Coordination of Pricing and Inventory Decisions in a Fresh-product Supply Chain Considering the Competition between New and Old Products

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ABSTRACT

Fresh product retailers often encounter excessive inventory at the end of each period. The leftover product has a lower perceived quality than the new product. Therefore, retailers try to influence consumers’ preferences through price differentiation that leads to an internal competition based on product age and prices. The price differentiation strategy affects the market demands, sales volume and consequently, the profitability of the whole fresh-product Supply Chain (SC) and its members. This paper addresses the pricing and inventory control problem for fresh products to examine the influence of this competition on the SC members’ decisions and profits. Accordingly, a new coordination model based on a return policy with the revenue and cost-sharing contract is developed to improve the profits of independent SC members. The fresh-product SC consists of one supplier and one retailer, where consumers are sensitive to the freshness and retail price of the product. Firstly, the retailer’s optimal decisions, including the order quantity and old product price, are made influenced by a decentralized decision-making structure. Then, a centralized approach to optimization of the SC decisions from the whole SC viewpoint is utilized. Finally, a new coordination contract is designed to motivate the members to participate in the coordination model. Numerical examples are presented to compare the performance of different decision-making approaches. The findings indicated that the proposed contract could effectively coordinate the fresh-product SC. Furthermore, the coordinated decision-making model is more profitable and beneficial for the whole SC compared to the decentralized one. The results also showed that when consumers were more sensitive to freshness, the simultaneous sale of multiple-aged products at different prices would be more profitable.

KEYWORDS: Fresh-product supply chain; Channel coordination; Pricing and inventory decisions; Cannibalization effect.

1. Introduction

Perishable inventory management is a challenging issue that retailers of perishable products are facing. It becomes more complicated for fresh products (e.g., fruits, vegetables, bread, meat, fresh milk, etc.) which have a very short lifetime and decaying utility over time [1]. The consumer’s tendency to purchase fresh products is a function of the product’s lifetime. Thus, the retailers have to provide new products for consumers; otherwise, they will lose considerable portions of their sales [2]. They refill the shelves with new products before the older ones are sold out to prevent empty shelves. Then, some shelves hold units of the same product with various expiration dates. In this situation, a challenge may arise due to the difference between consumer’s valuation of products and different remaining shelf life, which can result in an internal competition based on product age and prices. If the retailer charges the same price for all units of the same product, the consumers often prefer the newest products. In this case, new products cannibalize sales of older products, thus leading to increased waste of the products [3].

The retailer can use a markdown pricing policy in which a lower price is set on old units. The price differentiation can increase the total demand.
share by attracting consumers who only seek discounted products. On the contrary, too low prices on old units may result in the cannibalization of sales of new products [4]. The term ‘cannibalization’ describes how competition between products that are partial substitutes for each other results in a reduction of sales volume, sales revenue, or market share [5]. Therefore, the retailer faces the problem of finding optimal prices and order quantity jointly to manage fresh products better and increase its profitability. Moreover, it should be noted that the pricing and inventory replenishment problem, considering the internal competition between multiple-aged products, is no longer an internal problem in the retail store because its effects (such as increased demand share, reduced waste, etc.) extend to the other members of the fresh-product Supply Chain (SC). Moreover, the retailer’s decisions are influenced by supply contracts of fresh product suppliers. Therefore, a decentralized decision-making structure in which the retailer individually maximizes its profit might not be a proper approach. Hence, it is essential to coordinate the simultaneous pricing and inventory decisions for fresh products when new and old items coexist on the shelf, to improve the whole performance of the fresh-product SC and its members.

In the fresh-product SC literature, coordination models have attracted the attention of some scholars. Different issues including freshness-keeping efforts ([6], [7]), preservation technology investment [8], logistics outsourcing [9], etc. have been addressed. Some other researchers have focused on the problem of pricing and inventory replenishment considering the simultaneous sale of perishable products with different ages in the retail stores (see [4], [10], [11]). However, combining the abovementioned problem with coordination models in the fresh-product SC is neglected on which this study has focused.

The present study aims to investigate how the value reduction of older units and following cannibalization problem affect the retailer’s pricing and inventory decisions in coordination with other members of the supply chain. In addition, it is attempted to analyze the coordination of the fresh-product SC with respect to the internal competition between new and old products. Accordingly, mathematical models under both decentralized and centralized decision-making approaches are developed, and the optimal solutions are derived. Then, based on an analysis of the costs and benefits by members encounter, a hybrid incentive contract is developed to coordinate the fresh-product SC members. This contract motivates them to move from the decentralized decision-making approach to the centralized one.

This paper is structured as follows. Section 2 presents the literature review on the internal competition of multiple-aged inventories and the coordination models of fresh-product SCs as well. Section 3 describes the investigated problem. Section 4 develops the mathematical models under three approaches: (1) decentralized, (2) centralized, and (3) coordinated approach. Section 5 provides the numerical examples to test the proposed models and analyze the sensitivity of parameters. Finally, the Section 6 concludes the study and presents future research directions.

2. Literature Review

In this section, the most relevant studies used in this research are surveyed. The related literature of this research is reviewed in two main areas as follows.

2.1. Internal competition of multiple-aged inventories

The field of pricing and inventory management of perishable products has caught the attention of a number of researchers in the literature. In this area, the simultaneous sale of multiple-aged inventories and consumer valuation has been carefully studied by some scholars. Empirical studies on the consumers’ behavior in grocery retail implied that the customers’ willingness-to-pay reduced near the product’s expiration date and hence affected the demand. In this case, discounting is identified as an effective way to sell a perishable product close to its expiration date [12].

