waste removal cost per tonne.

$$\text{Cut-off grade} = \frac{(b + c_1)}{R\,(p - c_2)} \qquad (2)$$

$$\begin{aligned}&\text{Ore block economic value}\\&= \text{Tonnage per block} \times [\mathrm{Rg}(\mathrm{p} - \mathrm{c}_2) - (\mathrm{b} + \mathrm{c}_1)]\end{aligned} \qquad (3)$$

Also the inputs for starting the sensitivity analysis are included in Table 1.

**Tab. 1. The preliminary inputs of the sensitivity analysis**

| R | g (%) | p ($/tonne) | b = a ($/tonne) | $c_1$ ($/tonne) | $c_2$ ($/tonne) | BESR | $g_c$ (%) |
|---|---|---|---|---|---|---|---|
| 1 | 0.5 | 3000 | 1 | 3 | 300 | 9.5:1 | 0.15 |

The results are shown in Figures 2, 3, and 4. Through these figures it can be understood that the price of final product (p) is the most sensitive factor which has the positive effect (increasing effect) on BESR and ore block economic value and grade is the second, while in recent years more studies on the subject of uncertainty in mine planning were about grade uncertainty such as the studies of Rovenscroft in 1992 [27], Denby & Schofield in 1995 [28], Dowd in 1997 [29], Gody & Dimitrakopoulos in 2003 [30], Dimitrakopoulos & Ramazan in 2003 and Gholamnejhad & Osanloo in 2006.

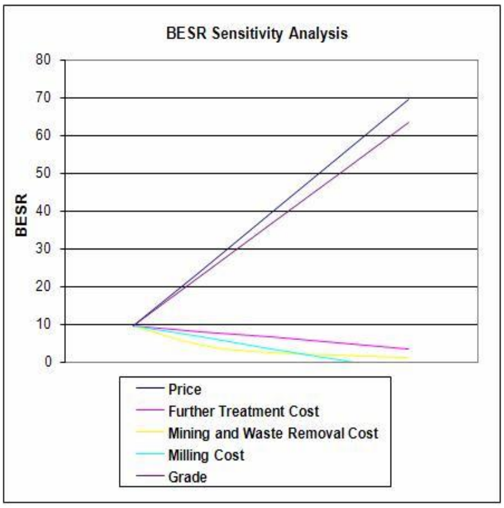

**Fig. 2. The results of BESR sensitivity analysis**

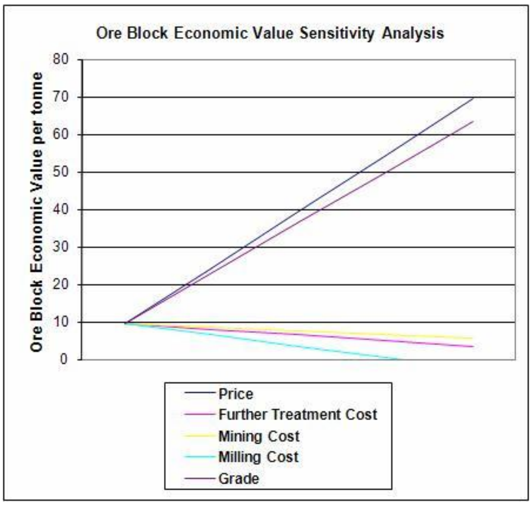

**Fig. 3. The results of ore block economic value sensitivity analysis**

It should be noted that among the effective factors on BESR, ore block economic value, and cut-off grade, except grade and price the others are cost factors. Cost factors within a geographical area like a country will have a relative constancy, while the nature of final product price is an international uncertain nature because of Political, Economical, Social, and Technological (PEST) changes. Yet Costa Lima & Suslick in 2006 developed their model for mine evaluation by ROV based on the uncertainty of operating cost and price. They assumed that the variation of operating cost and price revert to the Stochastic Differential Equation (SDE) of $dP = \alpha_P Pdt + \sigma_P Pdz_P$ and $dC = \alpha_C Cdt + \sigma_C Cdz_C$, where $\alpha_P$ and $\alpha_C$ are increase rate of price and operating cost respectively, P is the final product price, C is the operating cost, $\sigma_P$ and $\sigma_C$ are standard deviation of P and C, and $dz_P$ and $dz_C$ are Wiener increments of the Geometric Brownian Motion (GBM) for P and C.

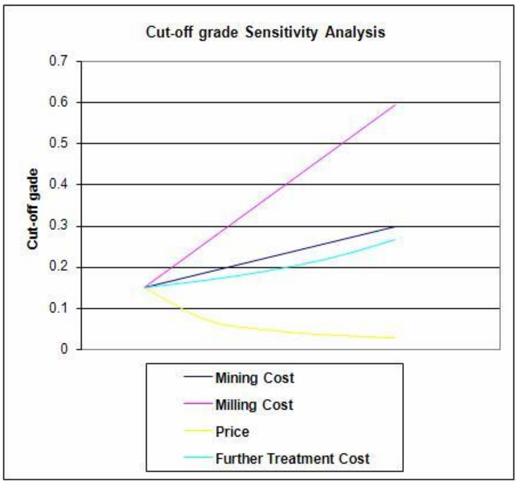

**Fig. 4. The results of cut-off grade sensitivity analysis**

In the Costa Lima & Suslick model the underlying asset through P and C follows the GBM which is a continuous-time stochastic process in which the logarithm of the randomly varying quantity follows a Brownian motion, or, perhaps more precisely, a Wiener process.

It is appropriate to mathematical modeling of some phenomena in financial markets. In other word GBM describes the movements in a variable or asset price when the proportional change in its value in a short period of time is normally distributed. The proportional changes in two non-overlapping periods of time are uncorrelated; hence the alternative name for the process is random walk. The term geometric refers to the fact that it is the proportional change in the asset price (not the absolute level) that is normally distributed. Obviously these assumptions could be correct about the final product price, but not about the operating cost and omitting the operating cost from the modeling processes, make it possible to develop a more applicable model.

## 3. The Concept of ROA with Regard to Mining

One of the most effective methods for managing price uncertainty in natural resources projects is real option method. Up to now, it was mostly used in the oil industry, but regarding recent metal price fluctuations, some authors such as Costa Lima & Suslick in 2006, Dimitrakopoulos & Abdel Sabour in 2007, note this theory is far more useful method than the classical NPV method for valuation of mining projects under condition of price uncertainty [25, 26]. The background of using real option theory recurs to Hotelling's ideas in the 1930s and the concept of convenience yield.

The concept of real option utilizes the financial option theory in the real investments such as natural resources or industrial projects or in their extension plans. The sense of option appears when the information obtained during the time can be effective on the investment decision, specially when decision making in presence of high degree of uncertainty, some managerial flexibilities and unawareness of all the facts. This is the right not an obligation to have a NPV from a cash flow through doing the investment in a suitable and optional specific time. Suppose a mining company achieve the right of exploration in a large area containing two large metal prospects with the similar geological conditions. The chance factor for both prospects is 20%, the exploration investment ($I_E$) in this area for exploratory drilling and detailed exploration is estimated to 8 million dollars for each of the cases and the NPV of both cases will be 35 million dollars after exploitation. The Economic Monetary Value (EMV) for each case based on NPV classical analysis will be as follows [31]:

$$\mathrm{EMV} = -\mathrm{I}_E + [\mathrm{CF.NPV}] = -8 + [0.2 \times 35] = -\$1\mathrm{million}$$

This result sentences not to carry out the project. But the question is; "Is the project really disqualified?" Analyzing this project but this time by ROV method will result as follows:

