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RESEARCH PAPER

The Interval Type-2 Fuzzy ELECTRE III Method to Prioritize Machines for Preventive Maintenance

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ABSTRACT
Preventive maintenance (PM) of machines and equipment plays the critical role in a factory or an organization. It can decrease the number of failures, increase reliability, and minimize total costs of production systems. The duty of maintenance section managers is to prioritize machines and then, implement PM programs for them. Since machines have different measures with respect to the maintenance costs, reliability, mean time between failures (MTBF), availability of spare parts, etc., the machines evaluation problem can be considered as a multiple criteria decision-making (MCDM) problem. Accordingly, the MCDM techniques can be applied to solve them. This paper is aimed at extending a version of the ELimination Et Choix Traduisant la REalité (ELECTRE III) method to interval type-2 fuzzy sets (IT2FSs) where curved membership functions (MFs) are applied. The extended ELECTRE III methodology is then utilized to a maintenance group MCDM (GMCDM) matrix including the quantitative (QNC) and qualitative criteria (QLC). In the proposed approach, the criteria weights, the assessment of alternatives with respect to criteria, and the thresholds are stated with Gaussian interval type-2 fuzzy sets (GIT2FSs). In order to show the effectiveness and applicability of the proposed approach, a case study and an illustrative example are exhibited using real decision-making problems. Due to the high correlation coefficients among our method and the others, it can be considered as a valid and reliable approach to prioritize machines for PM.

KEYWORDS: *ELECTRE III; Gaussian interval type-2 fuzzy sets; Alpha cuts; Preventive maintenance.*

1. Introduction

One of the costs imposed on the actual cost of manufactured goods is related to maintenance section. However, the structure's complexity of manufacturing systems and program of maintenance has transformed with a difficult problem. Therefore, after machines are purchased, they should be ranked and maintenance programs are then determined for them. In a usual classification, the maintenance activities can be categorized into two groups, namely corrective and preventive maintenance. Corrective maintenance (CM) is a situation in which system is stopped (due to a failure) and then, sub-systems or sub-components are replaced. In this case, the effective age of substituted sub-components is decreased to zero. On the other hand, preventive maintenance (PM) includes a set of pre-programmed actions for keeping sub-components in ideal conditions such that system is working [1, 2].

There are different machines in a factory. Each of them has distinguished properties than the others such that their devolution of work force, determination of maintenance activities, and budget allocation have transformed with a complex and time-consuming process. These properties are usually classified into two groups of criteria, namely the quantitative criteria (QNC) (such as investment measure, spare parts cost, capacity, etc.) and subjective or qualitative criteria (QLC) (such as attainability of spare parts, convenience, maintainability, etc.). Hence, such a problem may be considered as a multiple criteria decision-making (MCDM) problem in which alternatives are evaluated with respect to a set of QNC or QLC. Depending on nature of

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MCDM problem, ranking method, and ideal solutions, there are different methodologies for assessing the MCDM problems. The ELimination Et Choix Traduisant la REalité (ELECTRE) method was first presented by Roy [3]. Due to its various versions, it is one of the most practical techniques of MCDM. Because of the use of crisp data in the MCDM problem when using the classical ELECTRE method, solving ambiguous decision-making issues has transformed to a challenging process. Since the application of deterministic data to uncertain environments can impose irreparable costs [4], the fuzzy data significantly can help the MCDM techniques. However the evaluations and weights in ELECTRE III are expressed by the fuzzy numbers, a decision-maker (DM) may have doubt related to the measure of membership function (MF). Situations exist in which it is inconceivable to satisfactorily assess MF [5]. Hence, type-2 fuzzy sets were extended by Zadeh [6] for considering MF in the interval form. Since type-2 fuzzy sets are depicted in a three-dimensional space, type-2 fuzzy sets decrease uncertainty better than type-1 version. In fact, this is more rational to consider an interval type-2 fuzzy number (IT2FN) instead of the use of several type-1 fuzzy numbers with different MFs. There are different MFs for interval type-2 fuzzy sets (IT2FSs) like Gaussian interval type-2 fuzzy sets (GIT2FSs), trapezoidal interval type-2 fuzzy sets (TraIT2FSs), etc.

Generally, the main contributions of the present paper include:

- Since machines ranking problem is a group decision-making, the synthetic value method is used to integrate the interval type-2 fuzzy ratings of alternatives with respect to criteria and interval type-2 fuzzy weights of criteria.
- In addition to QNC, several QLC are merged with the ELECTRE III method based on IT2FSs.
- The aggregated weighted ratings-based the interval type-2 fuzzy ELECTRE III method is adopted to rank different machines.
- The new proposed methodology is usable to be applied to each type of MF (both straight and curve lines). Moreover, it can be utilized to rank other versions of fuzzy sets.

In summary, the principal goal of this paper is to present a group decision-making manner based on IT2FSs to rank machines for PM. According to the above arguments, the use of type-2 fuzzy data is more logical and suitable.

The rest of this paper is organized as follows: Section 2 presents the literature review regarding the MCDM techniques, ELECTRE III, and optimization models of machines. In Section 3, the arithmetic operations of GIT2FSs are reviewed. The proposed approach for ranking GIT2FNs is introduced in Section 4. In Section 5, our proposed methodology is integrated with the ELECTRE III approach. Section 6 includes the real case study and illustrative example in which our ranking methodology is integrated with the ELECTRE III approach and finally, conclusions are summarized in Section 7.

2. Literature Review

A MCDM problem can be expressed as the methodology of specifying the best option among all options with respect to criteria. Among the MCDM techniques, the ELECTRE III approach (alone or in integration with the others) has been applied to different decision-making scopes (due to property of its non-compensatory and the use of threshold). For example, Shafia et al. [7] prioritized scenarios according to fuzzy cognitive map by ELECTRE III in which scenarios help simulate future events. Noori et al. [8] evaluated water supply scenarios based on the fuzzy Delphi and ELECTRE III methods. Rodriguez et al. [9] utilized the simple additive weighting (SAW), ELECTRE, and vlsekriterijumska optimizacija i kompromisno resenje (VIKOR) methods to select the most adequate artificial lift system for crude oil production. Fei et al. [10] applied the ELECTRE method for supplier selection where the evaluation of information was expressed by the Dempster–Shafer theory. Bathrinath et al. [11] applied ELECTRE I to proritize the printing industries regarding risk factors. Akmaludin et al. [12] adopted the AHP and ELECTRE methods for choosing flight attendants.

Since the goal of present paper is to adopt the ELECTRE approach for the PM problems, it is necessary to study some applications of the maintenance problems or formulations in the literature. There are the different arrangements for machines including single machine, parallel machines, flow shop, job shop, open shop, and hybrid systems. The authors usually select one of them and it is then solve regarding one or multi-criteria such as cost, completion time, tardiness, due date, etc., where optimization programs or heuristic expressions (rules) are used. One of the most popular evaluation methods of machines or arrangements is to present single-objective or multi-objective optimization model (by considering the different assumptions) and it then is solved using the mathematics or metaheuristic methods. Moghaddam [13] presented the multi-objective and multi-objective nonlinear mixed-integer optimization models, in order to determine the optimal PM and replacement schedules in a repairable and maintainable multi-component system, respectively. Luan et al. [14] introduced a new mixed-integer linear programming (MILP) model that concurrently formulated and optimizes the train routes, orders, and passing times at each station. Vilarinho et al. [15] incorporated a new function into the computerized maintenance management to aid failure analysis and optimal periodicity definition of preventive interpositions. Khatab et al. [16] modeled a optimization formulation for the simultaneously determination of the optimal acquisition age, upgrade level, and imperfect PM strategy. Zhong et al. [17] modeled a fuzzy multi-objective nonlinear chance-constrained programming model for scheduling wind turbine maintenance. Navarro et al. [18] presented a maintenance programming model based on reliability regarding life cycle costs and environmental effects. Salmasnia et al. [19] extended a bi-objective optimization model which concurrently minimizes the manufacturer and buyer cost under a non-homogeneous Poisson process framework. Li and Zhang [20] introduced a multi-objective optimization model to determine the optimal PM interval. They then applied the Monte Carlo technique to check the behavior of the cluster system based on various PM intervals.

In spite of the fact that the different QLC may affect assessment process of machines, none of the above researches considered them. Unfortunately, they did not apply the fuzzy sets theory and MCDM techniques to evaluate and rank equipment or machines. There are few papers regarding ranking the machines for PM in the literature. However, these techniques have been handled in other applications of PM problems.