Ferguson and Koenigsberg [11] considered a two-period pricing and inventory problem under the consideration of internal competition between the old and new products. They used consumer’s utility functions to derive inverse demand functions for the old and new units. Li et al. [13] investigated a multi-period ordering and pricing problem under the simultaneous sale of new and out-of-season products. They proposed and compared different methods for solving the problem. Chintapalli [4] presented a combined pricing and inventory control model for perishable products with a two-period lifetime under stationary prices. Using Markov Decision Process (MDP), he proved that a myopic policy is an optimal steady-state policy over an infinite planning horizon. Sainathan [10] studied the
problem of competing perishable products with different ages under dynamic demand substitution. Recently, Fan et al. [14] developed a dynamic pricing policy based on the product’s real-time freshness. They analyzed the impacts of the age, freshness, and inventory level of the old product on the replenishment policy. The economic effect of Time-Temperature Indicator (TTI) technology on the effectiveness of price differentiation strategies was studied in [15-17]. This technology provides real-time tracking of product age and quality in stock. Herbon and Ceder [16] incorporated both dynamic pricing policy and TTI-based automatic device technology for monitoring a perishable inventory system with a random shelf-life. Herbon et al. [17] studied a price differentiation strategy jointly with a periodic-review replenishment policy and showed that the application of low-cost TTI tags could be very advantageous in dynamic stochastic environments.

In the literature, some studies proved that the retailer does not benefit from selling multiple-aged products at different prices when the market demand is deterministic (see [15], [18-20]). For example, Herbon [20] studied the replacement of a single-aged product selling policy with a multiple-aged product selling policy. He showed that the simultaneous sale of multiple-aged products reduces the retailer’s profit under deterministic demand. A majority of researches on the simultaneous sale of perishable products with different ages have investigated the problem in a retail store. However, the impacts of price differentiation strategy and consumer choice behavior are not limited to the retailer and extended to the upstream members of the supply chain, due to their influence on the market demand and waste reduction.

### 2.2. Coordination models of fresh-product SCs

Research on coordination mechanisms in fresh-product supply chain is both meaningful and critical. The fresh-product SC members face the challenge of maintaining product freshness. They are affected by consumer value, product freshness, and prices [6]. Coordinating the supply chain members’ decisions by considering the specific features of fresh-product SCs has recently attracted the attention of researchers. For example, some studies addressed freshness-keeping efforts throughout the supply chain (see [6], [7], [21-23]). Cai et al. [7] developed a fresh-product SC coordination model and attempted to preserve the product freshness to affect both the quality and quantity of product reaching the market. Mohammadi et al. [8] combined the preservation-technology investment problem with a coordination approach in the fresh-product SC context. Table 1 summarizes the results of related studies in the field of fresh-product SC coordination models.

The fresh-product SC members’ effort to maintain high availability of the fresh product on the shelves of the selling points leads to the coexistence of multiple-aged inventories [3]. In the fresh-product SCs, consumers are often sensitive to product quality (quality refers to the product freshness). Table 1 suggests that all previous studies in the related literature have considered the market demand as a function of the retail price, time, and/or other factors affecting the product freshness. To the best of the author’s knowledge, the difference in consumers’ valuation for fresh products of different ages has not yet reported in the literature of fresh-product SC coordination models.

According to Table 1, different coordination mechanisms were developed to encourage the fresh-product SC members to participate in the coordinated approach. Incentives suggested facilitating coordination include revenue and cost-sharing ([6], [23-25]), quantity discount contract [26], wholesale price discount [9], two-part tariff [22], call option contracts [27], etc. Moreover, the unsold inventory risk at the end of the selling season is considered by Wu et al. [28] who presented an incentive mechanism based on a price-discount and inventory-risk sharing contract. In addition, an unsellable production subsidy was embedded in the coordination mechanism proposed by Su et al. [29]. However, to our knowledge, return policies and buy-back contracts have not been considered in the literature of fresh-product SC coordination models. While in the real world, the suppliers offer a return policy for some types of fresh products such as pasteurized dairy products (see, for example, [21]) and buy the expired inventories to discount the risk of overstocking arising from uncertain demand that retailers face.

Accordingly, a new coordination mechanism based on return policy with revenue and cost-sharing contract is designed to improve the profitability of the whole fresh-product SC and its members. Although previous studies have considered consumer’s sensitivity to the product freshness, none of them considered the difference between consumers’ valuation of the products and different remaining shelf life. Hence, in this
study, a fresh-product SC coordination model under the simultaneous sale of multiple-aged products is proposed by adopting a price differentiation strategy.

3. Problem Definition
This paper studies a two-stage supply chain consisting of a single supplier and a single retailer with one type of fresh food product with a fixed shelf life. The supplier’s capacity is unlimited. The retailer employs an inventory system that is periodically replenished over an infinite planning horizon. The supplier visits the retailer at fixed intervals and delivers the order instantaneously. Due to the short lifetime of fresh products, the period length is short and predetermined by the supplier. The retailer faces a stochastic market demand, where consumers are sensitive to the product’s price and freshness degree.

The fresh product lasts for two periods and it is classified into new and old. Due to uncertainty about the demands, the retailer may encounter unsold new products at the end of each period; therefore, he begins the next period with an on-hand inventory of old items and orders new items at the start of the period. From the consumer’s view point, the old items have a lower perceived quality than the new items; therefore, the retailer provides lower valuations for the consumers that are sensitive to price and freshness of the product. A discount factor $\delta \in [0, 1]$ is utilized to represent the consumer’s valuation for the old units. Note that if $\delta=1$, consumers are indifferent about two products and view either type as identical.

The retailer tries to influence consumers’ preferences through price differentiation. The difference between new and old product prices leads to a migration of some consumers from purchasing the new product to the old one. This is called the cannibalization effect and depends on two measures: the price difference and the cannibalization amplitude parameter [31]. The cannibalization amplitude, i.e., $\gamma$, represents a decrease in new product’s marginal sales due to the price differentiation, and it is considered as a function of the discount factor $\delta$, as implied in references [10], [11], [32]. When the consumers’ valuation for the old units, i.e., $\delta$, is near to 1, the consumers have a low sensitivity to the freshness; therefore, a small discount on the old products leads to a large migration of consumers from purchasing new units to purchasing old units. In this case, the cannibalization amplitude $\gamma$ is high.