Regarding the similarity of geological conditions of the prospects, the exploration program can be planed in two stages. Under these circumstances, if the result of the first stage (exploration of the first prospect) is negative, the second stage of the operation (exploration of the second prospect) will be canceled, thus EMV of the first prospect will be $-\$1 million$ and EMV of the second prospect will be zero ($\mathrm{EMV}_1 = -1, \mathrm{EMV}_2 = 0$). But if the first stage has a positive result, the negative chance factor of the second stage will decrease to 10% based on expert opinion ($\mathrm{CF}_2^- = 10\%$). This 10% can be the maximum chance factor which a pessimistic geologist can consider regarding the similarity of the two prospects and the positive result achieved from the first one, and then $\mathrm{CF}_2^+$ will be 60% (Figure 5). The $\mathrm{EMV}_2^+$ under this circumstances will be:

$$EMV_2^+ = -I_E + [CF.NPV] = -8 + [0.6 \times 35] = \$13\ million$$

And the overall $\mathrm{EMV}_2$ (EMV [optional project 2|project 1 outcome]) regarding just that 20% considered chance factor is:

$$EMV_2 = [(CF^- \times EMV_2^-) + (CF^+ \times EMV_2^+)]$$
$$= [(0.8 \times 0) + (0.2 \times 13)] = \$2.6 million$$

And the overall EMV of the whole project in the worst conditions will be:

$$EMV = EMV_1 + EMV_2$$
$$= -\$1 million + \$2.6 million = \$1.6 million$$

Therefore regarding the existence managerial flexibilities which in this case were planning the program in two stages, the project can be carried out. ROA in addition to decision making about do or don't a project can be used for maximizing the NPV of a project.

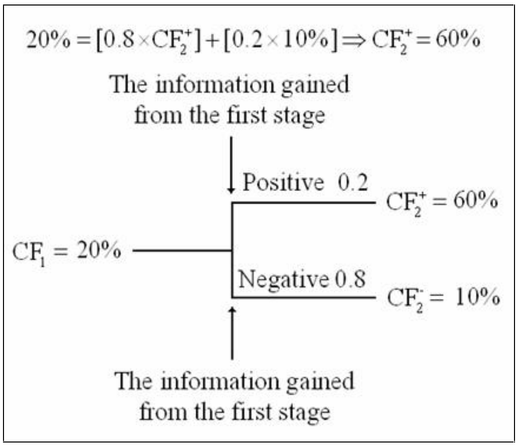

**Fig. 5. The calculation of the second stages chance factor**

Maximizing the NPV (typical objective function of mining projects) is limited to following instances:

- Related options (Managerial Flexibilities)
- Market uncertainties
- Technical uncertainties

By ROA we can maximize the NPV through finding the hidden values in the options. In other word this methodology by working with the existing options will result a better and more realistic NPV for natural resources projects in the presence of uncertainties. Sometimes the existing options are executive options called managerial flexibilities like the example above, but when the project is facing with market uncertainties such as commodity price and the demand, or technical uncertainties such as occurrence of orebody and its grade and quality, the planner should utilize strategic options to manage the uncertainties. There are different managerial flexibilities which can be useful for a mine planner in ROA. They are categorized in Table 2. Each of them can be used in their appropriate situations for making the NPV better.

## 4. Managing Market Uncertainties by ROA in a Mining Project

When facing the market uncertainties, the most important strategic options for the purpose of managing the effects of uncertainties are the time and suitable price for starting the project. The interaction of these two options can be expressed in a threshold investment graph. Shockley in 2007 evaluated a coal project as a practical example by ROV and presented a very simple example of such a graph in the form of a step shaded area which can be named exercise area [32].

Here his methodology is adjusted for a metal project just in order to give an example to facilitate the perception of ROV mechanism. But there are two fundamental differences between a coal project and a metal project and they cannot be evaluated in a same way. These differences are discussed in this research and will be considered in the proposed methodology by this paper in the next section. Also the goal of this research is not to evaluate a defined project; rather it is finding the best alternative for developing an open pit mine and determining its UPL in the condition of price uncertainty.

But at first and in this section the concept of ROV is described for a metal mining project in a simplistic way and in the next section the ROA will be used for finding the best alternative or option to develop an open pit mine in the condition of price uncertainty, regarding all specifications of a metal resource and considering all practical constraints in developing such a project. ROV of a natural resource project at first needs modeling the price uncertainty, this research does it by building a binominal tree of final product price. In order to do that following parameters are needed:

$\Delta t$ - Price data time step (for yearly price data $\Delta t=1$)

$T_o$ - Available time of option

$P_s$ - Spot price of the final product at the beginning of the available time of option ($/tonne)

ncy - Net convenience yield (decimal fraction) which is lost convenience cash flow

$\sigma$ - Observed volatility of changes in final product price

r - Risk free interest rate (decimal fraction)

Here the price uncertainty modeled with monthly steps ($\Delta t=1/12=0.0833$), assuming one and half a year available time for development decision from Jan. 2007 to Jun. 2008.

The rate of return considered 7% per year (r=0.07) and the spot copper price in Jan. 2007 was 6199 $/tonne based on the LME reports. By Jan. 2007 the observed volatility of changes in monthly copper prices was 40.25% ($\sigma=0.4025$) and the net convenience yield assumed 2.2%. It should be noted that one of the most evident differences between an oil or coal project and a metal mining project is in the concept of convenience yield which will be discussed later in this paper. The assumed deposit is a copper deposit with 30 million tonne of minable reserve. The specifications of the project to be evaluated are shown in Table 3 and it should be noted that it is assumed there is no inflation and no reclamation cost. The relationship between the spot/forward price of copper, and the value of a just-developed mine (the true underlying asset) can be seen in Table 4.

For modeling the price uncertainty through a binominal tree, the up step and the down step of the price for the assumed project is calculated as follows:

$$Up\ step\ (U) = e^{\sigma\sqrt{\Delta t}} = e^{0.4025\sqrt{0.0833}} = 1.123 \tag{4}$$

$$Down\ step\ (D) = \frac{1}{Up\ step} = \frac{1}{1.123} = 0.890 \tag{5}$$

These assumptions result a binominal tree of price looks like Table 5 in which the probability of up and down steps are:

$$\text{Up step probablity } (P) = \frac{e^{(r\text{-}ncy)\Delta t} - D}{U\ \text{-}D}$$
$$= \frac{e^{(0.07\text{-}0.022)} - 0.890}{1.123 - 0.890} = 0.488 \tag{6}$$

$$\text{Down step probablity } (Q) = 1 - P$$
$$= 1\text{-}0.488 = 0.512 \tag{7}$$

Based on the aforesaid information and the information shown in Table 3, 4 and 5 the optimal exercise boundary of the project will be achieved (Tables 5 and 6).

The mechanism of finding the exercise boundary can be described as follows:

- The PV of the project is calculated by prices of the last column of Table 5 (Jun. 2008) one by one from the top to the bottom regarding the gained cash flow of the project through these prices and the development cost ($129.911 million), when the PV of the project falls to zero, it reached to the exercise boundary in this column (Table 6).
- The next stage is going back to the previous column (May. 2008) and again the related PV of each price for the project will be calculated from the top to the bottom, but on each PV in this stage the question is: “should the project be developed immediately by this spot price and its related PV or it’s better to wait and postpone the development decision and keep the option alive? The answer of this question can be revealed by a comparison as following example:

The calculated PV of the project by the highest possible spot price of May. 2008 which locates in the top of the column ($39784) is 6.31E+09, but if the planner wait and keep the option alive to Jun. 2008, the spot price could be increased to 44685 $/tonne by the probability of 0.488 and the related PV of 7.14E+09 or decrease to 35419 $/tonne by the probability of 0.512 and the related PV of 5.58E+09 based on the built binominal tree of the price. So the value of keeping the option alive and postponing development given a spot price of 39784 $/tonne in May. 2008 is:

$$\text{Keeping option value} = \frac{P \times PV_{Up\ step} + Q \times PV_{Down\ step}}{e^{r\Delta t}}$$
$$= \frac{0.488 \times \$\ 7.14E+09 + 0.512 \times \$5.58E+09}{1.00585} \qquad (8)$$
$$= \$6.30E+09$$

Now making the comparison (getting $6.31E+09 or waiting for $6.30E+09), the decision is to exercise the project in May. 2008 and the value will be $6.31E+09 (Table 6).