To best of our knowledge, there is only one paper for ranking machines for PM. Khanlari et al. [21] prioritized equipment for PM works by fuzzy rules base. They constructed a fuzzy rule base for exploiting forty eight rules by data of six machines. In addition, this base has been applied to prioritize new three machines. Despite the novelty of the proposed method, it has challenged some situations as follows. For example, equipment 3 and 4 can be considered for comparisons. Although machine 4 has higher measures with respect to the five criteria, it has lower priority than machine 3. Moreover, the equal weights have been taken account for criteria in the data matrix. On the other hand, the linguistic variables of type-1 fuzzy sets instead of type-2 fuzzy sets have been used for evaluations that have less degree of freedom. Accordingly, in order to relieve all the limitations mentioned above, the current paper proposes a new generalized MCDM technique based on GIT2FSs to rank machines with respect to the QLC and QNC.

3. Type-2 Fuzzy Sets and Their Arithmetic Operations

Since the ELECTRE III approach should be extended to type-2 fuzzy sets [22], footprint of uncertainty (FOU) [23], normal GIT2FN [24], symmetric GIT2FN represented by Fig. 1 [25], alpha cut of type-2 fuzzy sets [26]. The arithmetic operations of type-2 fuzzy sets [27, 28] should be studied.

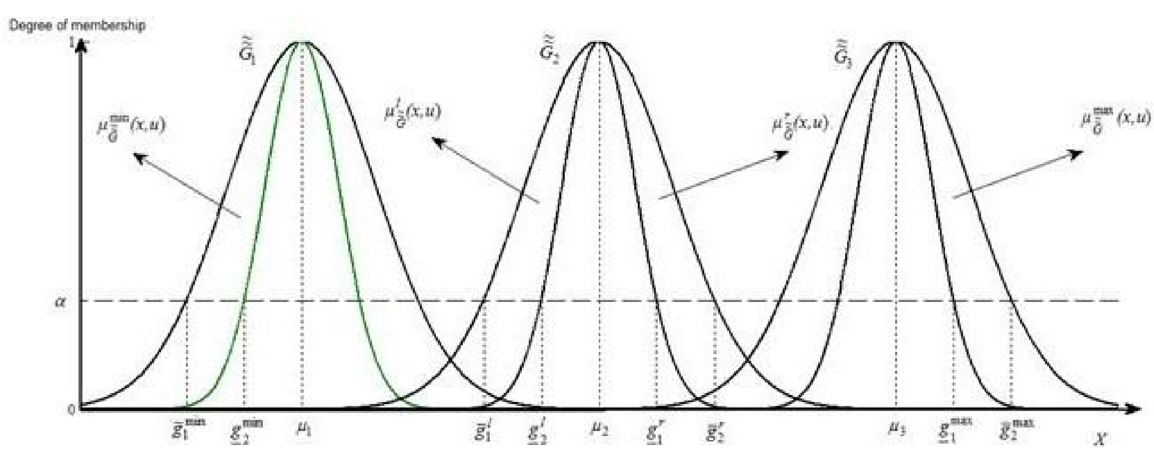

Fig. 1. The different reference limits (RL) of symmetric GIT2FNs.

Definition 3.1. Suppose that $\tilde{\tilde{A}}_1 = [\tilde{A}_1^L, \tilde{A}_1^U] = \left[\left(a_{1h}^L; H_{\tilde{A}_1}^L\right), \left(a_{1h}^U; H_{\tilde{A}_1}^U\right), h = 1,2,3,4\right]$ and $\tilde{\tilde{A}}_2 = [\tilde{A}_2^L, \tilde{A}_2^U] = \left[\left(a_{2h}^L; H_{\tilde{A}_2}^L\right), \left(a_{2h}^U; H_{\tilde{A}_2}^U\right), h = 1,2,3,4\right]$ be two non-negative trapezoidal interval type-2 fuzzy numbers (TraIT2FNs). The interested reader can refer to Chen [27] and Chen [28] in order to study arithmetic operations between them.

Definition 3.2. Let $\tilde{\tilde{G}}_1 = [\tilde{G}_1^L, \tilde{G}_1^U] = \left[\left(\mu_1^L; \sigma_1^L; H_{\tilde{G}_1}^L\right), \left(\mu_1^U; \sigma_1^U; H_{\tilde{G}_1}^U\right)\right]$ and $\tilde{\tilde{G}}_2 = [\tilde{\tilde{G}}_2^L, \tilde{\tilde{G}}_2^U] = \left[\left(\mu_2^L; \sigma_2^L; H_{\tilde{G}_2}^L\right), \left(\mu_2^U; \sigma_2^U; H_{\tilde{G}_2}^U\right)\right]$ be two non-negative normal GIT2FNs. In addition, let $H_{\tilde{G}_1}^L = H_{\tilde{G}_2}^L = H_{\tilde{G}_1}^U = H_{\tilde{G}_2}^U$, in which L and U are the lower and upper MFs, respectively. Also, assume that alpha cut of a normal GIT2FN is showed as follows:

$$\hat{G}_{t\alpha} = \left[\left[\overline{g}_{1t\alpha}^{l}, \underline{g}_{2t\alpha}^{l}\right], \mu_t, \left[\underline{g}_{1t\alpha}^{r}, \overline{g}_{2t\alpha}^{r}\right]\right], \tag{1}$$

where l and r represent the left and right MFs of $\tilde{\tilde{G}}$, respectively. Also, $[\overline{g}_1^l, \underline{g}_2^l]_\alpha$ and $[\underline{g}_1^r, \overline{g}_2^r]_\alpha$ are intervals produced from junction of level α with $\mu_{\tilde{\tilde{G}}}^l(x,u)$ (the left MF of $\tilde{\tilde{G}}$) and $\mu_{\tilde{\tilde{G}}}^r(x,u)$ (the right MF of $\tilde{\tilde{G}}$), respectively, as shown in Fig. 1.

Accordingly, the arithmetic operations between GIT2FNs based on representation of alpha cut, for each $\alpha = \alpha_1, \ldots, \alpha_N$ (N is the number of alpha cuts), are calculated as follows:

$$\begin{aligned}\hat{G}_{1\alpha} \oplus \hat{G}_{2\alpha} = &\left[\left(\underline{g}_{21\alpha}^{l} + \underline{g}_{22\alpha}^{l}, \mu_1 + \mu_2, \underline{g}_{11\alpha}^{r} + \underline{g}_{12\alpha}^{r}; \min\left\{H_{\tilde{G}_1}^L, H_{\tilde{G}_2}^L\right\}\right)\right], \\ &\left[\left(\overline{g}_{11\alpha}^{l} + \overline{g}_{12\alpha}^{l}, \mu_1 + \mu_2, \overline{g}_{21\alpha}^{r} + \overline{g}_{22\alpha}^{r}; \min\left\{H_{\tilde{G}_1}^U, H_{\tilde{G}_2}^U\right\}\right)\right].\end{aligned} \tag{2}$$

$$\begin{aligned}\hat{G}_{1\alpha} \ominus \hat{G}_{2\alpha} = &\left[\left(\underline{g}_{21\alpha}^{l} - \underline{g}_{12\alpha}^{r}, \mu_1 - \mu_2, \underline{g}_{11\alpha}^{r} - \underline{g}_{22\alpha}^{l}; \min\left\{H_{\tilde{G}_1}^L, H_{\tilde{G}_2}^L\right\}\right)\right], \\ &\left[\left(\overline{g}_{11\alpha}^{l} - \overline{g}_{22\alpha}^{r}, \mu_1 - \mu_2, \overline{g}_{21\alpha}^{r} - \overline{g}_{12\alpha}^{l}; \min\left\{H_{\tilde{G}_1}^U, H_{\tilde{G}_2}^U\right\}\right)\right]\end{aligned} \tag{3}$$

$$\begin{aligned}\hat{G}_{1\alpha} \otimes \hat{G}_{2\alpha} = &\left[\left(\underline{g}_{21\alpha}^{l} \times \underline{g}_{22\alpha}^{l}, \mu_1 \times \mu_2, \underline{g}_{11\alpha}^{r} \times \underline{g}_{12\alpha}^{r}; \min\left\{H_{\tilde{G}_1}^L, H_{\tilde{G}_2}^L\right\}\right)\right] \\ &\left[\left(\overline{g}_{11\alpha}^{l} \times \overline{g}_{12\alpha}^{l}, \mu_1 \times \mu_2, \overline{g}_{21\alpha}^{r} \times \overline{g}_{22\alpha}^{r}; \min\left\{H_{\tilde{G}_1}^U, H_{\tilde{G}_2}^U\right\}\right)\right]\end{aligned} \tag{4}$$

$$\begin{aligned}\hat{G}_{1\alpha} \oslash \hat{G}_{2\alpha} = &\left[\left(\frac{\underline{g}_{21\alpha}^{l}}{\underline{g}_{12\alpha}^{r}}, \frac{\mu_1}{\mu_2}, \frac{\underline{g}_{11\alpha}^{r}}{\underline{g}_{22\alpha}^{l}}; \min\left\{H_{\tilde{G}_1}^L, H_{\tilde{G}_2}^L\right\}\right)\right], \\ &\left[\left(\frac{\overline{g}_{11\alpha}^{l}}{\overline{g}_{22\alpha}^{r}}, \frac{\mu_1}{\mu_2}, \frac{\overline{g}_{21\alpha}^{r}}{\overline{g}_{12\alpha}^{l}}; \min\left\{H_{\tilde{G}_1}^U, H_{\tilde{G}_2}^U\right\}\right)\right].\end{aligned} \tag{5}$$