<table>
<thead>
<tr>
<th>Authors</th>
<th>SC structure</th>
<th>Product loss</th>
<th>Demand sensitive to</th>
<th>Demand decisions</th>
<th>Multi-aged products</th>
<th>Coordination mechanism</th>
</tr>
</thead>
<tbody>
<tr>
<td>Xiao et al. [25]</td>
<td>1 producer</td>
<td>Quality</td>
<td>Price, Time</td>
<td>Stochastic</td>
<td>Inventory, pricing</td>
<td>Cost sharing</td>
</tr>
<tr>
<td>Cai et al. [7]</td>
<td>1 producer</td>
<td>Quality</td>
<td>Price, Time</td>
<td>Stochastic</td>
<td>Inventory, pricing</td>
<td>Price discount sharing and compensation contract</td>
</tr>
<tr>
<td>Cai et al. [9]</td>
<td>1 supplier</td>
<td>Quantity</td>
<td>Price, Time</td>
<td>Stochastic</td>
<td>Inventory, pricing</td>
<td>Wholesale-market clearance &amp; wholesale price-discount sharing</td>
</tr>
<tr>
<td>Su et al. [29]</td>
<td>1 retailer</td>
<td>Quality</td>
<td>Price, Time</td>
<td>Stochastic</td>
<td>Inventory, pricing</td>
<td>Wholesale Price Discount &amp; Unsellable Produce Subsidy &amp; Cost Compensating</td>
</tr>
<tr>
<td>Wu et al. [28]</td>
<td>1 distributor</td>
<td>Quantity</td>
<td>Price, Logistics service level</td>
<td>Stochastic</td>
<td>Inventory, pricing, logistics service level</td>
<td>1. Revenue and service-cost sharing 2. Price-discount and inventory-risk sharing</td>
</tr>
<tr>
<td>Hou and Liu [30]</td>
<td>1 wholesaler</td>
<td>Quality</td>
<td>Price, quality</td>
<td>Stochastic</td>
<td>Inventory, pricing</td>
<td>Incentive contract based on an estimated quality and price</td>
</tr>
<tr>
<td>Zheng et al. [6]</td>
<td>1 supplier</td>
<td>Quality</td>
<td>Price, Time, Effort level</td>
<td>Deterministic</td>
<td>Pricing, fresh-keeping effort</td>
<td>Fresh-keeping cost and revenue-sharing</td>
</tr>
<tr>
<td>Yan et al. [24]</td>
<td>1 manufacturer</td>
<td>Quality</td>
<td>Price, Time</td>
<td>Stochastic</td>
<td>Inventory, pricing, transportation time</td>
<td>Cost and revenue sharing</td>
</tr>
<tr>
<td>Wang and Chen [27]</td>
<td>1 supplier</td>
<td>Quantity</td>
<td>-</td>
<td>Stochastic</td>
<td>Inventory, pricing</td>
<td>Call option contract</td>
</tr>
<tr>
<td>Moon et al. [21]</td>
<td>1 manufacturer</td>
<td>Quality</td>
<td>Price, freshness keeping effort</td>
<td>Deterministic</td>
<td>Pricing, freshness keeping investment</td>
<td>1. Revenue and investment cost sharing 2. Incremental quantity discount</td>
</tr>
<tr>
<td>Mohammadi et al. [8]</td>
<td>1 supplier</td>
<td>Quality</td>
<td>Price, Preservation technology investment</td>
<td>Stochastic</td>
<td>Inventory, pricing, Preservation technology investment</td>
<td>Revenue &amp; preservation Technology investment sharing</td>
</tr>
</tbody>
</table>

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Therefore, the two parameters have a direct relationship with each other. The new and old items compete for consumers’ interests with respect to their freshness and prices. The demands of the new and old products in an additive fashion are modeled and are linearly dependent on product prices. While the demand of the old items is assumed to be deterministic, it is assumed that the demand function of the new items has an independent random component, implying that the demands react to agents instead of prices and freshness degree. Such type of demand functions is common in the literature (for example, see [34], [35]) The demand functions for the new and old products are as follows:

\[
\xi_n(p_n - p_o) = a_1 - b_1 p_n - \gamma(p_n - p_o) + \bar{\xi}_0
\]
\[
D_o(p_n - p_o) = a_2 - b_2 p_o + \gamma(p_n - p_o)
\]

where \( p_o \) and \( p_n \) denote the retail price for the old unit and the new unit, respectively, and \( \bar{\xi}_0 \in [A_0, C_0] \) is the random component of demand. The parameters \( a_1 \) and \( a_2 \) represent the market potential of new and old products. \( b_1 \) and \( b_2 \) denote the consumer’s sensitivity to the prices of new and old units. \( \gamma \) is supposed that \( b_2 > b_1 \), showing that any marginal increase in the old unit’s price has a greater negative impact on the old unit’s demand, due to the value drop of the old units compared to new ones. \( \gamma(p_n - p_o) \) shows the cannibalization effect. At the end of each period, the retailer may incur the cost of preparing the remaining new units for sale in the next period as the old units. The cost includes the general cost of holding inventory, packaging, special storage, and re-tagging discounted items that imposes additional works on the personnel. Moreover, the retailer may encounter unsold old units. In this case, the supplier offers a return policy and buys back the expired products at a specified buyback price, \( b \), per unit. The expired returned products are salvaged by the supplier at a value \( g \) per unit and conveyed to other industries for recovery. In other words, the nature of fresh products and sanitary issues prohibit the returned product from being reused in the forward flow.

