An Augmented Sweep Clustering Method for the Inventory-Routing Problem with Simultaneous Pickup and Delivery Considering Lateral Transshipment Between Repair Centers

Vahid Babaveisi¹, Farnaz Barzinpour² & Ebrahim Teimoury*³

ABSTRACT
In this paper, an inventory-routing problem for a network of appliance repair service is discussed including several repair depots and customers. The customer in this network makes a demand to have his/her faulty appliance repaired. Then, the repairman is assigned to the demand based on the skill needed for repairing appliance differing for each one. The assigned repairman picks up the faulty appliance from the customer place using the vehicle for transferring faulty appliances to repair depot. The vehicle for picking up and delivering the appliances has a maximum capacity. Additionally, the repair depot needs spare parts to repair the faulty appliances that are supplied either by the supplier or lateral transshipment from the other depots. The capacitated vehicle inventory-routing problem with simultaneous pickup and delivery is NP-hard that needs a particular optimization procedure. Regarding the skill of repairman, it becomes more complex. Many solution approaches have been provided so far that have their pros and cons to deal with. In this study, an augmented angle-based sweep method is developed to cluster nodes for solving the problem. Finally, the heuristic is used in the main body of genetic algorithm with special representation.

KEYWORDS: Vehicle routing problem, Heuristic, Appliance repair service network, Genetic algorithm.

1. Introduction
This study deals with a repair service network that involves some repair center depots and customer points as a point of demand. Customers make a demand by calling the support center; then, based on the number of demands and distance from the customer, a repair center is assigned to demand to perform repair operations. There are different types of appliances, such as refrigerator, TV, etc., that need different repair skills. Therefore, skill is an important factor based on the complexity involved. The routes visiting the customers could have one or more purposes, e.g., the vehicle (repairman) visiting the first customer and returning to the depot, or visiting other customers for collecting or delivering the appliance and, then, going back home. All the repair centers work under the supervision of the manufacturer, which uses lateral transshipments in case there is no inventory on hand. The vehicle routing problem aims to minimize the total distance traveled by all the vehicles so that all the customers’ demand is satisfied regarding its capacity [1]. The problem of vehicle routing with simultaneous pickup and delivery was raised by Min in 1989 in a case study for the problem of book distribution that was in need of pickup and delivery between central and local libraries [2]. Several researchers provided a wide range of solutions for CVRP. Gendreau et al. studied the different problems of time-dependent VRP. According to the authors, the routing problem is a process of selecting the best routes of graph G=(V, A), where V is the set of nodes and A is the arc set. The travel time between two points is usually constant, though this time it can change in different situations such as weather conditions, traffic and speed of
Many popular approaches divide the problem into two types: assigning customers to vehicles and finding an optimal route for each vehicle. Among different solutions for customer assignment, Sweep Clustering Algorithm is of much interest due to its simplicity [4]. This algorithm calculates the polar angles of all the nodes, sorts them according to angles, and assigns to different clusters [5]. The direction of sweeping is clockwise or anticlockwise, and each cluster of customers will be specified when the vehicle reaches its capacity or when there are no more customers to assign. The general procedure of the sweep method is shown in Fig. 1 that clusters the customer nodes in the anticlockwise direction with a so-called angle $\theta_{\text{start}}$, which is the starting angle. In this study, the mechanism for solving the problem with more than one depot is defined in addition to diversifying the search to prevent early intensification by utilizing various vehicle and depot permutations and starting angle and direction of sweeping.

This paper is organized as in the following sections: first, the literature review of the recent studies on the vehicle routing problem is given. Next, a mathematical model is presented. Then, the sweep method as the core of the solution procedure is explained, followed by the Genetic Algorithm section. Based on the solution method provided, computations and results are provided. Finally, conclusion is presented.

2. Literature Review

Cho et al. introduced an adaptive genetic algorithm (AGA) for the problem of time-dependent inventory routing (TDIRP). This problem aims to minimize transportation costs and inventory while meeting customer demands. Since the problem is Np-hard, genetic algorithm is used to solve the problem [6].