Definition 3.3. Let there are U non-negative normal GIT2FNs, $\tilde{\tilde{G}}_u = [\tilde{G}_u^L, \tilde{G}_u^U] = \left[\left(\mu_u^L; \sigma_u^L; H_{\tilde{G}_u}^L\right), \left(\mu_u^U; \sigma_u^U; H_{\tilde{G}_u}^U\right)\right]$, such that $H_{\tilde{G}_u}^L = H_{\tilde{G}_u}^U$ $(u = 1, \ldots, U)$. According to Eq. (1), their mean operations, namely $\overline{\hat{G}}_\alpha = (\overline{\overline{g}}_{1\alpha}^{l}, \overline{\underline{g}}_{2\alpha}^{l}, \overline{\mu}_\alpha, \overline{\underline{g}}_{1\alpha}^{r}, \overline{\overline{g}}_{2\alpha}^{r})$ (at each $\alpha = \alpha_1, \ldots, \alpha_N$) are obtained by the following relations:

$$\overline{\overline{g}}_{1\alpha}^{l} = \left(\sum_{u=1}^{U} \overline{g}_{1u\alpha}^{l}\right) \Big/ U, \tag{6}$$

$$\overline{\underline{g}}_{2\alpha}^{l} = \left(\sum_{u=1}^{U} \underline{g}_{2u\alpha}^{l}\right) \Big/ U, \tag{7}$$

$$\overline{\mu}_\alpha = \left(\sum_{u=1}^{U} \mu_{u\alpha}\right) \Big/ U, \tag{8}$$

$$\overline{\underline{g}}_{1\alpha}^{r} = \left(\sum_{u=1}^{U} \underline{g}_{1u\alpha}^{r}\right) \Big/ U, \tag{9}$$

$$\overline{\overline{g}}_{2\alpha}^{r} = \left(\sum_{u=1}^{U} \overline{g}_{2u\alpha}^{r}\right) \Big/ U, \tag{10}$$

where $\left[\overline{\overline{g}}^{l}_{1ij\alpha}, \underline{g}^{l}_{2ij\alpha}\right]$ and $\left[\underline{g}^{r}_{1ij\alpha}, \overline{\overline{g}}^{r}_{2ij\alpha}\right]$ are the mean of intervals produced from junction of level α with $\mu^{l}_{\tilde{\tilde{G}}}(x,u)$ and $\mu^{r}_{\tilde{\tilde{G}}}(x,u)$, respectively, as shown in Fig .1.

In this section, some arithmetic operations (including addition, subtraction, multiplication, division, and mean) of GIT2FNs were presented.

4. Limit Distance Mean (*LDM*) for Ranking GIT2FNs

In this section, the arithmetic operations of IT2FSs presented in the previous section are used to make the proposed ranking method. There are the different studies for determining the distances in the literature. Ashtiani et al. [29] and Mokhtarian et al. [30] introduced the relations for calculating the distances between rating measures and ideal solutions ($A^{+}=(1,...,1)$ and $A^{-}=(0,...,0)$). These solutions might not be attainable in the MCDM matrix. In addition, they used only the reference points (RPs) in the above formulas. Chen and Lee [31] utilized TraIT2FSs to a MCDM matrix and then, they applied the heuristic expressions mean and standard deviation in order to prioritize the ratings. Rashid et al. [32] and Yang et al. [33] developed the generalized interval-valued fuzzy the technique for order of preference by similarity to ideal solution (TOPSIS) method. In its approach, the RPs were only considered for specifying the positive ideal (*PI*) and negative ideal (*NI*) solutions.

Suppose that $\mu_{\tilde{\tilde{G}}}(x,u)$ be divided into two distinctive MFs, namely $\mu^{l}_{\tilde{\tilde{G}}}(x,u)$ and $\mu^{r}_{\tilde{\tilde{G}}}(x,u)$. According to Fig. 1, let minimum and maximum RLs are $\mu^{\min}_{\tilde{\tilde{G}}}(x,u)$ and $\mu^{\max}_{\tilde{\tilde{G}}}(x,u)$, respectively. On the other hand, assume that $[\overline{g}^{\min}_{1}, \underline{g}^{\min}_{2}]_{\alpha}$, $[\underline{g}^{\max}_{1}, \overline{g}^{\max}_{2}]_{\alpha}$, $[\overline{g}^{l}_{1}, \underline{g}^{l}_{2}]_{\alpha}$, and $[\underline{g}^{r}_{1}, \overline{g}^{r}_{2}]_{\alpha}$ be intervals created from junction of level α with the $\mu^{\min}_{\tilde{\tilde{G}}}(x,u)$, $\mu^{\max}_{\tilde{\tilde{G}}}(x,u)$, $\mu^{l}_{\tilde{\tilde{G}}}(x,u)$, and $\mu^{r}_{\tilde{\tilde{G}}}(x,u)$, respectively. The interested researcher can refer to Mohamadghasemi et al. [25] for checking more explanations as compared to the proposed method and its properties. Hence, *LDM*s can be computed related to the *PI* and *NI* solutions for cost (*CC*) and benefit (*BC*) criteria using the following relations:

$$LDM_{PI,CC}(\tilde{\tilde{A}})=\frac{\left(\sum_{\alpha=0.1}^{1}\left|(\overline{g}^{l}_{1}-\underline{g}^{\min}_{2})_{\alpha}+(\underline{g}^{l}_{2}-\overline{g}^{\min}_{1})_{\alpha}\right|\right)}{\left(\sum_{\alpha=0.1}^{1}\left|(\overline{g}^{l}_{1}-\underline{g}^{\min}_{2})_{\alpha}+(\underline{g}^{l}_{2}-\overline{g}^{\min}_{1})_{\alpha}\right|\right)+\left(\sum_{\alpha=0.1}^{1}\left|(\underline{g}^{r}_{1}-\overline{g}^{\max}_{2})_{\alpha}+(\overline{g}^{r}_{2}-\underline{g}^{\max}_{1})_{\alpha}\right|\right)}, \tag{11}$$

$$LDM_{PI,BC}(\tilde{\tilde{A}})=\frac{\left(\sum_{\alpha=0.1}^{1}\left|(\underline{g}^{r}_{1}-\overline{g}^{\max}_{2})_{\alpha}+(\overline{g}^{r}_{2}-\underline{g}^{\max}_{1})_{\alpha}\right|\right)}{\left(\sum_{\alpha=0.1}^{1}\left|(\underline{g}^{r}_{1}-\overline{g}_{\max 2})_{\alpha}+(\overline{g}^{r}_{2}-\underline{g}^{\max}_{1})_{\alpha}\right|\right)+\left(\sum_{\alpha=0.1}^{1}\left|(\overline{g}^{l}_{1}-\underline{g}^{\min}_{2})_{\alpha}+(\underline{g}^{l}_{2}-\overline{g}^{\min}_{1})_{\alpha}\right|\right)}, \tag{12}$$

$$LDM_{NI,CC}(\tilde{\tilde{A}})=\frac{\left(\sum_{\alpha=0.1}^{1}\left|(\overline{g}^{l}_{1}-\overline{g}^{\max}_{2})_{\alpha}+(\underline{g}^{l}_{2}-\underline{g}^{\max}_{1})_{\alpha}\right|\right)}{\left(\sum_{\alpha=0.1}^{1}\left|(\overline{g}^{l}_{1}-\overline{g}^{\max}_{2})_{\alpha}+(\underline{g}^{l}_{2}-\underline{g}^{\max}_{1})_{\alpha}\right|\right)+\left(\sum_{\alpha=0.1}^{1}\left|(\underline{g}^{r}_{1}-\underline{g}^{\min}_{2})_{\alpha}+(\overline{g}^{r}_{2}-\overline{g}^{\min}_{1})_{\alpha}\right|\right)}, \tag{13}$$

$$LDM_{NI,BC}(\tilde{\tilde{A}})=\frac{\left(\sum_{\alpha=0.1}^{1}\left|(\underline{g}^{r}_{1}-\underline{g}^{\min}_{2})_{\alpha}+(\overline{g}^{r}_{2}-\overline{g}^{\min}_{1})_{\alpha}\right|\right)}{\left(\sum_{\alpha=0.1}^{1}\left|(\underline{g}^{r}_{1}-\underline{g}^{\min}_{2})_{\alpha}+(\overline{g}^{r}_{2}-\overline{g}^{\min}_{1})_{\alpha}\right|\right)+\left(\sum_{\alpha=0.1}^{1}\left|(\overline{g}^{l}_{1}-\overline{g}^{\max}_{2})_{\alpha}+(\underline{g}^{l}_{2}-\underline{g}^{\max}_{1})_{\alpha}\right|\right)}. \tag{14}$$

Unfortunately, none of the studies described above have considered the curved or stochastic MFs for IT2FSs. The proposed *LDM*s relieve all the above limitations. The authors have presented the above four expressions to determine the *PI* and *NI* solutions with respect to *CC* and *BC*. Moreover, Eq. (14) can be used to weight criteria.