In the investigated supply chain, the retailer decides on the order quantity of new units and the selling price of old units at the beginning of each period, by maximizing its profit function. It is assumed that the retailer’s decision variables are constant and do not change from one period to the next. In other words, they are independent of the inventory of the old product carried over from the previous period. This is a practical and simple policy to implement, which is prevalent at supermarkets that sell fresh products such as dairy, meat, vegetables, and bakery products. Such constancy in the discounted price is termed as markdown price stickiness [4].

Besides, several basic assumptions before modeling are posited in the following:

1. The random component \( \bar{\xi}_0 \) has a uniform distribution in the range \([A_0, C_0]\) with Probability density function \( f(x) \) and Cumulative distribution function \( F(x) \). The new random variable is defined as \( \bar{\xi}_1 = \bar{\xi}_0 - A_0 \), that \( \bar{\xi}_1 \in [0, B_0] \) where \( B_0 = C_0 - A_0 \). The demand for the new product can be written as \( \xi_n(p_n - p_o) = D_n(p_n - p_o) + \bar{\xi}_1 \) where \( D_n(p_n - p_o) = a_1 + A_0 - b_1 p_n - \gamma(p_n - p_o) \).

2. The retail price for the new products is assumed to be predetermined and fixed. This is quite usual in industries like foods, dairy products, etc.

3. Consumers do not substitute between new and old products during stock out of their preferred one. Of note, shortages result in lost sales with a negligible cost.

The supplier trades with the retailer under a return policy with predetermined wholesale and
buyback prices. The price differentiation strategy implemented by the retailer and the competition between new and old products can affect the supplier-retailer interaction. It may be beneficial for the whole supply chain, due to its effect on market demand. The supplier can take the opportunity of simultaneous sale of new and old products for increasing the market share. It can be achieved by coordination between the supplier and the retailer. However, in the decentralized approach, the members individually optimize their decisions without considering the fresh-product SC as a whole. This may not be a proper decision-making structure. In the centralized approach, the optimal values of decision variables are determined by maximizing the profitability of the entire fresh-product SC which may decrease the profitability of the retailer. Therefore, different incentive mechanisms should be used in a coordination structure to encourage the members to participate in the centralized approach.

4. Mathematical Models
In this section, the mathematical models for various decision-making approaches are developed. The following notations are used in the proposed models.

<table>
<thead>
<tr>
<th>Parameters:</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_m$</td>
<td>Supplier’s production cost per unit fresh-product</td>
</tr>
<tr>
<td>$\xi_0$</td>
<td>Random component of demand for the new unit that ranges in $[A_0, C_0]$ with Probability density function $f(x)$ and Cumulative distribution function $F(x)$</td>
</tr>
<tr>
<td>$\xi_n(p_n, p_o)$</td>
<td>Stochastic demand for the new product that is a function of the retail prices</td>
</tr>
<tr>
<td>$D_n(p_n, p_o)$</td>
<td>Deterministic demand for the new product that is a function of the retail prices</td>
</tr>
<tr>
<td>$D_o(p_n, p_o)$</td>
<td>Deterministic demand for the old product that is a function of the retail prices</td>
</tr>
<tr>
<td>$a_1, a_2$</td>
<td>market potential of new and old products</td>
</tr>
<tr>
<td>$b_1, b_2$</td>
<td>consumer’s sensitivity to the prices of new and old products</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Cannibalization amplitude</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Discount factor</td>
</tr>
<tr>
<td>$g$</td>
<td>Salvage value</td>
</tr>
<tr>
<td>$w$</td>
<td>Wholesale price of the supplier</td>
</tr>
<tr>
<td>$b$</td>
<td>Buy back price</td>
</tr>
<tr>
<td>$p_n$</td>
<td>Retail price for a new product</td>
</tr>
<tr>
<td>$h$</td>
<td>Holding cost per leftover new unit at the end of each period</td>
</tr>
<tr>
<td>$\Pi^N_M$</td>
<td>Total profit of M in N decision-making structure, where the suffixes R; S; SC indicate the retailer, supplier, and whole supply chain, respectively, and the prefixes dc; c; co indicate the decentralized, centralized, and coordinated structures.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Decision variables:</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_o$</td>
<td>Retail price for an old product</td>
</tr>
<tr>
<td>$q$</td>
<td>Order quantity of the retailer</td>
</tr>
<tr>
<td>$z$</td>
<td>Stocking factor</td>
</tr>
</tbody>
</table>

4.1. Decentralized decision-making
In the decentralized decision-making structure, each member optimizes the individual objective function, regardless of other chain members. In this problem, the retailer independently makes decisions about the order quantity of new units and selling price of old units by optimizing its profit function. The supplier has the choice to whether or not accept the retailer’s decisions. A myopic policy is taken into consideration that is a commonly followed policy for solving complex dynamic pricing, inventory, and scheduling problems [13]. In this policy, only the current period’s profit is accounted, and the myopic optimal solutions are derived. In the literature, the optimality of the myopic policy is proved for some stationary problems in which parameters and variables are time-invariant (For example, see [4], [36]). Given the Markov decision process theory, it has been proved that the myopic policy was certainly an optimal steady-state policy for an inventory-pricing
problem of products with two-period lifetime considering the policy of discounts and sticky pricing in an infinite planning horizon [4]. Accordingly, the retailer’s decisions are made to

\[
\Pi^r_\alpha(q, p_0) = \begin{cases} 
\frac{p_0}{1 + \frac{D_n(p_0) + a}{2b}} 
\end{cases}
\]

Profit function (3) includes the revenue of selling new products, revenue of selling old products, order cost, inventory holding cost, and buyback revenue of outdated units. To simplify the solving process, the stocking factor of new units as \( z = q - D_n(p_0) \) (Petruzzi and Dada [34]) is employed. Given that \( \xi_n = D_n(p_0) + \xi_1 \), the retailer’s expected profit function under the decentralized decision making can be written as:

\[
E[\Pi^r_\alpha(q, p_0)] = \begin{aligned}
p_0 \int_0^1 (D_n(p_0) + x) f_z(x) dx + \int_0^q (D_n(p_0) + z) f_z(x) dx 