- By continuing the process of comparison with the same method in the lower prices of this column, at last and at a particular price, the decision will be not to exercise the project. This point defines the boundary in this column (Table 6).

Going to the previous columns step by step and assuming get there without having the mine opened, the same procedure of comparison can be done. Then the optimal exercise boundary in all columns is determined one by one. These boundaries altogether form the exercise area of the project (shaded area in Table 6). But because the decision making is based on the spot price in a specific time, the exercise area should be copied on the spot prices tree (Table 5).

**Tab. 2. The executive options in a mining project**

| Project Stages | Existing Options | Goal |
|---|---|---|
| **Prospecting** | Proximately obligatory and without option, because the project must be born by carrying out this stage (But there is a cheap option in the form of gradual progress from method with lower cost to the method with higher cost and precision) | Find an exploration potential |
| **Exploration & Evaluation** | Drill or not to drill the core drills and after that making the drilling network more dense or not | Finding the resource and its quantitative evaluation, also decreasing the geological uncertainties (specially grade uncertainty) which these quantitative evaluation and decrease of grade uncertainty altogether can be named decrease of technical uncertainties |
| **Development** | Preparing the access roads and establishing the openings (in underground mining)/pre-production stripping (in open pit mining) or not to do them and waiting for a better situation/leaving the property | Investing under the most economic condition |
| **Production (Mining)** | The operation options in mining are limited and include short range production planning flexibilities in order to manage the natural and executive constraints. Therefore the suitable option must be selected during planning and design, before development stage. Among the effective optional parameters on planning and design, final product price is the most effective one, because mining is an inflexible process against increase or decrease of production planning while it is the price that defines the border of ore and waste and consequently the production planning | Making better the financial efficiency of the operation |
| **Reclamation** | The existing flexibilities in reclamation, such as the post mining land uses (agriculture, pasture, foresting, tourist attraction, constriction), the method of reclamation, and the starting time of reclamation process | |

**Tab. 3. The project assumptions**

| Tons/year (million) | Mining cost/tonne | Milling cost/tonne | FT cost/tonne of metal | Metal recovery | Development Duration | NCY | Marginal tax rate | Average grade | Taylor's mine life | Capital cost (million) | Depreciation Rate/year | OSR |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | $2 | $5 | $450 | 0.9 | 1 year | 2.2% | 10% | 0.5% | 15 | $130 | 6.66% | 1:1 |

**Tab. 4. The NPV of the just developed project by the spot copper price of Jan. 2007 (the true underlying asset)**

| Time Step | Jan. 2007 | Jan. 2008 | Jan. 2009 | Jan. 2010 | Jan. 2011 | Jan. 2012 | Jan. 2013 | Jan. 2014 | Jan. 2015 | Jan. 2016 | Jan. 2017 | Jan. 2018 | Jan. 2019 | Jan. 2020 | Jan. 2021 | Jan. 2022 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Forward price/tonne of metal (P) | 6199 | 6497 | 6808 | 7135 | 7478 | 7837 | 8213 | 8607 | 9020 | 9453 | 9907 | 10382 | 10881 | 11403 | 11950 | 12524 |
| Further treatment cost/tonne of metal (F) | | 450 | 450 | 450 | 450 | 450 | 450 | 450 | 450 | 450 | 450 | 450 | 450 | 450 | 450 | 450 |
| Net forward price/tonne of metal (P-F) | | 6047 | 6358 | 6685 | 7028 | 7387 | 7763 | 8157 | 8570 | 9003 | 9457 | 9932 | 10431 | 10953 | 11500 | 12074 |
| Gross income/ tonne R × ave. grade (P-F) | | 27.21 | 28.61 | 30.08 | 31.62 | 33.24 | 34.93 | 36.71 | 38.57 | 40.51 | 42.56 | 44.70 | 46.94 | 49.29 | 51.75 | 54.33 |
| Mining cost/tonne (b) | | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| Waste removal cost/tonne (a) | | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| Milling cost/tonne (c) | | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 |
| Marketing Margin (mm) | | 18.21 | 19.61 | 21.08 | 22.62 | 24.24 | 25.93 | 27.71 | 29.57 | 31.51 | 33.56 | 35.70 | 37.94 | 40.29 | 42.75 | 45.33 |
| Million tonne production | | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| Income ($) | | 3.64 E+07 | 3.92 E+07 | 4.22 E+07 | 4.52 E+07 | 4.85 E+07 | 5.19 E+07 | 5.54 E+07 | 5.91 E+07 | 6.30 E+07 | 6.71 E+07 | 7.14 E+07 | 7.59 E+07 | 8.06 E+07 | 8.55 E+07 | 9.07 E+07 |
| Less: depreciation ($) | | 7.87 E+06 | 7.87 E+06 | 7.87 E+06 | 7.87 E+06 | 7.87 E+06 | 7.87 E+06 | 7.87 E+06 | 7.87 E+06 | 7.87 E+06 | 7.87 E+06 | 7.87 E+06 | 7.87 E+06 | 7.87 E+06 | 7.87 E+06 | 7.87 E+06 |
| Taxable income ($) | | 2.86 E+07 | 3.14 E+07 | 3.43 E+07 | 3.74 E+07 | 4.06 E+07 | 4.40 E+07 | 4.75 E+07 | 5.13 E+07 | 5.52 E+07 | 5.92 E+07 | 6.35 E+07 | 6.80 E+07 | 7.27 E+07 | 7.76 E+07 | 8.28 E+07 |
| Less: Tax ($) | | 2.86 E+06 | 3.14 E+06 | 3.43 E+06 | 3.74 E+06 | 4.06 E+06 | 4.40 E+06 | 4.75 E+06 | 5.13 E+06 | 5.52 E+06 | 5.92 E+06 | 6.35 E+06 | 6.80 E+06 | 7.27 E+06 | 7.76 E+06 | 8.28 E+06 |
| Net Income(Cash Flow) ($) | | 3.36 E+07 | 3.61 E+07 | 3.87 E+07 | 4.15 E+07 | 4.44 E+07 | 4.75 E+07 | 5.07 E+07 | 5.40 E+07 | 5.75 E+07 | 6.12 E+07 | 6.50 E+07 | 6.91 E+07 | 7.33 E+07 | 7.77 E+07 | 8.24 E+07 |
| Free CF at risk-free rate ($) | | 3.14 E+07 | 3.37 E+07 | 3.62 E+07 | 3.88 E+07 | 4.15 E+07 | 4.44 E+07 | 4.73 E+07 | 5.05 E+07 | 5.37 E+07 | 5.72 E+07 | 6.08 E+07 | 6.46 E+07 | 6.85 E+07 | 7.27 E+07 | 7.70 E+07 |