5. The Application of New Ranking Method to ELECTRE III with GIT2FNs

The following stages show the proposed approach for symmetric normal GIT2FNs:

Step 1: Let there is the MCDM matrix $(X,\tilde{\tilde{G}})=[x_{rj},\tilde{\tilde{g}}_{rj}]_{R\times J}$ based on which DMs intends to assess R machines $M_{r}\,(r=1,...,R)$ with respect to

J criteria $C_j\,(j=1,\dots,m',m'+1,\dots,J)$, where $X=[x_{rj}]_{R\times(1,\dots,m')}$, $\tilde{\tilde{G}}=[\tilde{\tilde{g}}_{ij}]_{R\times(m'+1,\dots,J)}$, $C_j\,(j=1,\dots,m')$, and $\tilde{\tilde{C}}_j\,(j=m'+1,\dots,J)$ introduce the crisp values with respect to QNC, GIT2FNs with respect to QLC, the QNC, and the QLC, respectively. With these assumptions, create the Gaussian interval type 2 fuzzy MCDM (GIT2FMCDM) matrix as follows:

$$
\begin{array}{cc}
 & \begin{array}{ccccccccc} \overset{w_1}{C_1} & \overset{w_2}{C_2} & \overset{w_3}{C_3} & \cdots & \overset{w_{m'}}{C_{m'}} & \overset{w_{n'+1}}{C_{m'+1}} & \overset{w_{n'+2}}{C_{m'+2}} & \cdots & \overset{w_J}{CJ} \end{array} \\
X,\ \tilde{\tilde{G}} = \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ \vdots \\ A_R \end{array} & \left[\begin{array}{ccccccccc}
x_{11} & x_{12} & x_{13} & \cdots & x_{1m'} & \tilde{\tilde{g}}_{1m'+11}\cdots\tilde{\tilde{g}}_{1m'+1l} & \tilde{\tilde{g}}_{1m'+21}\cdots\tilde{\tilde{g}}_{1m'+2l} & \cdots & \tilde{\tilde{g}}_{1J1}\cdots\tilde{\tilde{g}}_{1Jl} \\
x_{21} & x_{22} & x_{23} & \cdots & x_{2m'} & \tilde{\tilde{g}}_{2m'+11}\cdots\tilde{\tilde{g}}_{2m'+1l} & \tilde{\tilde{g}}_{2m'+21}\cdots\tilde{\tilde{g}}_{2m'+2l} & \cdots & \tilde{\tilde{g}}_{2J1}\cdots\tilde{\tilde{g}}_{2Jl} \\
x_{31} & x_{32} & x_{33} & \cdots & x_{3m'} & \tilde{\tilde{g}}_{3m'+11}\cdots\tilde{\tilde{g}}_{2m'+1l} & \tilde{\tilde{g}}^{1}_{3m'+21}\cdots\tilde{\tilde{g}}^{l}_{3m'+2l} & \cdots & \tilde{\tilde{g}}_{3J1}\cdots\tilde{\tilde{g}}_{3Jl} \\
\vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \\
x_{R1} & x_{R2} & x_{R3} & \cdots & x_{Rm'} & \tilde{\tilde{g}}_{Rm'+11}\cdots\tilde{\tilde{g}}_{Rm'+1l} & \tilde{\tilde{g}}_{Rm'+21}\cdots\tilde{\tilde{g}}_{Rm'+2l} & \cdots & \tilde{\tilde{g}}_{RJ1}\cdots\tilde{\tilde{g}}_{RJl}
\end{array}\right],
\end{array}
\quad (15)
$$

where $\tilde{\tilde{g}}_{ijl}$ is GIT2FN [24] assigned by nth DM who assesses machine r $(r=1,\dots,R)$ with respect to criterion $C_j\,(j=m'+1,\dots,J)$.

Step 2: Introduce the verbal variables. The first type is applied to assess machines with respect to QLC and the second one is used to weight criteria.

Step 3: Integrate the type-2 fuzzy ratings $\tilde{\tilde{g}}_{rjl}$ ($l=1,\dots,L$, $r=1,\dots,R$, and $j=m'+1,\dots,J$) with the integrated type-2 fuzzy ratings $\tilde{\tilde{g}}_{ij}$ by equation:

$$\tilde{\tilde{g}}_{rj} = (1/L)\otimes(\tilde{\tilde{g}}_{rj1}\oplus\tilde{\tilde{g}}_{irj2}\oplus\dots\oplus\tilde{\tilde{g}}_{irjL}) \qquad r=1,\dots,R;\ \ j=m'+1,\dots,J. \tag{16}$$

In addition, let $\tilde{\tilde{w}}_{jl}$ be GIT2FN assigned by nth DM for weighting criteria. In a similar way, the type-2 fuzzy weights, $\tilde{\tilde{w}}_{jl}$ ($l=1,\dots,L$), for each criterion are merged with the integrated type-2 fuzzy weight $\tilde{\tilde{w}}_j$ as follows:

$$\tilde{\tilde{w}}_j = (1/L)\otimes(\tilde{\tilde{w}}_{j1}\oplus\tilde{\tilde{w}}_{j2}\oplus\dots\oplus\tilde{\tilde{w}}_{jL}) \qquad j=m'+1,\dots,J. \tag{17}$$

GIT2FNs at level α, $\tilde{\tilde{g}}_{rj\alpha}$, can be presented as the following expression [25]:

$$\hat{g}_{rj\alpha} = \left\{\left[\overline{g}^{\,l}_{1rjl\alpha},\underline{g}^{\,l}_{2rjl\alpha}\right],\mu_{rjl_\alpha},\left[\underline{g}^{\,r}_{1rjl\alpha},\overline{g}^{\,r}_{2rjl\alpha}\right]\right\} \qquad r=1,\dots,R;\ \ j=m'+1,\dots,J. \tag{18}$$

where μ_{rj_α} is the mean of GIT2FN while machine r $(r=1,\dots,R)$ is evaluated under C_j ($j=m'+1,\dots,J$). Similarly, GIT2FN chosen by nth DM at level α, $\tilde{\tilde{g}}_{rjl\alpha}$, can be given by:

$$\hat{g}_{rjl\alpha} = \left\{\left[\overline{g}^{\,l}_{1rjl\alpha},\underline{g}^{\,l}_{2rjl\alpha}\right],\mu_{rjl_\alpha},\left[\underline{g}^{\,r}_{1rjl\alpha},\overline{g}^{\,r}_{2rjl\alpha}\right]\right\} \qquad r=1,\dots,R;\ \ j=m'+1,\dots,J,;l=1,\dots,L. \tag{19}$$

The mean of RPs of GIT2FNs chosen by N DMs at each level α, $\overline{\hat{g}}_{rj\alpha} = \left\{\left[\overline{\overline{g}}^{\,l}_{1rj\alpha},\underline{g}^{\,l}_{2rj\alpha}\right],\mu_{rj_\alpha},\left[\underline{g}^{\,r}_{1rj\alpha},\overline{\overline{g}}^{\,r}_{2rj\alpha}\right]\right\}$ is calculated by the extension of Eqs. (6) to (10) (for $r=1,\dots,R;\ \ j=m'+1,\dots,J$) where $\left[\overline{g}^{\,l}_{1rj\alpha},\underline{g}^{\,l}_{2rj\alpha}\right]$ and $\left[\underline{g}^{\,r}_{1rj\alpha},\overline{\overline{g}}^{\,r}_{2rj\alpha}\right]$ are the mean of intervals created from junction of level α with left and right MFs of $\tilde{\tilde{G}}$, respectively.

In a similar way, the mean of RPs for weights of QLC selected by N DMs at each level α, $\overline{\hat{w}}_{j\alpha} = \left\{\left[\overline{\overline{w}}^{\,l}_{1j\alpha},\underline{w}^{\,l}_{2j\alpha}\right],\mu_{j_\alpha},\left[\underline{w}^{\,r}_{1j\alpha},\overline{\overline{w}}^{\,r}_{2j\alpha}\right]\right\}$, is calculated by the extension of Eqs. (6) to (10) ($j = m'+1,...,J$) in which $\left[\overline{\overline{w}}^{l}_{1j\alpha}, \underline{w}^{l}_{2j\alpha}\right]$ and $\left[\underline{\overline{w}}^{r}_{1j\alpha}, \overline{\overline{w}}^{r}_{2j\alpha}\right]$ are the mean of interval weights produced from junction of level α with left and right MFs of $\tilde{\tilde{G}}$, respectively.