+ p_0 \int_0^{D_n(p_0)} (D_n(p_0) + z - x) f_z(x) dx
\end{aligned}
\]

By simplifying (4), the mathematical model for the retailer can be written as follows:

\[
\max E[\Pi^r_\alpha(q, p_0)] = \begin{aligned}
-w (z + D_n(p_0)) + p_0 z 

+ p_0 D_n(p_0) - (p_0 + h) \int_0^{D_n(p_0)} F(x) dx 

+ p_0 \int_{-D_n(p_0)}^{D_n(p_0)} F(x) dx + b \int_0^{D_n(p_0)} F(x) dx
\end{aligned}
\]

where \( D_n(p_n, p_o) = A_0 - b(p_n - p_0) \) and \( D_o(p_n, p_o) = A_1 - b(p_n - p_0) \). Constraint (6) assures that there are consumers whose choice is buying the old product. The optimal decisions of the retailer are obtained through the following proposition:

**Proposition 1.** In the decentralized model, the retailer’s profit function is concave w.r.t. \( z \) for a given \( p_0 \).

Proof. We fix \( p_0 \) and get the first-order derivative and the second-order derivative of the retailer’s profit function (5) w.r.t. \( z \).

maximize the expected profit of the current period. Under the myopic policy, the retailer’s single-period profit in the decentralized supply chain is formulated as follows:

\[
\Pi^r_\alpha(q, p_0) = \begin{cases}
\frac{p_0}{1 + \frac{D_n(p_0) + a}{2b}}
\end{cases}
\]

From Eqs. (8) and (9), it can be concluded that \( \frac{\partial^2 \Pi^r_\alpha(z, p_0)}{\partial z^2} < 0 \). Because \( b < p_0 < p_n \); therefore, \( \Pi^r_\alpha \) is a concave function w.r.t. \( z \) for a given \( p_0 \) \( \square \)

Considering the uniform distribution for random components of demand, we can obtain the retailer’s expected profit function as:

\[
E[\Pi^r_\alpha(q, p_0)] = \begin{aligned}
-w (z + D_n(p_0)) + p_0 z 

+ p_0 D_n(p_0) - (p_0 + h) \int_0^{D_n(p_0)} F(x) dx 

+ b \int_0^{D_n(p_0)} F(x) dx
\end{aligned}
\]

The above function has exactly one maximum point \( z^*_\alpha(p_0) \) which can be calculated by \( \frac{\partial \Pi^r_\alpha(z, p_0)}{\partial z} = 0 \).

By substituting \( z^*_\alpha(p_0) \) into (10) and then, calculating the first-order derivative of \( \Pi^r_\alpha(p_0) \) w.r.t. \( p_0 \), the optimal price of the old product \( p_0^\alpha \) is obtained according to Proposition 2.

**Proposition 2.** In the decentralized structure, the retailer’s optimal price of the old units \( p_0^\alpha \) can be attained by comparing the stationary and boundary points of the retailer’s profit function. Thus:

\[
p_0^\alpha = \arg \max \{ \Pi^r_\alpha(0), \Pi^r_\alpha(p_0^\alpha), \Pi^r_\alpha(p_0^\alpha) \}
\]

By substituting \( z^*_\alpha(p_0) \) into (10) and then, calculating the first-order derivative of \( \Pi^r_\alpha(p_0) \) w.r.t. \( p_0 \), the optimal price of the old product \( p_0^\alpha \) is obtained according to Proposition 2.
Proposition 3. The expected profit function of the whole fresh-product SC is concave w.r.t. \( z \) for a given \( p_o \).

Proof. Similar to the proof of Proposition 1. \( \square \)

According to the uniform distribution for the random component of demand in the specified range, the function (16) can be written as:

\[
E[\Pi_{sc}(z, p_o)] = -c_w(z + D_o(p_o)) + p_o z + p_o D_o(p_o)
\]

\[
-\frac{p_o + h}{2B_o} z^2 + \frac{p_o}{2B_o} (z^2 - (z - D_o(p_o))^2) + \frac{g}{2B_o} (z - D_o(p_o))^2
\]

The optimal stocking factor of new units in the decentralized supply chain can be calculated by \( \partial \Pi_{sc}(z, p_o) / \partial z = 0 \),

\[
z_o^*(p_o) = \frac{B_o(p_o - c_w) + D_o(p_o)(p_o - g)}{p_o + h + g}
\]

Similar to the method in Section 4.1, by substituting Eq. (18) into Eq. (17), the total expected profit function of fresh-product SC w.r.t. the old unit’s price \( p_o \) can be obtained. Then, the \( p_o^* \) is obtained by means of Proposition 4.

Proposition 4. In the centralized supply chain, the optimal price of the old units \( p_o^* \) can be attained by comparing the stationary and boundary points of the fresh-product SC expected profit function. Therefore, \( p_o^* \) is the answer set of the equation \( \partial \Pi_{sc}(z, p_o) / \partial p_o = 0 \).