| | |
|---|---|
| Present Value ($) | 779328243 |
| NPV($) | 649417222 |

**Tab. 5. The binominal price model and the exercise area of the project through prices**

| Jan-07 | Feb-07 | Mar-07 | Apr-07 | May-07 | Jun-07 | Jul-07 | Aug-07 | Sep-07 | Oct-07 | Nov-07 | Dec-07 | Jan-08 | Feb-08 | Mar-08 | Apr-08 | May-08 | Jun-08 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 6199 | 6963 | 7821 | 8784 | 9867 | 11082 | 12448 | 13981 | 15704 | 17639 | 19812 | 22253 | 24995 | 28075 | 31534 | 35419 | 39784 | 44685 |
|  | 5519 | 6199 | 6963 | 7821 | 8784 | 9867 | 11082 | 12448 | 13981 | 15704 | 17639 | 19812 | 22253 | 24995 | 28075 | 31534 | 35419 |
|  |  | 4914 | 5519 | 6199 | 6963 | 7821 | 8784 | 9867 | 11082 | 12448 | 13981 | 15704 | 17639 | 19812 | 22253 | 24995 | 28075 |
|  |  |  | 4375 | 4914 | 5519 | 6199 | 6963 | 7821 | 8784 | 9867 | 11082 | 12448 | 13981 | 15704 | 17639 | 19812 | 22253 |
|  |  |  |  | 3895 | 4375 | 4914 | 5519 | 6199 | 6963 | 7821 | 8784 | 9867 | 11082 | 12448 | 13981 | 15704 | 17639 |
|  |  |  |  |  | 3467 | 3895 | 4375 | 4914 | 5519 | 6199 | 6963 | 7821 | 8784 | 9867 | 11082 | 12448 | 13981 |
|  |  |  |  |  |  | 3087 | 3467 | 3895 | 4375 | 4914 | 5519 | 6199 | 6963 | 7821 | 8784 | 9867 | 11082 |
|  |  |  |  |  |  |  | 2748 | 3087 | 3467 | 3895 | 4375 | 4914 | 5519 | 6199 | 6963 | 7821 | 8784 |
|  |  |  |  |  |  |  |  | 2447 | 2748 | 3087 | 3467 | 3895 | 4375 | 4914 | 5519 | 6199 | 6963 |
|  |  |  |  |  |  |  |  |  | 2179 | 2447 | 2748 | 3087 | 3467 | 3895 | 4375 | 4914 | 5519 |
|  |  |  |  |  |  |  |  |  |  | 1940 | 2179 | 2447 | 2748 | 3087 | 3467 | 3895 | 4375 |
|  |  |  |  |  |  |  |  |  |  |  | 1727 | 1940 | 2179 | 2447 | 2748 | 3087 | 3467 |
|  |  |  |  |  |  |  |  |  |  |  |  | 1537 | 1727 | 1940 | 2179 | 2447 | 2748 |
|  |  |  |  |  |  |  |  |  |  |  |  |  | 1369 | 1537 | 1727 | 1940 | 2179 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  | 1219 | 1369 | 1537 | 1727 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 1085 | 1219 | 1369 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 966 | 1085 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 860 |

**Tab. 6. The exercise area of the project through values**

| Jan-07 | Feb-07 | Mar-07 | Apr-07 | May-07 | Jun-07 | Jul-07 | Aug-07 | Sep-07 | Oct-07 | Nov-07 | Dec-07 | Jan-08 | Feb-08 | Mar-08 | Apr-08 | May-08 | Jun-08 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 6.49E8 | 7.78E8 | 9.23E8 | 1.09E9 | 1.27E9 | 1.47E9 | 1.70E9 | 1.96E9 | 2.25E9 | 2.58E9 | 2.95E9 | 3.36E9 | 3.82E9 | 4.34E9 | 4.92E9 | 5.58E9 | 6.31E9 | 7.14E9 |
|  | 4.67E8 | 6.49E8 | 7.78E8 | 9.23E8 | 1.09E9 | 1.27E9 | 1.47E9 | 1.70E9 | 1.96E9 | 2.25E9 | 2.58E9 | 2.95E9 | 3.36E9 | 3.82E9 | 4.34E9 | 4.92E9 | 5.58E9 |
|  |  | 4.33E8 | 5.35E8 | 6.49E8 | 7.78E8 | 9.23E8 | 1.09E9 | 1.27E9 | 1.47E9 | 1.70E9 | 1.96E9 | 2.25E9 | 2.58E9 | 2.95E9 | 3.36E9 | 3.82E9 | 4.34E9 |
|  |  |  | 3.42E8 | 4.33E8 | 5.35E8 | 6.49E8 | 7.78E8 | 9.23E8 | 1.09E9 | 1.27E9 | 1.47E9 | 1.70E9 | 1.96E9 | 2.25E9 | 2.58E9 | 2.95E9 | 3.36E9 |
|  |  |  |  | 2.61E8 | 3.42E8 | 4.33E8 | 5.35E8 | 6.49E8 | 7.78E8 | 9.23E8 | 1.09E9 | 1.27E9 | 1.47E9 | 1.70E9 | 1.96E9 | 2.25E9 | 2.58E9 |
|  |  |  |  |  | 1.89E8 | 2.61E8 | 3.42E8 | 4.33E8 | 5.35E8 | 6.49E8 | 7.78E8 | 9.23E8 | 1.09E9 | 1.27E9 | 1.47E9 | 1.70E9 | 1.96E9 |
|  |  |  |  |  |  | 1.25E8 | 1.89E8 | 2.61E8 | 3.42E8 | 4.33E8 | 5.35E8 | 6.49E8 | 7.78E8 | 9.23E8 | 1.09E9 | 1.27E9 | 1.47E9 |
|  |  |  |  |  |  |  | 6.74E7 | 1.25E8 | 1.89E8 | 2.78E8 | 3.42E8 | 4.33E8 | 5.35E8 | 6.49E8 | 7.78E8 | 9.23E8 | 1.09E9 |
|  |  |  |  |  |  |  |  | 1.67E7 | 6.74E7 | 1.25E8 | 1.89E8 | 2.61E8 | 3.42E8 | 4.33E8 | 5.35E8 | 6.49E8 | 7.78E8 |
|  |  |  |  |  |  |  |  |  | -2.85E7 | 1.67E7 | 6.74E7 | 1.25E8 | 1.89E8 | 2.61E8 | 3.42E8 | 4.33E8 | 5.35E8 |
|  |  |  |  |  |  |  |  |  |  | -6.88E7 | -2.85E7 | 1.67E7 | 6.74E7 | 1.25E8 | 1.89E8 | 2.61E8 | 3.42E8 |
|  |  |  |  |  |  |  |  |  |  |  | -1.05E8 | -6.88E7 | -2.85E7 | 1.67E7 | 6.74E7 | 1.25E8 | 1.89E8 |
|  |  |  |  |  |  |  |  |  |  |  |  | -1.37E8 | -1.05E8 | -6.88E7 | -2.85E7 | 1.67E7 | 6.74E7 |
|  |  |  |  |  |  |  |  |  |  |  |  |  | -1.65E8 | -1.37E8 | -1.05E8 | -6.88E7 | -2.85E7 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  | -1.90E8 | -1.65E8 | -1.37E8 | -1.05E8 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | -2.13E8 | -1.90E8 | -1.65E8 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | -2.33E8 | -2.13E8 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | -2.51E8 |

Shaded area in Table 5 shows the exercise area. Table 5 says by the actual spot copper price of 6199 $/tonne in Jan 2007, the best decision is to wait and see and this will be the decision made till Mar. 2007.

In Apr. 2007 if the copper price reaches to about 8500 $/tonne, the decision will be to exercise the project; otherwise, the decision again is to wait and see. After that and in May 2007 if the mine hasn't been opened yet and if the copper price reaches to about 9500 $/tonne, the decision will be exercising the project, but if the price is less than 9500 $/tonne, it will be better to wait and see. In Jun. 2007 the threshold price is again about $8500 and for the less prices than that the decision is to wait and see and for the prices more than that the project will be exercised, if it wasn't exercised earlier. Then in Jul. 2007 the threshold price is again about \$9500. Table 5 shows the exercise boundary for the other specific times with the same configuration. But if it comes to Jun. 2008 without exercising the project the threshold price will be just about \$2500, because it is the point of now or never.

## 5. Finding the UPL Considering Price Uncertainty

It seems that the first person who paid attention to the price changes and the problem of price uncertainty was Groze (1969), but the first applicable consideration of price uncertainty belongs to Whittle (1988), who indicated it indirectly. Whittle after Lerchs-Grossmann nested algorithm by his 4D package generates a series of nested pits in which each pit is optimal for a different set of economic ratios. His primary goal was presenting a production plan in which the nested pit show the gradual development of the open pit mine in accordance with the economical growth and price increase, but in practice each nested pit could be the representative open pit mine of a certain economic condition. From Whittle point of view a block economic value can be calculated through the following equation.

$$\text{Block Economic Value} = [(M \times R \times P) - (T_o \times C_p)] - (T \times C_m) \quad (9)$$

Where T is tonnes of rock in the block, $T_o$ is tonnes of ore in the block, M is the metal content of the block, P is the final product price (metal price), R is the portion of recoverable metal, $C_m$ is the mining cost per tonne, and $C_p$ is the processing cost of ore per tonne. Among these seven factors T , $T_o$ and M are categorized as the geological factors depending on the geological logic of block modeling of the resource. R is the technological representative and just P, $C_m$ and $C_p$ are economical factors.