Step 4: Determine thresholds q_j, p_j, and v_j.

The maximum and the minimum measures of ratings under the QNC are calculated by expressions:

$$A_j^{+} = \{\max_i (x_{ij})\} \qquad j = 1,..., n', \qquad (20)$$

$$A_j^{-} = \{\max_i (x_{ij})\} \qquad j = 1,..., n'. \qquad (21)$$

Accordingly, the threshold measures under the QNC are given by:

$$q_j = (A_j^{+} - A_j^{-}) * \beta_{1j} \qquad j = 1,..., n', \qquad (22)$$

$$p_j = (A_j^{+} - A_j^{-}) * \beta_{2j} \qquad j = 1,..., n', \qquad (23)$$

$$v_j = (A_j^{+} - A_j^{-}) * \beta_{3j} \qquad j = 1,..., n', \qquad (24)$$

where values β_{1j}, β_{2j}, and β_{3j} with respect to quantitative criterion j are specified by DMs.

Moreover, $\tilde{\tilde{q}}_j$, $\tilde{\tilde{p}}_j$, and $\tilde{\tilde{v}}_j$ with respect to QLC are chosen based on GIT2FNs.

Step 5: Obtain the concordance index of machine A_i versus machine A_t by the following equation:

$$\hat{C}(A_i, A_t) = \frac{\sum_{j=1}^{m'} LDM_W(\tilde{\tilde{w}}_j) * c_j(A_i, A_t) + \sum_{j=m'+1}^{J} LDM_W(\tilde{\tilde{w}}_j) * \hat{c}_j(A_i, A_t)}{\sum_{j=1}^{J} LDM_W(\tilde{\tilde{w}}_j)} \qquad i,t = 1,..., R;\ i \neq t, \qquad (25)$$

where $LDM_W(\tilde{\tilde{w}}_j)$, $c_j(A_i, A_t)$, and $\hat{c}_j(A_i, A_t)$ are weight of criterion j by using Eq. (14), the preferable measure of machine A_i versus machine A_t with respect to QNC, and the preferable measure of machine A_i versus machine A_t with respect to QLC, respectively. $\hat{c}_j(A_r, A_k)$ is computed with respect to the benefit and cost QLC, respectively, by using the following relations:

$$\hat{c}_j(A_r, A_k) = \begin{cases} 0 & LDM'_{PI,B}(\tilde{\tilde{g}}_{kj}) - LDM'_{PI,B}(\tilde{\tilde{g}}_{rj}) \geq LDM'_{PI,B}(\tilde{\tilde{p}}_j), \\ \dfrac{LDM'_{PI,B}(\tilde{\tilde{g}}_{rj}) - LDM'_{PI,B}(\tilde{\tilde{g}}_{kj}) + LDM'_{PI,B}(\tilde{\tilde{p}}_j)}{LDM'_{PI,B}(\tilde{\tilde{p}}_j) - LDM'_{PI,B}(\tilde{\tilde{q}}_j)} & otherwise, \\ 1 & LDM'_{PI,B}(\tilde{\tilde{g}}_{kj}) - LDM'_{PI,B}(\tilde{\tilde{g}}_{rj}) \leq LDM'_{PI,B}(\tilde{\tilde{q}}_j), \end{cases} \qquad (26)$$

and

$$\hat{c}_j(A_r, A_k) = \begin{cases} 0 & LDM_{PI,C}(\tilde{\tilde{g}}_{rj}) - LDM_{PI,C}(\tilde{\tilde{g}}_{kj}) \geq LDM_{PI,C}(\tilde{\tilde{p}}_j), \\ \dfrac{LDM_{PI,C}(\tilde{\tilde{g}}_{kj}) - LDM_{PI,C}(\tilde{\tilde{g}}_{rj}) + LDM_{PI,C}(\tilde{\tilde{p}}_j)}{LDM_{PI,C}(\tilde{\tilde{p}}_j) - LDM_{PI,C}(\tilde{\tilde{q}}_j)} & otherwise, \\ 1 & LDM_{PI,C}(\tilde{\tilde{g}}_{rj}) - LDM_{PI,C}(\tilde{\tilde{g}}_{kj}) \leq LDM_{PI,C}(\tilde{\tilde{q}}_j), \end{cases} \qquad (27)$$

where $LDM'_{PI,B}(\tilde{\tilde{g}}) = 1 - LDM_{PI,B}(\tilde{\tilde{g}})$.

Step 6: Determine the discordance index ($\hat{d}_j(A_r, A_k)$) of machine A_r versus machine A_k related to benefit and cost QLC ($j = n'+1,...,n$) by:

$$\hat{d}_j(A_r,A_k)=\begin{cases}0 & LDM'_{PI,B}(\tilde{\tilde{g}}_{kj})-LDM'_{PI,B}(\tilde{\tilde{g}}_{rj})\le LDM'_{PI,B}(\tilde{\tilde{p}}_j),\\ \dfrac{LDM'_{PI,B}(\tilde{\tilde{g}}_{kj})-LDM'_{PI,B}(\tilde{\tilde{g}}_{rj})-LDM'_{PI,B}(\tilde{\tilde{p}}_j)}{LDM'_{PI,B}(\tilde{\tilde{v}}_j)-LDM'_{PI,B}(\tilde{\tilde{p}}_j)} & otherwise,\\ 1 & LDM'_{PI,B}(\tilde{\tilde{g}}_{kj})-LDM'_{PI,B}(\tilde{\tilde{g}}_{rj})\ge LDM'_{PI,B}(\tilde{\tilde{v}}_j),\end{cases} \tag{28}$$

and

$$\hat{d}_j(A_r,A_k)=\begin{cases}0 & LDM_{PI,C}(\tilde{\tilde{g}}_{rj})-LDM_{PI,C}(\tilde{\tilde{g}}_{kj})\le LDM_{PI,C}(\tilde{\tilde{p}}_j),\\ \dfrac{LDM_{PI,C}(\tilde{\tilde{g}}_{rj})-LDM_{PI,C}(\tilde{\tilde{g}}_{kj})-LDM_{PI,C}(\tilde{\tilde{p}}_j)}{LDM_{PI,C}(\tilde{\tilde{v}}_j)-LDM_{PI,C}(\tilde{\tilde{p}}_j)} & otherwise,\\ 1 & LDM_{PI,C}(\tilde{\tilde{g}}_{rj})-LDM_{PI,C}(\tilde{\tilde{g}}_{kj})\ge LDM_{PI,C}(\tilde{\tilde{v}}_j).\end{cases} \tag{29}$$

Step 7: Determine credit degree index of machine A_r versus machine A_k ($\hat{S}(A_r,A_k)$) under QNC ($j=1,\ldots,n'$) and QLC ($j=n'+1,\ldots,n$) by the following relation:

$$\hat{S}(A_r,A_k)=\begin{cases}\hat{C}(A_r,A_k) & if\ \hat{d}_j(A_r,A_k)\le\hat{C}(A_r,A_k)\quad \forall j,\\ \hat{C}(A_r,A_k)\displaystyle\prod_{j\in J:\hat{d}_j(A_r,A_k)>\hat{C}(A_r,A_k)}\frac{1-\hat{d}_j(A_r,A_k)}{1-\hat{C}(A_r,A_k)} & otherwise,\end{cases} \tag{30}$$

where $\hat{S}(A_r,A_k)$ and J are the degree of outranking machine A_r versus machine A_k and the set of criteria for which $\hat{d}_j(A_r,A_k)>\hat{C}(A_r,A_k)$, respectively.

Step 8: Calculate the final scores and rank machines:

In present paper, the net credibility value method [34] is applied to determine the final scores. Thus, the concordance credibility and the discordance credibility value should first be computed as follows:

I. The concordance credibility value is calculated by the following expression:

$$\theta_r^c=\sum_{k=1,k\neq r}^{m}\hat{S}(A_r,A_k)\qquad r=1,\ldots,m. \tag{31}$$

The discordance credibility value is calculated by the following relation:

$$\theta_r^d=\sum_{k=1,k\neq r}^{m}\hat{S}(A_k,A_r)\qquad r=1,\ldots,m. \tag{32}$$

The net credibility value is calculated as:

$$\theta_r=\theta_r^c-\theta_r^d\qquad r=1,\ldots,m. \tag{33}$$

The following algorithm shows the steps mentioned above. The algorithm includes five sections. The first section consists of inputs. The second section is related to calculate $\hat{C}(A_i,A_t)$. The third section is applied to compare $\hat{d}_j(A_i,A_t)$ and $\hat{C}(A_i,A_t)$. The fourth section calculates $\hat{S}(A_i,A_t)$ and finally, the fifth section determines the concordance credibility value. In each section, the several loops and conditional commands may be used.