One of the prominent problems of capacitated routing is simultaneous pick-up and delivery (VRPSPD). Sayyah et al. presented an efficient Ant Colony Algorithm to solve the problem. Two feasibility conditions should be considered: solution feasibility and acceptable sequence of routes. To avoid early intensification, various local searches are implemented in this study [7].

Deng et al. examined a closed-loop location-routing-inventory problem and solved it using a hybrid ant colony algorithm. Given the forward and reverse movements in the network, the objective function is defined by minimizing total costs in the network whole [8]. Iassinovskaia et al. examined an inventory-routing problem for a returnable item with time windows and simultaneous pick-up and delivery. They focused on the environmental sustainability of the problem by sharing returnable items among different locations. A heuristic of clustering is used to solve the problem that was compared with the branch-and-cut solution method [9]. The reviewed paper is about the production routing problem in a reverse Logistics network written by

Fig. 1. Sweep clustering method
Qiu et al. They developed a mixed-integer programming model that is solved by the branch-and-cut algorithm. The algorithm shows better results in cases that the number of pickups is large [10]. Cho et al. presented the problem of routing for ships and inventory control of liquefied gas considering weather disruptions. They developed a mathematical model for the problem under uncertainty. Since the complexity of this combinatorial problem is too high, a technique is developed to reduce the solution space by removing infeasible and non-optimal variable. The computation results show a logical amount of time to solve the problem [11]. Sun et al. studied the problem of time-dependent capacitated profitable tour. In this problem, departure time will be determined such that the profit is maximized where some precedence limitations are defined. A dynamic programming method called Tailored Labeling Algorithm is used [12]. Zhang et al. studied a many-to-many VRPSPD in which other similar problems become too complex in high dimensions. In this paper, an adaptive memory programming (AMP) is used to solve the NP-hard problem. VNS algorithm is implemented for the neighborhood search of optimal solutions [13]. A summary of literature review is shown in Tab. Gajpal and Abad [14] used a multi-ant colony system to solve VRP with backhauls. Zachariadis et al. [15] developed an adaptive memory methodology to solve VRP with simultaneous pickup and delivery. A cluster and route method with first-priority delivery was provided [16] that generated good results regarding vehicle capacity, but the distance traveled was too far, which needs to be modified. Kanthavel and Prasad [17] considered the maximum utilization of loading capacity and obtained the optimum tours for CVRP by a nested particle swarm optimization. The algorithm was used as the master PSO and, also, as the slave PSO for obtaining a list of routes. The recent research studies are shown in Table 1.

### 3. Mathematical Model

A mathematical formulation is provided in five sections that include indices, parameters, decision variables, objective function, and constraints.

#### 3-1. Indices

- \( r \in \{1,...,R\} \) Index of repair center (depot)
- \( c \in \{R+1,...,R+C\} \) Index of customer
- \( v' \in \{1,...,R+C\} \) Index of nodes (customers & depots)
- \( v \in \{1,...,V\} \) Index of vehicles (repairman)
- \( t \in \{1,...,T\} \) Index of planning periods
- \( e \in \{1,...,E\} \) Index of skills (expertise)

#### 3-2. Parameters

- \( mc \) Vehicle cost per kilometer
- \( transp \) Income from moving appliances
- \( b_{rc} \) Transshipment cost from repair depot \( r \) to repair depot \( c \)
- \( h_t \) Holding cost in period \( t \)
- \( pr_{et} \) Income from operation \( e \) in period \( t \)
- \( de_{cet} \) 1, if customer \( c \) demands skill \( e \) in period \( t \); 0, otherwise
- \( z_{ve} \) 1, repairman (vehicle) \( v \) demands skill \( e \); 0, otherwise
- \( G_c \) 1, if demand of customer \( c \) is in guarantee period; 0, otherwise
- \( l_{cet} \) Diagnosis of faulty appliance of customer \( c \) with expertise \( e \) in period \( t \)
- \( dis_{rc} \) Distance matrix between repair center \( r \) and collection center \( c \)
- \( rm_{vr} \) 1, repairman \( v \) is assigned to repair center \( r \); 0, otherwise
- \( op_{vect} = z_{ve} \times de_{cet} \) 1, if repairman \( v \) is able to perform the demand of customer \( c \) in period \( t \); 0, otherwise
- \( vol_e \) Percentage of occupied capacity for appliance with skill \( e \)
- \( I_0^{er} \) Inventory of spare parts for expertise (skill) \( e \) in repair center \( r \) for the first period
- \( cos_{let} \) Purchase cost of spare part for expertise \( e \) in period \( t \)
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\[ \text{num}_{\text{cert}} = 1 \text{, if } l_{\text{cert}} > 1; 0, \text{otherwise} \]