As it discussed before in this paper $C_m$ and $C_p$ are sensitive compared to P. Whittle thought so and focused on P and for making his equation simple divided it by $C_m$ which is more sensitive compared to $C_p$, the result was the equation below.

$$Value = \{[(M \times R \times (P/C_m)] - [T_o \times (C_p/C_m)]\} - T \quad (10)$$

In equation 10 $C_p/C_m$ is dimensionless and indifferent against any inflation factor and if $P/C_m$ is converted to $C_m/P$, the calculated value will be indicative of the amount of product gained from each tonne of ore (Equation 11).

$$\text{Value} = \{[(M \times R)/(C_m/P)] - [T_o \times (C_p/C_m)]\} - T \quad (11)$$

Paying attention to the equation 11 it could be realized that by decreasing $C_m/P$, the gained value will increase where the decrease of $C_m/P$ in its turn depends on the increase of P. Therefore considering 40 P from $P - 20\%P$ to $P + 20\%P$, 40 nested pit will be formed which each one could be suitable for a certain economic condition and altogether are the incarnation of a Time-Volume or a 4D open pit mine, but in the economic climate of the last two decades of 20th century, the time can be named the period of relative price constancy. But in nowadays economic condition and in the age of astonishing metal price growth this methodology is incapable.

For example copper price from 2003 to 2005 doubled (from 1819 \$/tonne to \$3679 \$/tonne) and again from 2005 to 2006 became 1.8 times more (from 3679 \$/tonne to \$6758 \$/tonne), it means 360% growth during just in 3 years. As a mater of fact the considered flexibility by Whittle against price uncertainty which is $\pm 20\%$ won't be enough at all. The required flexibility for covering such a price uncertainty is about $\pm 400\%$. This means 800 nested pits for a time period of just 3 years instead of 40 for at least 15 years. Such a variation during such a short period of time would be unmanageable and the difference between the basic pit and the desired pit after price growth would be too great to develop. Such a condition just could be managed if the price growth can be predicted; otherwise some kind of option should be available to the mine planner in order to make his decision on the policy of developing the pit or the wait and see policy. Making the decision to develop a pit or to wait and see can be defined as the concept of ROA in the mine planning procedure and determination of the UPL.

Before starting the explanation of suggested real option methodology for finding the UPL, it is necessary to highlight the two particular differences between a metal mining project and other natural resources projects such as coal or oil.

- The convenience yield on a productive asset looks and works just as known cash yield on a financial asset. It drives down the value and cause to consider exercising earlier. Some industries used to store some natural resources commodities as their feeds. These commodities give them the continuation of cash flow, they achieved by transforming the commodities. Some of these industries are electricity producers which store coal or oil, oil refineries which store crude, producers of manufacturing metal parts which store metals, and smelters which store metal concentrate. Storing such commodities causes storage cost, but selling or lending these commodities could cause losing the continuation of the cash flow and benefits. But the point is whether the producers of the raw materials and the inferior industries which are their consumers are financially related or not? In other words, do the superior industries and inferior industries organize in a specific firm or not? When a firm owns both the raw material producing facilities and the transforming facilities of the raw material (such as an electricity producer which owns a coal mine, or a metal concentrate producer which owns smelter and refinery), stores a commodity out land, there are both storage cost and convenience yield. But if the raw material producing facilities and the transforming facilities belong to different firms (such as a metal concentrate producer doesn't have smelter and refinery against a smelter company, or a crude producing company against a refining company), the raw material producers accepting some storage cost can take benefits from storing the extracted material outland or even the material inland without any storage cost by postponing the exercise time of the project, disregard to the interruption of the cash flow of the inferior industries. In such condition there will be no convenience yield. The overall frame of this research is creating value; hence it contemplates about the concept of value as a leagued concept through the whole production path from mining to processing, to smelting and refining up to producing metal for using in different industries. Therefore it must be careful about the lost value in the inferior industries due to storing the resources in land or the run-of-mine or concentrate outland. Based on such a point of view the metal miners and metal producers are interrelated and they must be accounted as a unique firm with final product of metal. But on the other hand a metal miners face simultaneously with the considerable storage cost of metal concentrate and the good market condition for the metal as the final product. Regarding the both sides, the assumed net convenience yield in metal projects should be an average rate, while in other natural resources projects such as coal and oil, usually it is above average or even high.

- Despite the coal and oil deposits, metal deposits are not constant against price variation. In case of coal or oil the amount of reserve won't change, if the price increases or decreases and it is just the revenue that will increase or decrease. But a metalliferous deposit expands or reduces in land by increasing or decreasing of the price. It means by any increase in the price, the cut-off grade decreases, hence more materials can be defined as ore and by any decrease in the price, vice versa. It should be noted that while by price increase the related revenue of the previously defined ore increases, some new low grade ores are added to the production planning which their related revenues are low. Therefore the average unit of revenue for the orebody could be reduced. Here this fact is revealed through the assumed deposit of Figure 6 which is a cylindrical copper deposit with the grade distribution described in Table 7.

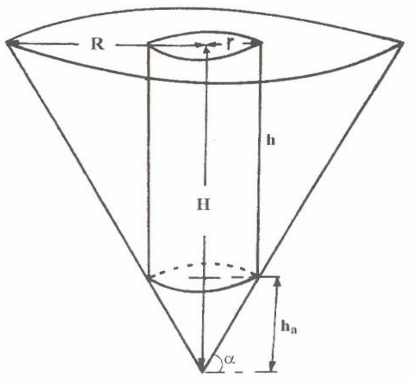

**Fig. 6. Open pit mining design parameters of a cylindrical deposit**

The high grade ore zone is located in the central cylinder and when it goes to the outer periderms, it becomes poorer. The other concerned assumptions are as follows:

- Diameter of horizontal section is about 160 meters
- Vertical expansion is about 217 Meters
- The Specific Gravity of ore and waste is nearly the same (SG=2.292)

**Tab. 7. Grade distribution of the assumed deposit**

| Tonnage | Grade |
|---|---|
| 1000000 | 0.0 – 0.1 |
| 1000000 | 0.1 – 0.2 |
| 1000000 | 0.2 – 0.3 |
| 1000000 | 0.3 – 0.4 |
| 1000000 | 0.4 – 0.5 |
| 1000000 | 0.5 – 0.6 |
| 1000000 | 0.6 – 0.7 |
| 1000000 | 0.7 – 0.8 |
| 1000000 | 0.8 – 0.9 |
| 1000000 | 0.9 – 1.0 |

In order to use ROA for determining UPL in mine planning process, at first it is necessary to take a methodology for managing the problem of having an unsteady resource against price fluctuations. For doing that, this study divides the prices within the uncertain bounds to some spans, then considers some development alternatives for each price span and defines the concept of alternative quality for them. It should be noted that here each development alternative, in fact is an option. Modeling procedure starts from defining NPV according to price as follows:

$$NPV_j = DCF_j - CC_j \qquad (12)$$

Where $NPV_j$ is the net present value of the mine when it's developed through alternative or option j, $CC_j$ is the capital cost of option j including the cost of mining machinery, processing machinery, all necessary roads and buildings, infrastructures and pre-mining waste removal, and $\mathrm{DCF_j}$ is the discounted cash flow of the project after development through option j, while its cash flow can be shown by the following equation.

$$CF_j = \{[R \times SL \times RL \times f\ (P - S - r - F)\bar{g}_j] - (b + c) - OSR_j \times a\} T_j \tag{13}$$

Where $\mathrm{CF_j}$ is the gained cash flow of the mine when developed by option j, r is the concentrating recovery, SL is the smelting loss, RL is refining loss, f is money factor, P is the price of metal per tonne, S is the smelting cost per tonne of metal, R is the refining cost per tonne of metal, F is the freight cost from concentrator to smelting and refining plant, b is mining cost per tonne, c is concentrating cost per tonne of ore, a is waste removal cost per tonne. It should be noted that the cost factors such as b, c, and a depend on production rate which defers from a specific development alternative to another, but as the main part of mining and mineral processing currently are taking place in developing countries with low rate of operating cost, this tolerance can be disregarded. The $\bar{\mathrm{g}}_\mathrm{j}$ is the average grade of the located ore in the determined limits of the mine when developed by option j, also $\mathrm{OSR_j}$ is overall stripping ratio of the open pit mine when developed by option j and $\mathrm{T_j}$ is tonnes of ore located in the determined limits of the mine when developed by option j.