Begin

Inputs: $A_r\,(r=1,\ldots,R)$, alternatives.

$C_j\,(j=1,\ldots,m')$, QNC.

$\tilde{\tilde{C}}_j\,(j=m'+1,\ldots,J)$, QLC.

x_{rj}, the evaluation measure of machine i with respect to criterion j.

$LDM(\tilde{\tilde{x}})$, LDM of IT2FSs.

$LDM'_{PI,B}(\tilde{\tilde{x}})=1-LDM_{PI,B}(\tilde{\tilde{x}})$

$\tilde{\tilde{g}}_{rj}$ and $LDM(\tilde{\tilde{g}}_{rj})$, the integrated type-2 fuzzy evaluation (Eq. (16)) and crisp measure of machine r $(r = 1,..., R)$ with respect to the QLC, respectively.

$\tilde{\tilde{w}}_j$ and $LDM_W(\tilde{\tilde{w}}_j)$, the integrated type-2 fuzzy weight (Eq. (17)) and crisp weight of criterion *j*, respectively.

A_j^+ and A_j^-, the largest and the smallest evaluations with respect to the QNC (Eqs. (20) and (21)).

β_{1j}, β_{2j}, and β_{3j}, the threshold scalars assigned by DM with respect to criterion *j*.

q_j, p_j, and v_j, the threshold measures with respect to the QNC (Eqs. (22) to (24)).

$\tilde{\tilde{q}}_j, \tilde{\tilde{p}}_j$, and $\tilde{\tilde{v}}_j$, the threshold measures chosen by DM with respect to the QLC.

$c_j(A_i, A_t)$ and $\hat{c}_j(A_i, A_t)$, the preferable measure of machine A_i versus machine A_t for QNC and the preferable measure of machine A_i versus machine A_t for QLC (Eqs. (26) and (27)), respectively.

$d_j(A_i, A_t)$ and $\hat{d}_j(A_i, A_t)$, the discordance index of machine A_i versus machine A_t with respect to QLC (Eqs. (28) and (29)), respectively.

for $i, t = 1,..., R;\ i \neq t$ **do**

$$\hat{C}(A_i, A_t) = \frac{\sum_{j=1}^{m'} LDM_W(\tilde{\tilde{w}}_j) * c_j(A_i, A_t) + \sum_{j=m'+1}^{J} LDM_W(\tilde{\tilde{w}}_j) * \hat{c}_j(A_i, A_t)}{\sum_{j=1}^{J} LDM_W(\tilde{\tilde{w}}_j)}$$ by using Eq. (25)

if $i \neq t$ **then** $\hat{C}(A_i, A_t) = 0$
end
for $i, t = 1,..., R$ **do**

$d_j(A_i, A_t)$ and $\hat{d}_j(A_i, A_t)$) by using Eqs. (28) and (29)

if $\hat{d}_j(A_i, A_t) \leq \hat{C}(A_i, A_t)$, $\forall j$, **then** $\hat{S}(A_i, A_t) = \hat{C}(A_i, A_t)$

else

$$\hat{S}(A_i, A_t) = \hat{C}(A_i, A_t). \prod_{j \in J: \hat{d}_j(A_i, A_t) > \hat{C}(A_i, A_t)} \frac{1 - \hat{d}_j(A_i, A_t)}{1 - \hat{C}(A_i, A_t)}, j \in J : \hat{d}_j(A_i, A_t) > \hat{C}(A_i, A_t)$$

end
end
for $r = 1,..., R$ **do**

$\theta_i^c = \sum_{k=1, k \neq i}^{R} \hat{S}(A_i, A_k)$

$\theta_i^d = \sum_{k=1, k \neq i}^{R} \hat{S}(A_k, A_i)$

$\theta_i = \theta_i^c - \theta_i^d$

end
end

Since $\hat{C}(A_i, A_t)$, $d_j(A_i, A_t)$, $\hat{d}_j(A_i, A_t)$, $\hat{S}(A_i, A_t)$, and θ_i are the finite and definite based on the above algorithm, it is convergent. In other words, since the denominator of Eqs. (25) and (30) is the opposite of zero and also, Eqs. (26) to (29) lie at interval [0, 1], the infeasible and unbounded solutions do not obtain by applying the above algorithm. Therefore, the MCDM problem has the finite optimal solution.

In this section, the ELECTRE III approach is extended to GIT2FNs using *LDM*s formulated in Section 4 where GIT2FMCDM includes both *CC* and *BC*. Although, the method is explained for GIT2FNs, it can also be adopted to TraIT2FNs or TriIT2FNs. At the end, the above eight steps were presented in an algorithmic form.

6. Case Study and Illustrative Example

6.1. Case study

In this section, a case study is presented to show the effectiveness of the proposed approach using GIT2FNs, *LDM*s, and the ELECTRE III approach presented in the previous sections. Let five alternatives (single machine) M_i $(i = 1,2,3,4,5)$ are to be evaluated with respect to five criteria C_j $(j = 1,2,3,4,5)$ for maintenance services where total cost, mean time between failure (MTBF), and reliability function (R(T)) are the QNC, as well as the availability of spare parts and repairability are the QLC. The maintenance managers of a fishing net factory (located in Zahedan, Iran) desire to prioritize the fishing net machines for maintenance services. Prioritization of these machines can help them for the better implementation of maintenance activities (like procuring spare parts, decision-making for CM or PM, forecasting the costs of maintenance section, etc.). A machine with the larger precedence needs more inspection and control. The total cost consists of the mean of PM, CM, spare parts, and human resource costs per year. The failure rate of the Weibull distribution ($\lambda(t)$ [35]), MTBF [36], and R(T) [37] are given by:

$$\lambda(t) = \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1}, \tag{34}$$

$$MTBF = \int_0^T t\,\frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1} dt, \tag{35}$$

$$R(\mathrm{T}) = 1 - \int_0^T \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1} dt, \tag{36}$$

where β, θ, and T are the shape parameter, the scale parameter of Weibull distribution, and planning horizon, respectively. The interested reader can refer to Appendix A for showing definitions of linguistic variables and the performance measures of machines with respect to criteria.

Table 1 shows measures *LDM*s with respect to QLC.

Tab. 1. The measures *LDM*s of machines with respect to the QLC

Machines	Criteria				
	MTBF (hr.)	R(T)	Total cost ($/per maintenance)	Availability of spare parts	Repairability
M_1	225.00	0.875	650.8	0.357	0.899
M_2	50.810	0.972	310.0	0.996	0.300
M_3	711.11	0.555	680.0	0.571	0.699
M_4	120.00	0.937	230.6	0.003	0.994
M_5	509.11	0.646	580.5	0.357	0.005

Table 2 demonstrates threshold scalars and the linguistic variables with respect to QNC and QLC, respectively. The linguistic variables are specified based on DMs' preferences using the data of Table A1 in Appendix A.

Tab. 2. Threshold scalars and the linguistic variables with respect to the different criteria

scalars	Criteria				
	MTBF (hr.)	R(T)	Total cost ($/per maintenance)	Availability of spare parts	Repairability
β_1	0.15	0.15	0.05	VL	L
β_2	0.30	0.40	0.50	M	M
β_3	0.60	0.65	0.70	H	VH

Based on the data of Table 2, Table 3 represents the threshold measures with respect to QNC by using Eqs. (22) to (24) of Step 4 in Section 5 and QLC by using Eq. (14) calculated in Section 4.

Tab. 3. The threshold measures with respect to the different criteria

	Criteria				
	MTBF (hr.)	R(T)	Total cost ($/per maintenance)	Availability of spare parts	Repairability
q_j	99.040	0.062	22.460	0.071	0.200
p_j	198.09	0.166	224.70	0.500	0.500
v_j	396.18	0.271	314.58	0.714	0.916

According to Eqs. (25) to (27) of Step 5 in Section 5, the concordance matrix (Table 4) is obtained based on the comparison of the alternatives.

Tab. 4. The concordance matrix

	M_1	M_2	M_3	M_4	M_5
M_1	1.000	0.769	0.847	0.800	0.791
M_2	0.374	1.000	0.449	0.760	0.260
M_3	0.705	0.740	1.000	0.545	0.638
M_4	0.661	0.837	0.500	1.000	0.454
M_5	0.705	0.785	0.832	0.627	1.000

After calculating the discordance matrices for each criterion, the comparisons between the concordance and discordance matrices are carried out, as shown in Table 5.