### Tab. 1. Classification of related literature review

<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Mathematical model</th>
<th>Type of Routing problem</th>
<th>Vehicle capacity</th>
<th>Inventory</th>
<th>Solution method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cho et al. [6]</td>
<td>MIP</td>
<td>TDIRP</td>
<td>*</td>
<td>*</td>
<td>GA</td>
</tr>
<tr>
<td>Sayyah et al. [7]</td>
<td>MIP</td>
<td>CVRPSPD</td>
<td>*</td>
<td>*</td>
<td>ACO</td>
</tr>
<tr>
<td>Deng et al. [8]</td>
<td>MILP</td>
<td>LIRP</td>
<td>*</td>
<td>*</td>
<td>ACO</td>
</tr>
<tr>
<td>Iassinovskaia et al. [9]</td>
<td>MILP</td>
<td>PDIRPTW</td>
<td>*</td>
<td>*</td>
<td>BC</td>
</tr>
<tr>
<td>Qiu et al. [10]</td>
<td>MILP</td>
<td>PRPRPD</td>
<td>*</td>
<td>*</td>
<td>BCGS</td>
</tr>
<tr>
<td>Cho et al. [11]</td>
<td>MIP</td>
<td>CVRP1RP</td>
<td>*</td>
<td>*</td>
<td>-</td>
</tr>
<tr>
<td>Sun et al. [12]</td>
<td>MILP</td>
<td>TDVRP</td>
<td>*</td>
<td>*</td>
<td>TLA</td>
</tr>
<tr>
<td>Zhang et al. [13]</td>
<td>NLMIP</td>
<td>M-M-VRPSPD</td>
<td>*</td>
<td>*</td>
<td>AMP</td>
</tr>
<tr>
<td>Gajpal and Abad</td>
<td>MIP</td>
<td>CVRP</td>
<td>*</td>
<td>*</td>
<td>GA</td>
</tr>
<tr>
<td>Zachariadis et al. [15]</td>
<td>MIP</td>
<td>VRPSD</td>
<td>*</td>
<td>*</td>
<td>TS</td>
</tr>
<tr>
<td>Senthil Kumar [16]</td>
<td>MIP</td>
<td>CVRP</td>
<td>*</td>
<td>*</td>
<td>Heuristic</td>
</tr>
<tr>
<td>Kanthavel and Prasad [17]</td>
<td>MIP</td>
<td>CVRP</td>
<td>*</td>
<td>*</td>
<td>PSO</td>
</tr>
<tr>
<td>This paper</td>
<td>MIP</td>
<td>CVRPSPD</td>
<td>*</td>
<td>*</td>
<td>GA-Augmented Sweep</td>
</tr>
</tbody>
</table>

BCGS: Brach & Cut Guided Search, BC: Branch & Cut, AMP: Adaptive Memory programming

#### 3-3. Decision variables

\( l_{\text{ert}} \) Inventory of spare parts for expertise e in repair center c in period t

\( x_{ijvt} \) 1, if the path between nodes i and j is served by vehicle v in period t; 0, otherwise

\( w_{ijet} \) 1, if lateral transshipment of spare parts with expertise e is done between repair centers (depots) i and j in period t; 0, otherwise

\( op_{vec} \) 1, if customer c is served by repairman v for expertise e in period t; 0, otherwise