As the price in use for valuation in mineral industry is the price of processed material (metal) despite the prices in use in coal or oil industry which are the price of raw material (coal or crude oil), the cash flow model here is a little more complex. Hence for making the model simpler, the orebody's quality is defined as follows:

$$q_j = R \times SL \times RL \times f \tag{14}$$

And FTC can be defined as the further treatment costs by following equation.

$$FTC = S + r + F \tag{15}$$

And $\mathrm{OC_j}$ as the operating cost of the plant when developed by option j can be defined as follows:

$$OC_j = b + c + OSR_j \times a \tag{16}$$

Then $\mathrm{DCF_j}$ can be calculated by Equation 17.

$$DCF_j = \sum_{n=1}^{n_j} [((q_j (P - FTC)\bar{g}_j - OC_j)\frac{T_j}{n_j}) \times (\frac{1}{(1+i)^n})] \tag{17}$$

Where $\mathrm{n_j}$ is the mine life when developed by option j and i is interest rate. Obviously the NPV of the mine through option j can be calculated as follows:

$$NPV_j = \{\sum_{n=1}^{n_j} [(q_j \bar{g}_j \frac{T_j}{n_j}(P - FTC) - \frac{T_j}{n_j} OC_j) \times (\frac{1}{(1+i)^n})]\} - CC_j \tag{18}$$

If the aforesaid concept of alternative quality (option quality) is $\mathrm{Q_j = q_j \bar{g}_j \frac{T_j}{n_j}}$, the NPV of option j can be calculated as follows:

$$NPV_j = (Q_j (P - FTC) - OC_j \frac{T_j}{n_j})(\frac{(1+i)^{n_j} - 1}{i(1+i)^{n_j}}) - CC_j \tag{19}$$

$$NPV_j = (Q_j (P - FTC)(\frac{(1+i)^{n_j} - 1}{i(1+i)^{n_j}})) - (OC_j \frac{T_j}{n_j} \frac{(1+i)^{n_j} - 1}{i(1+i)^{n_j}}) - CC_j \tag{20}$$

$$NPV_j = Q_j P (\frac{(1+i)^{n_j} - 1}{i(1+i)^{n_j}}) - [(Q_j FTC (\frac{(1+i)^{n_j} - 1}{i(1+i)^{n_j}})) + (OC_j \frac{T_j}{n_j} (\frac{(1+i)^{n_j} - 1}{i(1+i)^{n_j}})) + CC_j] \tag{21}$$

Equation 21 reveals that the NPV of the option j depends on $P$ as the independent variable, $Q_j (\frac{(1+i)^{n_j} - 1}{i(1+i)^{n_j}})$ as the gradient and $[(Q_j FTC (\frac{(1+i)^{n_j} - 1}{i(1+i)^{n_j}})) + (OC_j \frac{T_j}{n_j} (\frac{(1+i)^{n_j} - 1}{i(1+i)^{n_j}})) + CC_j]$ as the y-intercept. The option j should be considered for a price span in which the $NPV_j$ can be developed depending on the price (P). In this span $\mathrm{Q_j}$ and $\mathrm{n_j}$ are the variables which determine the NPV gradient which is shown in Figure 8 by $\theta_\mathrm{j}$. Regarding Equation 21 it is difficult to predict a general arrangement for the graphs which can describe the changes of the options' NPVs, because $\mathrm{T_j}$, $\mathrm{n_j}$, $\bar{\mathrm{g}}_\mathrm{j}$ and in consequence $\mathrm{Q_j}$ show different behavior based on the tonnage-grade configuration. But there is an overall shape which the arrangement of the graphs is mostly similar to it (Figure 7). The red areas in Figure 7 show despite the price alteration dictates using a new development option, the development option mustn't be changed. It must be changed just after the red area disappeared. The red area could be seen in the preliminary stages of using a development option. It is because of paying extra development cost for a new development option while the price hasn't increased much yet. Figure 8 shows the options' NPVs graphs for the assumed deposit of Table 7.

The above mentioned methodology can manage the problem of having an unsteady resource against price fluctuations and prepares the background in order to develop the uncertainty management methodology through finding the exercise boundaries.

This is these boundaries which can guide the mine planner in an uncertain situation. Having these boundaries the planner can decide when and on what condition from price point of view can start the project and by which alternative or option should develop the mine. Hence, here finding the exercise boundaries will be a complicated problem against the example mentioned through Tables 4, 5 and 6 or against an application of ROA in oil or coal projects.

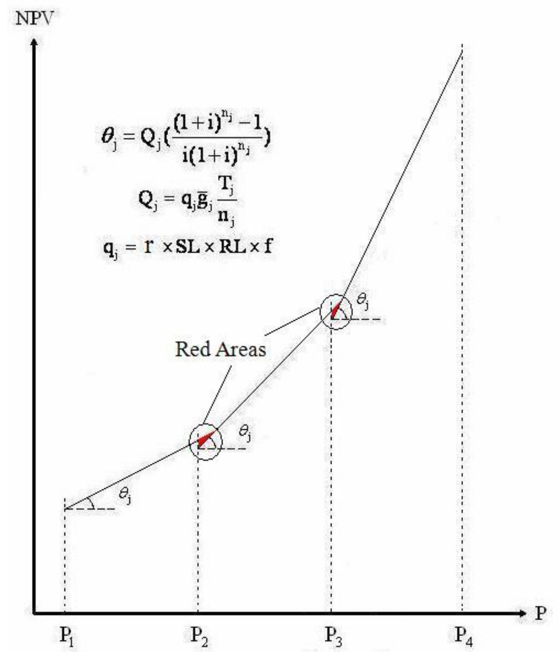

**Fig. 7. The general arrangement of NPV behavior of the alternatives**

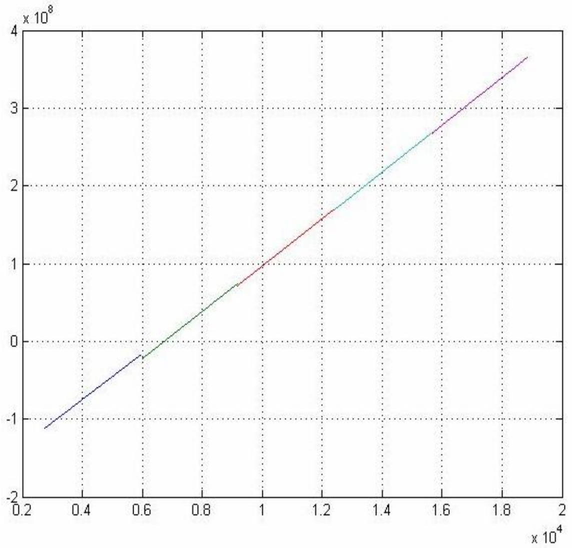

**Fig. 8. The NPV behavior of the alternatives for the resource with the tonnage-grade configuration of the table 7**

On that example the amount of metalliferous ore resource was assumed steady against price fluctuations just like a coal or oil resource and there was just an alternative for developing the mine. But here for controlling the problem of having an unsteady metalliferous ore resource against price fluctuations, some executive alternative or in fact some development options were formed. Also, as the price volatility in metals usually is more than coal or oil, using discrete time steps in uncertainty modeling procedure results an imprecise stairs form exercise boundary which in some cases cannot exactly show the acceptable threshold price for investment, therefore it's needed to do the calculations based on a time continuum. The combination of multiplicity of the alternatives and the continuity of the time structure make it necessary to find the exercise boundaries by software.