Tab. 5. The comparisons the concordance and discordance measures

	M_1	M_2	M_3	M_4	M_5
M_1	-	$c(1,2) > d(1,2)$	$c(1,3) < d_2(1,3)$	$c(1,4) > d(1,4)$	$c(1,5) < d_5(1,5)$
M_2	$c(2,1) < d_{3,4}(2,1)$	-	$c(2,3) < d_{2,3}(2,3)$	$c(2,4) < d_4(2,4)$	$c(2,5) < d_{2,3,4}(2,5)$
M_3	$c(3,1) < d_1(3,1)$	$c(3,2) < d_1(3,2)$	-	$c(3,4) < d_1(3,4)$	$c(3,5) > d\ (3,5)$
M_4	$c(4,1) < d_3(4,1)$	$c(4,2) > d\ (4,2)$	$c(4,3) < d_{2,3}(4,3)$	-	$c(4,5) < d_{2,3,5}(4,5)$
M_5	$c(5,1) > d\ (5,1)$	$c(5,2) < d_1(5,2)$	$c(5,3) > d\ (5,3)$	$c(5,4) < d_1(5,4)$	-

Now, the results of credit degree, $\hat{S}(A_r, A_k)$, between machines are presented in Table 6 by using Eq. (30). Then, the net credibility matrix is created by Eqs. (31) to (33), as represented in Table 7. In the final step, the ranking results specified by the ELECTRE III [38], TOPSIS [32], VIKOR [39] based on GIT2FNs, and ELECTRE III based on type-1 fuzzy sets are shown in Table 8.

Tab. 6. The credit degree matrix

	M_1	M_2	M_3	M_4	M_5
M_1	-	0.769	0	0.800	0.192
M_2	0	-	0	0	0
M_3	0	0	-	0	0.638
M_4	0	0.837	0	-	0
M_5	0.705	0	0.832	0.060	-

Tab. 7. The net credibility matrix

	M_1	M_2	M_3	M_4	M_5
θ_r^c	1.762	0	0.638	1.291	1.597
θ_r^d	0.705	1.606	0.832	0.860	1.284
θ_r	1.057	-1.606	-0.194	-0.023	0.766

Tab. 8. The ranking results obtained by the GIT2FNs-based ELECTRE III , TOPSIS, and VIKOR methods

	Rankings				
Machines	The proposed method (ELECTRE III using *LDM*)	The ELECTRE III method [38]	TOPSIS [32]	ELECTRE III based on type-1 fuzzy sets	VIKOR based on type-2 fuzzy sets [39]
M_1	1	1	1	1	1
M_2	5	5	5	5	4
M_3	4	4	4	3	5
M_4	3	3	2	4	2
M_5	2	2	3	2	3

Based on the descending order obtained by the above five methods, Machine 1 is the most important machine for PM. There are the similar ranking order between the proposed method (ELECTRE III using *LDM*) and the ELECTRE III method [38] (the columns two and three). This conclusion makes the validation transparent. However, it should be noted that the partial difference between the ELECTRE III and TOPSIS methods can be arisen from the impact of the threshold measures on the results of ELECTRE III. In other words, inexistence of thresholds in the TOPSIS method is the main reason of these differences. On the other hand, the situations of Machines 3 and 4 are the different of our approach when applying the ELECTRE III method based on type-1 fuzzy sets in which MF is stated as a specified number at interval [0, 1]. Based on Table 8, Fig. 2 shows the ranking results as bar chart. As represented in Fig. 2, the results are roughly the same when applying the above five methods for machines 1 and 2. However, there is the partial difference between the ranking results of the others, due to the different ranking programs. On the other hand, Table 9 presents the correlation coefficients between the different approaches. As you can see in this table, there are high correlation coefficients between our approach and the others. It implies that the proposed method presents the stable results.

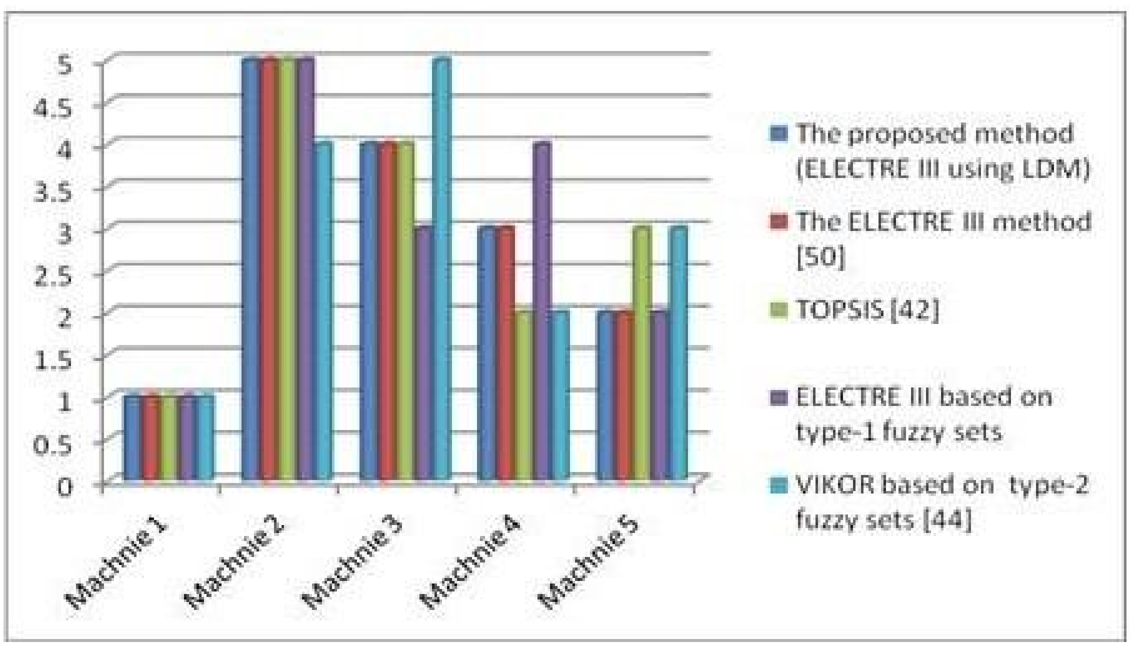

Fig. 2. The ranking results using the different methods

Tab. 9. The correlation measures between the various approaches

Correlation coefficients	ELECTRE III (type 2 fuzzy sets)	TOPSIS	ELECTRE III (type 1 fuzzy sets)	VIKOR (type 2 fuzzy sets)
ELECTRE III (type 2 fuzzy sets)	-	0.9	0.9	0.8
TOPSIS	0.9	-	0.7	0.9
ELECTRE III (type 1 fuzzy sets)	0.9	0.7	-	0.5
VIKOR (type 2 fuzzy sets)	0.8	0.9	0.5	-

6.2. Illustrative example

Here, an illustrative example proposed by Zhong and Yao [40] is adopted to check the ranking order of our method with the others. The managers of a high-technology company desire to prioritize five suppliers under seven criteria. The weights of these criteria are 0.0562, 0.1234, 0.0452, 0.1245, 0.2325, 0.1380, and 0.2801, respectively. The interested reader can refer to Zhong and Yao [40] for studying other measures and data of the above problem.Table 10 shows the concordance matrix between alternatives (suppliers). By constructing the discordance matrices under each criterion, Table 11 represents the accommodations between the concordance and discordance matrices. The measures of credit degree of suppliers are presented in Table 12 by using Eq. (30). Finally, Table 13 shows the net credibility matrix. Moreover, Table 14 presents the ranking results of the different methods, namely the proposed method, TOPSIS with IT2FSs [32], VIKOR with IT2FSs [39], ELECTRE III with IT2FSs [38], ELECTRE I [40], weighted aggregated sum-product assessment (WASPAS) with IT2FSs [41], and SAW with IT2FSs [42]. Accordingly, $A_5 > A_4 > A_1 > A_2 > A_3$ is the results ranking order using the proposed approach and the ELECTRE III method [38] (the columns two and three). This conclusion makes the validation transparent. As a result, alternative 5 is chosen as the optimal option that is also the most significant option according to the last four methods. It proves the validity and stability of our results. In addition, the ranking order attained by the ELECTRE I method [40] is different from the others. This conclusion shows that the mentioned approach cannot be the desirable method for prioritizing alternatives.