\( Am_{ert} \) Amount of spare parts for expertise e needed for repair center r in period t

\( u_t \) Subtour elimination variable

#### 3-4. Model formulation

The objective function in Eq. (1) aims to maximize the total profit of all repair centers (depots). The first term is the income of repairing faulty appliances paid by the customers to repairmen. The second term includes transportation income. The third term is the purchase costs of the spare parts. The fourth term involves total holding costs. The fifth term is the total transportation costs. The last term is the total transshipment costs among repair centers (depots).
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\[ \text{Max } z = \sum_{e} \sum_{i} \sum_{j} \sum_{c} \sum_{v} \sum_{t} p_{e} G_{j} x_{ijvt} + \sum_{i} \sum_{j} \sum_{c} \sum_{v} \sum_{t} \text{transp} \times x_{ijvt} - \sum_{e} \sum_{r} A_{ert} \times \text{cost}_{et} - \sum_{e} \sum_{r} \sum_{t} h_{e} l_{ert} - \sum_{i} \sum_{j} \sum_{c} \sum_{v} \sum_{t} m_{c} \times d_{ij} \times x_{ijt} - \sum_{i} \sum_{j} \sum_{e} b_{ij} w_{jiet} \]  
(1)

Equation (2) limits the maximum vehicle input of each node to one. Equation (3) is the subtour elimination that makes the tours include depot node. Equation (4) ensures that there is no path among depots (not regarding lateral transshipments). Equation (5) does not allow the cycle on the depot node. Equation (6) ensures that each node has equal input and output. Equation (7) assigns one vehicle at maximum (repairman) to each customer. Equation (8) dedicates the repairman assignment matrix to customers based on their skill (expertise). Equation (9) assigns repairman to depots regarding the assignment matrix of a repairman. Equation (10) composes the path according to repairman skills and customer demand. Equation (11) makes the path based on the demand. Equation (12) considers the maximum total capacity of the vehicle to be below 1 (100 percent). Equations (13), (14) control the spare part inventory that is needed for each type of expertise in the first period and a period more than one. Equation (15) specifies the domain of the decision variables.

\[ \sum_{v \in V} x_{ijvt} \leq 1 \]  
\( u_{i} - u_{j} + n x_{ijvt} \leq n - 1 \)  
\( \sum_{i} \sum_{j} x_{ijvt} = 0 \)  
\( \sum_{i} \sum_{j} x_{ijvt} = 0 \)  
\( \sum_{j \in V', j \neq i} x_{ijvt} = \sum_{j \in V', j \neq i} x_{jvit} \)  
\( \sum_{v \in V} x_{ijvt} \leq 1 \)  
\( o_{\text{ve} \text{vect}} = z_{\text{ve} \text{dec} t} \)  
\( \sum_{j \in C} x_{ijvt} = r_{m_{vi}} \)  
\( \sum_{e \in E} x_{icvt} \leq \sum_{e \in E} o_{\text{ve} \text{vect}} \)  
\( \sum_{v \in V \setminus C} \sum_{e} x_{icvt} = \sum_{e} d_{cet} \)  
\( \sum_{i \in V' \setminus C} \sum_{e} \sum_{c} v_{\text{le} \text{e} \text{xijvt}} \leq 1 \)  
\( l_{ert} = l_{ert}^{0} + A_{ert} - \sum_{c} \sum_{j \neq r} w_{jret} + \sum_{j \neq r} w_{jret} \)  
\( l_{ert} = l_{ert-1} + A_{ert} - \sum_{c} \sum_{j \neq r} w_{jret} + \sum_{j \neq r} w_{jret} \)  
\( I_{ert} \geq 0 \)  
\( x_{ijvt} \in \{0,1\} \)  
\( w_{jret} \geq 0 \)  
\( A_{ert} \geq 0 \)  
\( u_{i} \geq 0 \)  
\( \forall j \in C, \forall t \in T \)  
\( \forall i, j \in C, \forall t \in T, \forall v \in V \)  
\( \forall i, j \in R, \forall t \in T, \forall v \in V \)  
\( \forall t \in T, \forall v \in V \)  
\( \forall i \in V', \forall t \in T, \forall v \in V \)  
\( \forall i, j \in V', \forall t \in T \)  
\( \forall c \in C, \forall t \in T, \forall v \in V, \forall e \in E \)  
\( \forall v \in V, \forall i \in R, \forall t \in T \)  
\( \forall v \in V, \forall t \in T, \forall c \in C \)  
\( \forall v \in V, \forall t \in T \)  
\( \forall v \in V, \forall t \in T \)  
\( \forall e \in E, \forall r \in R, \forall t = 1 \)  
\( \forall e \in E, \forall r \in R, \forall t \in T \)  
\( \forall i, r, t \)  
\( \forall i, j, v, t \)  
\( \forall i, j, e, t \)  
\( \forall e, r, t \)  
\( \forall i \)
4. Node Clustering Through Sweep