## 6. Practical Description of the Methodology

In order to give a practical comprehensive description about the presented methodology, this study has utilized it for the assumed resource which its general shape was shown in Figure 6, its grade distribution was shown in Table 7. This time it is assumed that the project can be started from Jan. 2008 up to Dec. 2008. The observed copper price volatility by Jan. 2008 is 0.3022, hence the results of copper price uncertainty modeling through binominal method show the copper price won't exceed 18840 \$/tonne while won't fall below 2521 \$/tonne. The Overall Stripping Ratio (OSR) of such a deposit for open pit mining can be calculated as follows:

$$OSR = \frac{\left[\frac{1}{3}\pi R^2 H - \frac{1}{3}\pi r^2 h_a\right] - \pi r^2 h}{\pi r^2 h} \tag{22}$$

Where R is the radius of the biggest section of the pit, H is the height of the upside down cone which pit is a part of it in an unfinished manner, r is the radius of the horizontal section of cylindrical deposit, $h_a$ is height of the out of pit section of upside down cone. These parameters are calculated as follows:

$$h_a = r \times \tan\alpha \tag{23}$$

$$H = h + h_a + Over\ Burden\ Thickness \tag{24}$$

$$R = \frac{H}{\tan\ \alpha} \tag{25}$$

Where α is the stable slope angle of the pit's wall. As it can be seen through equations 22 to 25, all the above mentioned design parameters which are the design parameters of such a deposit for open pit mining will change if r (radius of the horizontal section of cylindrical deposit) changes, while r is continuously changing based on price fluctuations. Here this r parameter makes the project undefined and the pit limits unknown. Before this study ROA just used for evaluation a defined project with defined limitations. But this study uses it for pit limits determination of an undefined project.

For managing the problem of having an unsteady resource against price fluctuations and prepares the background to develop the exercise boundaries, at first a price span with the limitation of the modeled price uncertainty should be considered. Then this main span regarding its extent ($2570-$18840) divided to five sub-spans in order to define the executive alternatives (development options). Depending on the price in the sub-spans, their related cut-offs, the resultant r, and regarding Equation 21 the development options are defined. The specifications of these options are shown in Table 8 while their related NPV graphs depended on price were shown before through Figure 8. As a matter of fact Figure 8 is prepared based on the information presented in Table 8. The assumptions were impressive in achieving the data of Table 8 are as follows: Mining cost per tonne (b) is 2 USD, waste removal cost per tonne (a) is 2 USD, milling cost per tonne (c) is 5 USD, further treatment cost (FTC) is 450 USD, the interest rate is 7%, the development duration is one year while there is one extra year for decision making about the best time for starting the project, and the capital costs of the development options were calculated by modified O’Hara estimator [34].

For finding the threshold curves, the first development option (Alt. 1) and the last development option (Alt. 5) are disregarded, because the relative NPV of the Alt. 1 is negative in return for all the appropriate prices of Alt. 1 and the appropriate prices of the Alt. 5 are too high to consider (15622-18840 USD), hence the engineering judgment would be to omit it. The result of multiple and time continuum analysis of the development options by ROA is the threshold investment curves in Figure 10. Based on these curves the UPL of the copper deposit of Table 7 can be achieved for every point of price-time plane, if this point of the plane is principally included in the exercise areas and isn’t included in the area of waiting (Figure 10).

As a matter of fact the curves of Figure 10 are clearly the answer to the questions of UPL determination but in order to give some explanative examples, suppose the mine planner reaches to Jul. 2008 without having the mine opened before and the copper price on that point of the time be 8000 USD, in this situation the decision of the planner will be wait and see, but if the copper price on that time be 9000 USD, the decision will be developing the mine by development option 2, while if the price would be 10000 USD at the same time, the decision will again be wait and see, hence the price-time point of 8000 USD-Jul. 1, 2008 and 10000 USD-Jul. 1, 2008 means nothing except waiting, while 9000 USD-Jul. 1, 2008 means developing the mine by the alternative 2 and recovering 89.09% of the deposit. Reaching the price to 12000 USD at that time will make the planner to develop the mine by the development option 3 and if at that time the price is 13000 USD the best choice is developing the mine by development option 4, therefore the price-time point of 12000 USD-Jul. 1, 2008 and 13000 USD-Jul. 1, 2008 means developing the mine by the alternative 3 and 4 respectively and recovering 92.48% and 94.27% of the deposit.

## 7. Conclusions

The carried out studies showed that non of the existing algorithm of UPL determination consider the metal price uncertainty while it was shown that the price is the most sensitive factor in mine planning procedure with regard to UPL determination. In this study ROA was used as an effective tool of decision making in the condition of uncertainty in order to prepare a methodology for determining the UPL, whereas ROA were just used for evaluating of defined natural resources projects before. The proposed methodology is based on determination of the exercise boundaries for a metal project to be developed. But as the magnitude of metal resources in spite of oil or coal resources is not steady against price fluctuations and it varies in concordance with the variation of price and cut-offs, it is necessary to manage this problem at first. Managing this problem was done in this study by considering some adequate executive alternatives as the development options for the project.

**Tab. 8. The specification of the defined executive alternatives for managing the problem of having an unsteady resource against price fluctuations**

| Development Options | Alt 1 | Alt 2 | Alt 3 | Alt 4 | Alt 5 |
|---|---|---|---|---|---|
| Price span (USD) | 2750-5968 | 5968-9186 | 9186-12404 | 12404-15622 | 15622-18840 |
| r (m) | 71.62 | 75.53 | 76.95 | 77.69 | 78.14 |
| (m) $h_a$ | 71.62 | 75.53 | 76.95 | 77.69 | 78.14 |
| H (m) | 288.62 | 292.53 | 293.95 | 294.69 | 295.14 |
| R (m) | 288.62 | 292.53 | 293.95 | 294.69 | 295.14 |
| Ore Tonnage ( $T_j$ ) by tonne | 8010290 | 8908688 | 9248161 | 9426544 | 9536513 |

| Development Options | Alt 1 | Alt 2 | Alt 3 | Alt 4 | Alt 5 |
|---|---|---|---|---|---|
| Cumulative Ore Tonnage (%) | 80.10 | 89.09 | 92.48 | 94.27 | 95.37 |
| Average Grade (%) | 0.599 | 0.555 | 0.538 | 0.529 | 0.523 |
| Life ( $n_j$) by year | 11 | 11 | 11 | 11 | 11 |
| Production per year (tonne) | 728208 | 809881 | 840742 | 856959 | 866956 |
| (USD) $CC_j$ | 74965338 | 79902402 | 81715566 | 82657656 | 83234875 |
| (USD) $OC_j$ | 19.18039 | 18.24949 | 17.94104 | 17.7871 | 17.69483 |
| $Q_j$ | 3929 | 4042 | 4068 | 4077 | 4082 |
| $q_j$ | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 |
| OSR:1 | 6.09 | 5.62 | 5.47 | 5.39 | 5.35 |
| Waste (tonne) | 48784222 | 50109119 | 50592233 | 50842544 | 50995675 |
| NPV Gradient $Q_j(\frac{(1+i)^{n_j}-1}{i(1+i)^{n_j}})$ | 29461.93 | 30311.05 | 30502.99 | 30575.52 | 30610.50 |
| NPV y-intercept $[(Q_j FTC(\frac{(1+i)^{n_j}-1}{i(1+i)^{n_j}}))+(OC_j\frac{T_j}{n_j}(\frac{(1+i)^{n_j}-1}{i(1+i)^{n_j}}))+CC_j]$ | 192959559 | 204372138 | 208550274 | 210717499 | 212043993 |