By substituting \( p_o^* \) and \( z_o^* \) into (17), the optimal expected profit function of fresh-product SC in the centralized structure can be written as (19).

\[
\Pi_{sc}^*(z, p_o) = -c_w(z_o^* + D_o(p_o^*)) + p_o z_o^* + p_o D_o(p_o^*)
\]

\[
-\frac{p_o + h}{2B_o} z_o^*^2 + \frac{p_o}{2B_o} (z_o^*^2 - (z_o^* - D_o(p_o^*))^2) + \frac{g}{2B_o} (z_o^* - D_o(p_o^*))^2
\]

Since \( p_o^* \) and \( z_o^* \) are globally optimized, then

\[
\Pi_{sc}^*(z, p_o^*) \geq \Pi_{sc}^*(z_o^*, p_o^*)
\]

from Eq. (20), it is clear that the centralized decision-making model leads to the best performance of the whole fresh-product SC. However, the centralized solution might decrease the retailer’s profitability. As a result, the retailer...
refuses to join the centralized decision-making approach. Hence, proposing an incentive mechanism to motivate the members to participate in the centralized approach is required. In the next section, an incentive mechanism based on a return policy with the revenue and cost-sharing contract is designed to coordinate the channel.

4.3. Coordination model and incentive scheme

Although the profitability of the entire fresh-product supply chain increases under the centralized decision-making model in comparison with the decentralized system, the members’ profitability might decrease. In this section, a coordination plan is designed not only to maximize the profit of the whole fresh-product SC but also to ensure both members’ participation. Accordingly, an incentive mechanism based on return policy with revenue and cost-sharing contract is proposed.

In this mechanism, the revenue and cost-sharing coefficient \( \varphi \), considered between 0 and 1, is used to coordinate the channel. This hybrid contract consists of three parts: (1) a wholesale price determined based on cost-plus pricing method, (2) unsellable product compensation, and (3) salvage revenue sharing.

Specifically, the supplier determines the wholesale price based on the cost-plus pricing method in which the selling price is set by adding a fraction \( \varphi \) of marginal profit per unit product to the product’s unit cost. Therefore, the wholesale price of the supplier can be expressed as \( w = c_m + \varphi(p_n - c_m) \).

Uncertain market demand results in a considerable risk of unsellable product [28]. In each period, every unsold new product which loses the opportunity of sales with price \( p_n \) can be carried over to the next period and resold as an old product with price \( p_o \). Further, the retailer pays the holding cost to prepare and carry the leftover items for sale in the next period. Therefore, the retailer loses \( (p_n - p_o + h) \) per unit. Under the proposed contract, the supplier pays compensation to share some loss of unsellable new products as \( s_1 = \varphi(p_n - p_o + h) \) per unit. In addition, the whole carried-over units may be more than the demand for old products. Then, for every unsold old unit, the retailer will lose \( p_o \) and the supplier recompenses \( s_2 = \varphi p_o \).

On the contrary, the expired returned products are salvaged by the supplier at a value \( g \) per unit and sent to other industries for recovery. Under the coordinating contract, the salvage revenue is shared among members. Accordingly, the supplier keeps the fraction \( \varphi \) of the salvage revenue and shares the fraction \( (1 - \varphi) \) of that with the retailer.

Given that the compensation payment for the returned products together with the salvage revenue sharing in the contract is similar to the traditional buyback arrangement, it can be well implemented based on the number of returned products.

In coordination with the proposed contract, the retailer’s expected profit function can be calculated as:

\[
E[\Pi^*_o(z, p_o)] = (p_o - w)(z + D_o(p_o)) - p_o \int_0^z F(x)dx + \varphi \int_{s_1(z, p_o)}^{D_o(p_o)} F(x)dx
\]

\[
= (p_o - w)(z + (s_1 - h)) (1 - \varphi)h \int_0^z F(x)dx + (s_1 + (1 - \varphi)g) \int_{s_1(z, p_o)}^{D_o(p_o)} F(x)dx
\]

(21)

The Function (21) can be expanded as:

\[
E[\Pi^*_o(z, p_o)] = (p_o - w)(z + D_o(p_o)) - p_o \int_0^z F(x)dx + \varphi \int_{s_1(z, p_o)}^{D_o(p_o)} F(x)dx
\]

\[
+ (p_o - p_n) \int_{s_1(z, p_o)}^{D_o(p_o)} F(x)dx
\]

(22)

Moreover, the supplier’s profit function after the acceptance of the contract is:

\[
E(\Pi^*_s) = (w - c_o)(z + D_s(p_s)) + \varphi \int_0^{s_2} F(x)dx - \varphi p_o \int_{s_1(z, p_o)}^{D_o(p_o)} F(x)dx
\]

Proposition 5. By considering the coordinated decision-making model, the retailer’s optimal order quantity and optimal price of the old product are compatible with the optimal decisions in the centralized system.

Proof. In the coordinated approach, the expected profit function of the retailer, Eq. (22), can be rewritten as:
From Eq. (24), it can be concluded that
\[ E[\Pi^C_0(z, p_o)] = (1 - \varphi)E[\Pi^C_{SC}(z, p_o)]. \]
Clearly, under the proposed contract, the retailer’s optimal decisions are the same as those in the centralized system that are \( z^*_c \) and \( p^*_o \). Similarly, based on Eq. (23), it can be demonstrated that the supplier’s expected profit function, under the coordinated system, is \( E[\Pi^C_0] = \varphi E[\Pi^C_{SC}(z, p_o)]. \) Hence, the retailer’s optimal profit is \((1 - \varphi)\Pi^C_{SC}\), and the supplier’s optimal profit is \( \varphi \Pi^C_{SC}. \) Indeed, \( \varphi \) indicates the proportion that the supplier gains of the total fresh-product SC profit. The value of the sharing rate should be considered such that it is admissible for both members.