Sweep clustering method is used to sweep the demand nodes based on their angle. This method is suitable for assigning nodes to vehicles (repairmen) with a limited capacity and continues until all nodes are assigned. The assignment is finished for a vehicle when it is a full load. The Angle-based sweep is shown in Fig. 2.

![Angle-based sweep](image)

The method includes many different disadvantages that may lead to non-optimality. Difficulties result from a specific starting angle (zero degree), a fixed vehicle, and depot assignment sequence to customers.

In the problem proposed in this paper, the idea of diversifying the search is proposed in the following to enhance the search quality and speed:

1) A different starting angle is considered and is optimized to obtain an optimized cluster of the nodes,
2) defining different sequences of vehicles to assign to customers,
3) specifying different sequences of depots to assign.

This heuristic involves the following steps to cluster the nodes:

1) Determine the position of customers and depots,
2) Translate all points of customers to the depot as a coordinated origin,
3) Calculate the angle of points according to Eq. (16):

\[
\theta = \tan^{-1}\left(\frac{y_i}{x_i}\right)
\]

(16)
4) Sort the angles as desired (clockwise or anti-clockwise)
5) Cluster the customer points for vehicles according to the starting angle, direction of sweep, and capacity of the vehicle.

In this method, it is very important to consider the permutation of assignment so that it can diversify the search for depot and vehicle assignment. The augmented sweep could be implemented independently; however, to optimize the solution, it is preferable to use an iterative as a wrapper. Here, Genetic Algorithm (GA) is selected to be the wrapper.

5. Genetic Algorithm

The proposed heuristic is implemented in a Genetic Algorithm (GA) by using a specific representation and the framework of the proposed heuristic. GA is a population-based algorithm that uses chromosomes as populations.

Fig. 3 shows a chromosome and its gene. Each gene contains some information. This study inserts a random value in these genes. It can be a value such as angle, priority of the depot, vehicle, etc. The selected populations that are different chromosomes are reproduced by applying crossover and mutation operators.

![Chromosome and Gene](image)
5-1. Representation
Chromosome representation is a matrix of \( \text{depot} \times (3 \times \text{period} + \text{vehicle} + \text{depot}) \) columns shown in Fig. 4.
The formula calculates the number of columns that involves five parts. From the left side, the first part is a random starting angle between \([0, 360]\); the second part is the clockwise angle, or that anti-clockwise angle is +1 or -1.
The third part is the vehicle assignment sequence to tours. The fourth part shows the depot assignment priority, i.e., which depot is assigned first and the like. The last part is the lateral transshipment probability.

![Fig. 4. Chromosome representation for Sweep](Image)

5-2. Crossover and mutation
Crossover and mutation are specifically designed for each part. Crossover is of two-point type. Mutation is defined as reversion. Fig. 5 shows the crossover and mutation procedures.