Tab. 10. The concordance table

	A_1	A_2	A_3	A_4	A_5
A_1	1.0000	0.7697	0.8475	0.7478	0.6719
A_2	0.4563	1.0000	0.4494	0.7607	0.2607
A_3	0.7853	0.7404	1.0000	0.5460	0.6380
A_4	0.6618	0.8376	0.5001	1.0000	0.4540
A_5	0.7054	0.7853	0.8324	0.6276	1.0000

Tab. 11. The comparisons table

	A_1	A_2	A_3	A_4	A_5
A_1	-	$c(1,2) < d_1(1,2)$	$c(1,3) > d(1,3)$	$c(1,4) < d_1(1,4)$	$c(1,5) > d(1,5)$
A_2	$c(2,1) > d(2,1)$	-	$c(2,3) > d(2,3)$	$c(2,4) > d(2,4)$	$c(2,5) > d(2,5)$
A_3	$c(3,1) > d(3,1)$	$c(3,2) < d_1(3,2)$	-	$c(3,4) < d_1(3,4)$	$c(3,5) < d_1(3,5)$
A_4	$c(4,1) > d(4,1)$	$c(4,2) > d(4,2)$	$c(4,3) > d(4,3)$	-	$c(4,5) > d(4,5)$
A_5	$c(5,1) > d(5,1)$	$c(5,2) > d(5,2)$	$c(5,3) > d(5,3)$	$c(5,4) > d(5,4)$	-

Tab. 12. The credit degree table

	A_1	A_2	A_3	A_4	A_5
A_1	0.0000	0.4820	0.8475	0.5376	0.6719
A_2	0.4563	0.0000	0.4494	0.7607	0.2607
A_3	0.7853	0.0000	0.0000	0.0000	0.3381
A_4	0.6618	0.8376	0.5001	0.0000	0.4540
A_5	0.7054	0.7853	0.8324	0.6276	0.0000

Tab. 13. The net credibility table

	A_1	A_2	A_3	A_4	A_5
θ_r^c	2.5389	1.9271	1.1234	2.4535	2.9507
θ_r^d	2.6088	2.1049	2.6294	1.9259	1.7247
θ_r	-0.0699	-0.1778	-1.5060	0.5276	1.2260

Tab. 14. The ranking results of different approaches

Suppliers	The proposed method (ELECTRE III using *LDM*)	The ELECTRE III method [38]	The ELECTRE I method [40]	TOPSIS with IT2FSs [32]	WASPAS with IT2FSs [41]	VIKOR with IT2FSs [39]	SAW with IT2FSs [42]
A_1	3	3	3	2	2	2	2
A_2	4	4	5	5	5	5	5
A_3	5	5	4	3	3	3	3
A_4	2	2	1	4	4	4	4
A_5	1	1	2	1	1	1	1

In this section, a case study and an illustrative example were presented in order to show the effectiveness of our approach. The results showed that the proposed approach is the effective approach for solving the MCDM problems. In general, it can be deduced that the difference reason of ranking results between ELECTRE III with the others is the existence of thresholds.

7. Conclusion and Discussion

The present paper applies a new ranking methodology to rank the IT2FNs. It is then utilized to rank machines. In order to show the effectiveness of the proposed methodology, it has been adopted to a real case study. In this case study, the authors proposed an integrated group MCDM (GMCDM) approach to prioritize machines where the standpoints of DMs are represented as GIT2FNs. In our suggested methodology, the type-2 fuzzy weights of criteria and type-2 fuzzy assessments of machines with respect to criteria are first merged with the integrated type-2 fuzzy weights and evaluations, respectively, where the concept of alpha cuts is used. Afterwards, the crisp weights of criteria and assessments are determined by *LDM*s. According to the proposed method, thresholds with respect to QLC are stated as GIT2FNs. In decision-making problems, it may be a case where a DM may consider identical importance with respect to measures of some ratings. Thus, the use of thresholds can help DM when dealing with such situations. In other words, a manager may have no preference for prices of $500 and $1000. These arguments show which the ELECTRE III approach with GIT2FNs than some approaches (such as TOPSIS and VIKOR) has more realistic vision when facing with the GMCDM problems. There is not such privilege in some methods like TOPSIS and VIKOR. In general, it can be deduced that the difference argument of ranking results between the ELECTRE III approach with the others is the existence of thresholds. However, machine 1 is selected as the most important option, implying that the above approach is the effective approach for solving the MCDM problems. On the other hand, there are the following conclusions when comparing the results of the proposed method with the others regarding the illustrative example:

- Ranking order is as $A_5 > A_1 > A_3 > A_4 > A_2$ based on the proposed method, The ELECTRE III method [38], TOPSIS [32], and type-1 fuzzy sets-based ELECTRE III.
- The proposed method has the ranking results similar to the ELECTRE III method presented by Selvaraj and Jeon [38]. It proves the results validity.
- Supplier 5 is chosen as the most important supplier based on *LDM*-based ELECTRE III, ELECTRE III with IT2FSs [38], TOPSIS with IT2FSs [32], WASPAS with IT2FSs [41], VIKOR with IT2FSs [39], and SAW with IT2FSs [42]. Exceptionally, the ranking order obtained by ELECTRE I [40] has different results than the others. Accordingly, Supplier 4 is chosen as the most suitable item. It shows that the mentioned approach is not the suitable method for prioritizing alternatives.

  Generally, it is worth noting be mentioned that the use of the threshold measures in ELECTRE III is the main reason of difference ranking results.

This approach has some important drawbacks which are the determination of expert DMs and problems regarding defining variables. On the other hand, the proposed method can be led to improper, imperfect, and incompatible results when DMs have very contradiction standpoints with respect to ratings or weights.

Some important cases for further studies are given as:

1. The proposed method was applied only to a type of machines' arrangements (single machine). However, one can use it to the others such as parallel machines, flow shop, etc.
2. Other QLC or QNC can be considered in other manufacturing industries.
3. The proposed method is usable to be applied to other MCDM methods.
4. The proposed approach can be applied to other fuzzy environments.

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Appendix A

The evaluations of machines with respect to the QLC and the weights of criteria are stated as GIT2FNs using linguistic variables, as indicated in Table A1.

Tab. A1. Definitions of linguistic variables for the performance evaluations and weights of criteria

		$\left[\left(\mu^{L},\sigma^{L};H_{\tilde{G}}^{L}\right),\left(\mu^{U},\sigma^{U};H_{\tilde{G}}^{U}\right)\right]$
Absolutely Low (AL)	Absolutely unimportant (AU)	[(3,0.5;1) , (3,1;1)]
Very Low (VL)	Very unimportant (VU)	[(5,0.5;1) , (5,1;1)]
Low (L)	Unimportant (U)	[(7,0.5;1) , (7,1;1)]
Medium (M)	Medium (M)	[(9,0.5;1) , (9,1;1)]
High (H)	Important (I)	[(11,0.5;1) , (11,1;1)]
Very high (VH)	Very important (VI)	[(13,0.5;1) , (13,1;1)]
Absolutely high (AH)	Absolutely important (AI)	[(15,0.5;1) , (15,1;1)]

Table A2 represents the linguistic variables assigned by DMs for criteria using data of Table A1. They have been working in department of maintenance (sections of maintenance engineering, maintenance executive affairs, and spare parts warehouse) for several years such that have enough experience in the field of maintenance. These variables are first merged with the integrated type-2 fuzzy weight by using Eqs. (6) to (10) in Definition 3.3 and the weights of criteria (as presented in Table A3) are then calculated based on Eq. (14) calculated in Section 4. Table A4 shows the measures of machines with respect to the QNC for one year (300 working days or 2400 hours) by using Eqs. (34) to (36) and the evaluation of machines with respect to QLC using linguistic variables of Table A1. In fact, this table is GIT2FMCDM matrix. The data of this matrix have obtained by staff of maintenance section.

Tab. A2. Linguistic variables assigned by DMs with respect to criteria

DMs	Criteria				
	MTBF (hr.)	*R*(T)	Total cost ($/per maintenance)	Availability of spare parts	Repairability
DM 1	I	M	AI	VI	U
DM 2	I	U	VI	I	U
DM 3	M	M	VI	M	VU

Tab. A3. The weights of criteria

weights				
c_1	c_2	c_3	c_4	c_5
0.530	0.348	0.833	0.591	0.166

Tab. A4. The performance measures of machines with respect to different criteria

Machines	Criteria								
	MTBF (hr.)	*R*(T)	Total cost ($/per maintenance)	Availability of spare parts			Repairability		
				DMs			DMs		
				1	2	3	1	2	3
Machine 1	225.00	0.8750	650.8	L	M	L	H	VH	H
Machine 2	50.810	0.9728	310.0	AH	VH	VH	L	M	L
Machine 3	711.11	0.5556	680.0	M	M	H	H	M	H
Machine 4	120.00	0.9375	230.6	VL	VL	AL	VH	H	VH
Machine 5	509.11	0.6464	580.5	L	M	L	VL	L	VL

The linguistic variabl es with respect to QLC are merged with an integrated type-2 fuzzy evaluation by using Eqs. (6) to (10) in Definition 3.3 and their *LDMs* are then obtained by using Eq. (11) calculated in Section 4.

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