By taking the rational behavior of the supply chain members into account, it can be comprehended that both retailer and supplier accept to participate in the coordination structure if and only if their profits under the contract are more than the decentralized approach. The satisfaction condition for the retailer is:
\[ \Pi^C_{R}(z, p_o) \geq \Pi^d_{R}(z, p_o) \]  
(25)

By substituting the Eq. (13) in (25), the maximum admissible sharing rate from the retailer point of view is:
\[ \varphi_{\text{min}} = \frac{(p_o - w)(z^*_c + D_s(p^*_o)) - (p_o + h)\int_0^{z^*_c} F(x)dx + p^*_o \int_{-D_o(p^*_o)}^{z^*_c} F(x)dx + b \int_{z^*_c}^{D_s(p^*_o)} F(x)dx}{(p_o - c_o)(z^*_c + D_s(p^*_o)) - (p_o + h)\int_0^{z^*_c} F(x)dx + p^*_o \int_{-D_o(p^*_o)}^{z^*_c} F(x)dx + g \int_{z^*_c}^{D_s(p^*_o)} F(x)dx} \]
(26)

The condition for the supplier to do is:
\[ \Pi^C_{S} \geq \Pi^d_{S} \]
(27)

By substituting Eq. (15) into (27), the minimum acceptable sharing rate from the supplier’s viewpoint is obtained as follows:
\[ \varphi_{\text{min}} = \frac{(w - c_o)(z^*_c + D_s(p^*_o)) + (g - b)\int_{-D_o(p^*_o)}^{z^*_c} F(x)dx}{(p_o - c_o)(z^*_c + D_s(p^*_o)) - (p_o + h)\int_0^{z^*_c} F(x)dx + p^*_o \int_{-D_o(p^*_o)}^{z^*_c} F(x)dx + g \int_{z^*_c}^{D_s(p^*_o)} F(x)dx} \]
(28)

If the interval \([\varphi_{\text{min}}, \varphi_{\text{max}}]\) is non-empty, the channel coordination is achievable as shown below:
\[ \varphi_{\text{max}} - \varphi_{\text{min}} = 1 - \frac{\Pi^c_{S}}{\Pi^C_{SC}} - \frac{\Pi^d_{S}}{\Pi^C_{SC}} = 1 - (\frac{\Pi^c_{S}}{\Pi^C_{SC}} + \frac{\Pi^d_{S}}{\Pi^C_{SC}}) \geq 0 \]
(29)

Then, from Eq. (29), the interval \([\varphi_{\text{min}}, \varphi_{\text{max}}]\) exists. Therefore, the proposed contract can achieve perfect coordination of fresh-product SC when \( \varphi \) belongs to \([\varphi_{\text{min}}, \varphi_{\text{max}}]\). At the lower bound \( \varphi_{\text{min}} \), all increased profits in the coordinated approach are gained by the retailer.

On the contrary, at the upper bound \( \varphi_{\text{max}} \), all coordination profits are achieved by the supplier. The relative bargaining power of the members determines the practical value of \( \varphi \).

5. Numerical Examples and Discussions

In this section, the performance of the proposed approaches is analyzed using four test problems presented in Table (2). It is assumed that the relationship between parameters \( \delta \) and \( \gamma \) is given by \( \gamma = 1/(1 - \delta) \).
The results of (1) decentralized, (2) centralized, and (3) coordinated models for the four test problems are given in Table (3). As seen in Table (3), the retailer’s order quantity and profitability of the whole fresh-product SC in the centralized structure are higher than those in the decentralized structure. In addition, the retail price of the old units in the centralized structure is lower than that in the decentralized structure. Thus, the retailer’s profit may decrease and the retailer refuses to participate in the centralized structure without a proper incentive mechanism. Besides, according to Table (3), the optimal order quantity, the retail price of the old units, and the whole fresh-product SC profit in the coordinated model are equal to those values in the centralized one. However, in the coordinated structure, each member’s profit depends on the sharing rate φ. According to Eqs. (26) and (28), the feasible intervals of the sharing rate in the proposed coordination model are also provided in Table (3) for test problems. In these intervals, the proposed contract can be adopted by both supplier and retailer. The sensitivity of both supplier’s and retailer’s profit with regard to sharing rate φ in TP#3 is shown in Fig. 1. According to Fig. 1, with an increase in the sharing rate φ, the retailer’s profit in the coordinated approach is decreasing while the supplier’s profit is increasing; in other words, the increased profit of the whole fresh-product SC under coordination transfers from the retailer to the supplier gradually. Fig. 1 shows that as the value of φ is greater than almost 0.57, the supplier’s profit in the coordinated structure is higher than that in the decentralized structure. In addition, the retailer’s profit in the coordinated structure is lower than the decentralized structure when φ is higher than almost 0.62. As a result, if φ ∈ [0.57, 0.62], the proposed coordination contract can be acceptable to both retailer and supplier, and the overall profit of the centralized supply chain can be achieved.

**Tab. 2. Data for four test problems**

<table>
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<tr>
<td>TP#1</td>
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<tr>
<td>TP#3</td>
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<tr>
<td>TP#4</td>
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**Tab. 3. Results of models for four investigated test problems**

<table>
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<tr>
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<th>TP#2</th>
<th>TP#3</th>
<th>TP#4</th>
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</thead>
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<td>38.5</td>
<td>74</td>
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<tr>
<td>( q_c^{c} )</td>
<td>252</td>
<td>311</td>
<td>447</td>
<td>912</td>
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<tr>
<td>( \Pi_{SC}^{c} )</td>
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<td>4132</td>
<td>12954</td>
<td>41146</td>
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<td>Coordinated approach (φ=0.6)</td>
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<td>( p_0^{co} )</td>
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<td>Coordinated approach (φ=0.7)</td>
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<td>447</td>
<td>912</td>
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</table>

The proposed coordination model can be adopted by both supplier and retailer. The sensitivity of both supplier’s and retailer’s profit with regard to sharing rate φ in TP#3 is shown in Fig. 1. According to Fig. 1, with an increase in the sharing rate φ, the retailer’s profit in the coordinated approach is decreasing while the supplier’s profit is increasing; in other words, the increased profit of the whole fresh-product SC under coordination transfers from the retailer to the supplier gradually. Fig. 1 shows that as the value of φ is greater than almost 0.57, the supplier’s profit in the coordinated structure is higher than that in the decentralized structure. In addition, the retailer’s profit in the coordinated structure is lower than the decentralized structure when φ is higher than almost 0.62. As a result, if φ ∈ [0.57, 0.62], the proposed coordination contract can be acceptable to both retailer and supplier, and the overall profit of the centralized supply chain can be achieved.
Coordination of Pricing and Inventory Decisions in a Fresh-product Supply Chain Considering the Competition between New and Old Products

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</table>

<table>
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<tbody>
<tr>
<td>φ_min</td>
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<td>0.65</td>
<td>0.57</td>
<td>0.77</td>
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</table>

Fig. 1. The sensitivity of each fresh-product SC members’ profit with respect to the sharing rate φ

In the following, sensitivity analyses on some key parameters are conducted to better show the performance of the proposed models and provide some management insights for the fresh-product SC in practice. To perform the sensitivity analyses, TP#3 is applied.