![Fig. 5. Mutation and Crossover](Image)

5-3. Parameter tuning
GA operates using specific parameters that need to be tuned. Crossover rate is generally between \([0, 3, 0.9]\), while mutation rate changes in \([0.001, 0.01]\) interval. The mutation rate is usually initiated by \(\frac{1}{k}\), where k is the number of decision variables. The number of populations is also at \([20, 100]\) interval [18]. Three levels are defined for parameter tuning.
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Tab. 2. Parameter levels

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population</td>
<td>20</td>
<td>25</td>
<td>40</td>
</tr>
<tr>
<td>Iteration</td>
<td>50</td>
<td>100</td>
<td>150</td>
</tr>
<tr>
<td>Crossover rate</td>
<td>0.3</td>
<td>0.5</td>
<td>0.7</td>
</tr>
<tr>
<td>Mutation rate</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Parameters are tuned by designing experiments using Minitab software. According to this software, L9 Design, shown in Tab. 3, is suggested. In this design, nine runs are recommended that should be used for each instance.

Tab. 3. L9 design (levels of parameters)

<table>
<thead>
<tr>
<th>Design number</th>
<th>Population</th>
<th>Crossover rate</th>
<th>Mutation rate</th>
<th>Iteration</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
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<td>3</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>1</td>
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<td>8</td>
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<td>2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

6. Computations and Results

Random instances are designed based on Tab. 4. Five parameters that include the dimensions of the instances are considered. Tab. 4 shows the dimension of instances. Given the designed experiments, each instance is solved nine times.

Tab. 4. dimensions of Instances

<table>
<thead>
<tr>
<th>Instance</th>
<th>Nodes (n)</th>
<th>Skills (e)</th>
<th>Vehicles (v)</th>
<th>Periods (t)</th>
<th>Repair centers (r)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50</td>
<td>2</td>
<td>10</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>50</td>
<td>4</td>
<td>15</td>
<td>2</td>
<td>2</td>
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<tr>
<td>3</td>
<td>50</td>
<td>5</td>
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<td>5</td>
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</tr>
<tr>
<td>6</td>
<td>100</td>
<td>6</td>
<td>25</td>
<td>2</td>
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<td>7</td>
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<td>10</td>
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<td>9</td>
<td>250</td>
<td>6</td>
<td>25</td>
<td>6</td>
<td>5</td>
</tr>
</tbody>
</table>

Test problems were carried using MATLAB R2014a on a Laptop with 8 GB RAM and Intel® Core™ i7-6500U @ 2.5 GHz CPU. The values of the objective functions are presented in Tab. 5. Since each instance is run nine times, an average is calculated.

Tab. 5. CPU time comparison

<table>
<thead>
<tr>
<th>Instance</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>Average (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.63</td>
<td>4.5</td>
<td>10.32</td>
<td>7.19</td>
<td>3.72</td>
<td>7.65</td>
<td>9.2</td>
<td>13.33</td>
<td>5.85</td>
<td>7.04</td>
</tr>
</tbody>
</table>
CPU time is changing over different instances, as shown in Fig. 6. The maximum CPU time is observed in Designs 3 and 8, and the minimum is observed in 5. The trend of CPU time is incremental after Design 5 and is decremental in Designs 3 to 5. CPU time for each instance is shown in Fig. 6. The changes in CPU time are obvious for different instances.

Fig. 6. CPU time changes vs. instances

Fig. 7 shows the intensification of the objective function. As it is clear, objective function intensifies in all cases.
To validate the obtained results from the Genetic Algorithm, results are compared with the exact method from IBM ILOG CPLEX Optimization. The gap between the two methods is calculated through Eq. (17).

\[
Gap = \frac{|\text{Meta- heuristic obj.} - \text{Exact obj.}|}{\text{Exact obj.}} \quad (17)
\]

Since the inventory-routing problem is NP-hard, a small-sized instance is tested by two methods, as shown in Tab. 6, to validate the results obtained by the sweep method in meta-heuristic algorithm.