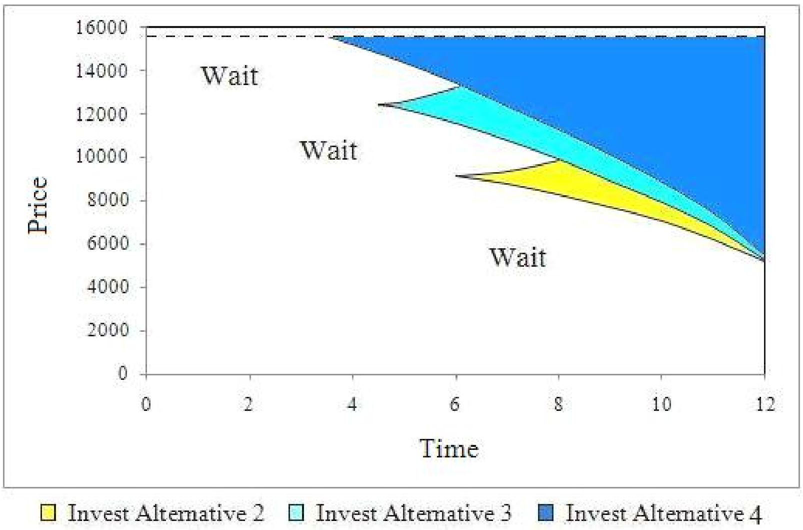

**Fig. 10. The threshold investment curves for the copper deposit of table 7**

These development options are defined through their related gradients and y-intercepts which in their turns are depended on the orebody quality, the interest rate, the life of the alternatives, the available ore tonnage when the mine developed by an specific option, the related capital and operating costs of the options and some secondary variables which were used in the modeling in this study.

Before managing the aforesaid problem through defining the development options, the exercise boundary was determined for a simple case assuming just one executive alternative in order to give an idea of how to achieve such a boundary in mine planning procedure, but ultimately considering the multiplicity of the alternatives and the continuity of the time structure the exercise boundaries has been developed in the shape of threshold curves for a cylindrical copper deposit. Although the resultant curves are adequate for the supposed case, the proposed methodology can be used for all other deposits in order to find their relative investment threshold curves. Through these curves a mine planner can determine the UPL in different times and with different prices or in other word in any price-time point of the price-time plane which is described through the min-max amplitude of the modeled price uncertainty and the available time of decision making.

## References

[1] Lillico, T.M., *Economics and Open Pit Design - Beware of Break-Even Ratios*. International Journal of Rock Mechanics, Mining Science & Geomechanics Abs., 1974, 11(8), A17.

[2] Koskiniemi, B.C., *Hand methods. In Open Pit Mine Planning and Design,* ed. Crawford & Hustrulid. SME/AIME Transactions, 1979, pp. 187-195.

[3] Lerchs, H., Grossmann, L., *Optimum Design of Open-Pit Mines*. CIM Bulletin, 1965, 58, pp. 47-54.

[4] Lerchs, H., Grosmann, L., *Optimum Design of Open Pit Mines.* CIM Transaction, 1965, 68, pp.17-24.

[5] Zhao, Y., Kim, Y.C., *A New Ultimate Pit Limit Design Algorithm.* 23rd APCOM, 1992, pp. 423-434.

[6] Yamatomi, J., Mogi, G., Akaike, A., Yamaguchi, U., Selective extraction dynamic cone algorithm for three dimensional open pit designs. 25th APCOM, 1995, 267-274.

[7] Picard, J.C., *Maximum Closure of a Graph and Applications to Combinatorial Problems*. Management science, 1976, 22, pp. 1268-1272.

[8] David, M., Dowd, P.A., Korobov, S., Forcasting departure from planning in open pit design and grade control. 12th APCOM, 1974, F131-F142.

[9] Dowd, P.A., Onur, A.H., Optimizing open pit design and scheduling. 23rd APCOM, 1992, 411-422.

[10] Dowd, P.A., Onur, A.H., Open pit optimization – Part 1: Optimal open pit design. IMM Transactions, 1993, 102, A95-A104.

[11] Gershon, M.E., Alinear programming approach to mine scheduling optimization. 17th APCOM, 1982, 483-493.

[12] Huttagosol, P., Cameron, R.E., *A Computer Design of Ultimate Pit Limit by Using Transportation Algorithm.* 23rd APCOM, 1992, 443-460.

[13] Denby, B., Schofield, D., *Open Pit Design and Scheduling by Use of Genetic Algorithms.* IMM Transactions, 1994, 103, A21-A26.

[14] Tolwinski, B., Underwood, R., *An Algorithm to Estimate the Optimal Evolution of an Open Pit Mine*. 23rd APCOM, 1992, 399-409.

[15] Gershon, M.E., *Heuristic Approaches for Mine Planning and Production Scheduling*. Intentional journal of mining and geological engineering, 1987, 5, 1-13.

[16] Wang, q., Sevim, H., *Enhance Production Planning in Open pit Mining Through Intelligent Dynamic Search*. 23rd APCOM, 1992, pp. 461-471.

[17] Whittle, J., Beyond optimization in open pit design, 1st CAMI, 1988.

[18] Whittle, J., The facts and facilities of open pit design. Whittle programming Pty Ltd, 1989.

[19] Williams, J.C., Wright, B.D., *Storage and Commodity Markets.* Cambridge University Press, Cambridge, England. 1991.

[20] Deaton, A., Laroque, G., *Competitive Storage and Commodity Price Dynamics*. Journal of Political Economy, 1996, CIV, pp. 896 - 923.

[21] Chambers, M.J., Bailey, R.E., *A Theory of Commodity Price Fluctuations*. Journal of Political Economy, 1996, CIV, pp. 924 - 957.

[22] Litzenberger, R.H., Rabinowitz, N., *Backwardation in Oil Futures Markets: Theory and Empirical Evidence. Journal of Finance*, 1995, L, pp. 1517-1545.

[23] Considine, T.J., Larson, D.F., *Uncertainty and the Convenience Yield in Crude Oil Price Backwardations*. Energy Economics, 2001, 23, pp. 533 - 548.

[24] Dias, M.A.G., *Valuation of Exploration and Production Assets: an Overview of Real Options Models*, Journal of Petroleum Science and Engineering, 2004, 44, pp. 93-114.

[25] Costa Lima, Gabriel A., Suslick, Saul, B., *Estimating the Volatility of Mining Projects Considering Price and Operating Cost Uncertainties.* Resources Policy, 2006, 31 (2), pp. 86-94.

[26] Dimitrakopoulos, R., Abdel Sabour, S., *Evaluating mine Plans Under Uncertainty: Can the Real Options Make a Difference? Resources Policy*, 2007, 32(3), pp. 116-125.

[27] Rovenscroft, P.J., *Risk Analysis for Mine Scheduling by Conditional Simulation*. IMM Transactions, 1992, 101, A82-A88.

[28] Denby, B., Schofield, D., *Inclusion of Risk Assessment in Open pit Design and Scheduling*. IMM Transactions, 1995, 104, A67-A71.

[29] Dowd, P.A., *Risk in Mineral Projects: Analysis, Perception and Management*. IMM Transactions, 1997, 106, A9-A18.

[30] Godoy, M., Dimitrakopoulos, R., *Managing Risk and Waste Mining in Long Term Production Scheduling of Open Pit Mine.* SME Transactions, 2004, 316, pp. 43-50.

[31] Akbari, A.D., Osanloo, M., Shirazi, M.A., *Real Option Theory and Some Key Points for Using it in Mining.* 16th MPES, 2007, pp. 1-12.

[32] Shockly, R., *Applied Course in Real Options Valuation*, Thomson Learning, London, UK, 2007.

[33] Groze, R.W., *A changing Economics of Surface Mining: a Case History (Decade of Digital Computing in the Mineral Industry*, ed. A. Weiss), 1969. SME, pp. 401-440.

[34] Akbari, A.D., Osanloo, M., *An Updated an Modified O'Hara Cost Estimating Model Based on World and Iran Economic Condition*. 32nd APCOM, 2005, pp. 3-18.