5.1. The impact of γ on decision making

The product cannibalization phenomenon is one of the most relevant results of simultaneous sale of new and old products. In the investigated models, the cannibalization amplitude γ plays a significant role in the fresh-product SC members’ decisions. Recall that the cannibalization amplitude has a direct relationship with the discount factor δ that is consumers’ valuation for the old units. In this sub-section, the impact of parameter γ on decision-making results in the decentralized and coordinated models is studied when other parameters in the models remain unchanged.

Figs. 2 and 3 indicate changes in the optimal decision variables by increasing the cannibalization amplitude γ under two different decision-making structures. As observed in Figs. 2 and 3, in both decentralized and coordinated models, by increasing the parameter γ, the optimal price of the old product increases, and the optimal order quantity q decreases.

By analyzing the results, it can be concluded that when the consumers’ valuation for the old units δ increases, the consumers’ sensitivity to freshness decreases, and the sensitivity to the price difference between new and old products increases. As a result, a small discount on the old products leads to a large migration of consumers from purchasing new units to purchasing old units. In this case, the cannibalization amplitude γ is high; therefore, the retailer prefers to set a lower price difference between new and old products to decrease the cannibalization threat. Moreover, it is expected from the retailer to order fewer units of the new products because of the reduction in demand for new products.

Fig. 2. Changes of old unit’s price by increasing the cannibalization amplitude γ
Moreover, by increasing the cannibalization amplitude $\gamma$, the whole fresh-product SC profit decreases in both models, as shown in Fig. 4. Accordingly, when consumers are more sensitive to freshness, and the cannibalization amplitude $\gamma$ is low, the implementation of price differentiation strategy and simultaneous sale of the new and old products are more profitable. The above results are concordant with the study done by Herbon et al. [37]. They addressed the price differentiation effect in a deteriorating inventory system with three types of customers. They found that the retailer’s profit is more for freshness-oriented than price-oriented and indifference-oriented customers.

According to Fig. 4, the coordinated model is more profitable and beneficial for the whole supply chain in comparison with the decentralized one. In the integrated optimization of the supply chain, the retail price of the old units is less than that in the decentralized one (see Fig. 2). In this case, the implied cannibalization threat becomes a business opportunity for reducing the expired returned products, increasing the market share, thus increasing the whole fresh-product SC profit.
Based on these figures, the profit of the entire fresh-product SC decreases by increasing the price elasticity coefficients, $b_1$ and $b_2$, in both decentralized and coordinated structures. Furthermore, the whole fresh-product SC profit is more sensitive to the parameter $b_1$ than $b_2$ because a more dramatic decrease in the whole fresh-product SC profit occurs when the same increase is made in these parameters. Further, the figures show that the coordinated decision-making structure creates more profitability for the whole fresh-product SC compared to the decentralized approach.

**Fig. 5. Changes of whole fresh-product SC profit by increasing the price elasticity $b_1$**

6. **Conclusion**

The present study aims to coordinate the pricing and inventory replenishment decisions in the context of a fresh-product supply chain considering the simultaneous sale of products of different ages in the retail store. The investigated supply chain consists of one supplier and one retailer with one type of fresh product with a fixed shelf life. The stochastic market demand is dependent on the product’s retail price and freshness degree. The retailer decides on the price of the old units and the order quantity of the new units. An internal competition between new and old units based on products’ age and price can affect the sales volume, sales revenue, or market share, and its effects extend to the upstream member of the fresh-product SC. Therefore, a coordination model is proposed to improve the fresh-product SC members’ profitability with regard to the product cannibalization threat. In the fresh-product SC literature, such issue has not been addressed to the best of author’s knowledge. Decentralized and centralized decision-making models are developed to determine the optimal decisions. By analyzing the numerical test problems, it can be concluded that the retailer orders less and sets a higher price for the old units in the decentralized approach compared with the centralized one. Accordingly, there is a considerable profit loss for the whole supply chain in the decentralized system. Hence, a new incentive scheme based on a return policy with...
the revenue and cost-sharing contract is proposed that leads to coordination between the two members so that the optimality of the centralized approach can be obtained. The findings demonstrate that the proposed contract can coordinate the fresh-product SC effectively, and the coordinated model is more profitable and beneficial for the whole supply chain in comparison with the decentralized one. Moreover, in this study, sensitivity analyses based on some key parameters in the various decision-making approaches are conducted. Numerical results show that by increasing the consumer’s sensitivity to the freshness, the product cannibalization adverse effects are reduced, and the whole fresh-product SC profit increases.

The proposed models can be extended by relaxing some of the assumptions such as markdown price stickiness that complicates the problem and makes the old units’ price of each period depending on the period’s initial inventory. As a more realistic case, it can be considered that consumers can substitute between the new and old products during stock-outs. Furthermore, using other coordination mechanisms such as the target sales rebate contract for coordinating the investigated fresh-product SC is another area for further studies.

References


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Considering the Competition between New and Old Products


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URL: http://ijiepr.iust.ac.ir/article-1-1078-en.html