<table>
<thead>
<tr>
<th>Solution method</th>
<th>Objective function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exact (CPLEX)</td>
<td>807,000</td>
</tr>
<tr>
<td>GA</td>
<td>777,173</td>
</tr>
<tr>
<td>Gap</td>
<td>3.7%</td>
</tr>
</tbody>
</table>

Fig. 7. Intensification of the objective function

Taguchi analysis for each instance is shown in Fig. 8. Left column shows the signal to noise, and the right column shows the mean of means. Considering that the objective function is maximization, the maximum objective function is selected from the right column figures and the maximum from the signal to noise figures. All the experiments are verified according to the latter explanation. This study compared Sweep and Augmented Sweeps. In the sweep, the objective functions are the same; however, regarding the augmented sweep, it outperforms the sweep and improves the quality of results. The optimal results of the augmented sweep come from diversifying the search and using different starting sweep angles rather than 0 degree. It this case, many solutions examine the search space through different sequences of vehicles, depots in addition to different angles, and lateral transshipments. Table 7 shows the effect of the starting angle on the enhancement of solution quality.
Fig. 8. Taguchi analysis

Tab. 7. Augmented Sweep vs. Sweep

<table>
<thead>
<tr>
<th>Instance</th>
<th>Sweep $\theta_s = 0$</th>
<th>Augmented Sweep</th>
<th>Starting Angle ($\theta_s$)</th>
<th>Improvement (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>782,276</td>
<td>156.7</td>
<td></td>
<td>41.5</td>
</tr>
<tr>
<td>2</td>
<td>782,276</td>
<td>101.4</td>
<td></td>
<td>41.5</td>
</tr>
<tr>
<td>3</td>
<td>733,309</td>
<td>90.6</td>
<td></td>
<td>37.5</td>
</tr>
<tr>
<td>4</td>
<td>801,096</td>
<td>91.5</td>
<td></td>
<td>42.8</td>
</tr>
<tr>
<td>5</td>
<td>457,606</td>
<td>54.4</td>
<td></td>
<td>38.5</td>
</tr>
<tr>
<td>6</td>
<td>744,168</td>
<td>85.5</td>
<td></td>
<td>41.1</td>
</tr>
<tr>
<td>7</td>
<td>777,173</td>
<td>41.2</td>
<td></td>
<td>40.3</td>
</tr>
<tr>
<td>8</td>
<td>780,782</td>
<td>55.2</td>
<td></td>
<td>41.3</td>
</tr>
<tr>
<td>9</td>
<td>772,555</td>
<td>77.6</td>
<td></td>
<td>40.7</td>
</tr>
</tbody>
</table>

Fig. 9 shows the changes in the objective function considering different starting angles. It is obvious that, at a specific angle, the maximum objective function is achieved, while other angles cause non-optimality. Another finding is that we can track objective function changes during angle shifts to find the optimal angle.
7. Conclusion
In this paper, a network of inventory-routing for repairing faulty appliance was examined. This problem aims to determine the optimal tour of vehicles considering maximum capacity of vehicles in addition to repairman skill for repairing faulty appliances. The problem becomes more complex when these constraints are applied. Therefore, it cannot be dealt with exact methods. In this case, heuristic and meta-heuristic methods are advantageous.

Sweep is one of the heuristics that is helpful for solving routing problems; however, the starting angle causes non-optimality. Since the inventory and skills are added to the problem, the complexity increases and the sweep will not result in good solutions with a good objective function value. Therefore, an augmented sweep with different starting angles was proposed that considered sequence of vehicle assignment and depots in addition to lateral transshipment. To test the functionality of this heuristic, GA is used with a specific representation based on the augmented sweep framework.

The results of the algorithm outperform ordinary sweep due to diversifying the search to enhance the solution quality. Another achievement is to prevent early intensification by creating diversification in comparison to the sweep. Therefore, it can be declared that different sequences of assignment and starting angle could enhance the functionality of the sweep by improving the local search mechanism.

It is recommended that enthusiasts work to enhance the droplet shape of the tours if required. In practice, it is also suggested that researchers work to develop applications to operate independently on the systems for operational purposes. It is also helpful if other researchers think of using another meta-heuristic algorithm based on the presented augmented sweep.

References


An Augmented Sweep Clustering Method for the Inventory-Routing Problem with Simultaneous Pickup and Delivery Considering Lateral Transshipment Between Repair Centers


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